ZDM Mathematics Education (2013) 45:425–440
DOI 10.1007/s11858-012-0476-0

ORIGINAL ARTICLE

Teachers' gestures and speech in mathematics lessons: forging common ground by resolving trouble spots

Martha W. Alibali · Mitchell J. Nathan · R. Breckinridge Church · Matthew S. Wolfgram · Suyeon Kim · Eric J. Knuth

Accepted: 18 November 2012 / Published online: 31 January 2013

Abstract This research focused on how teachers establish and maintain shared understanding with students during classroom mathematics instruction. We studied the micro-level interventions that teachers implement spontaneously as a lesson unfolds, which we call *micro-interventions*. In particular, we focused on teachers' micro-interventions around *trouble spots*, defined as points during the lesson when students display lack of understanding. We investigated how teachers use gestures along with speech in responding to such trouble spots in a corpus of six middle-school mathematics lessons. Trouble spots were a regular occurrence in the lessons ($M = 10.2$ per lesson). We hypothesized that, in the face of trouble spots, teachers might increase their use of gestures in an effort to re-establish shared understanding with students. Thus, we predicted that teachers would gesture more in turns immediately following trouble spots than in turns immediately preceding trouble spots. This hypothesis was supported with quantitative analyses of teachers' gesture frequency and gesture rates, and with qualitative analyses of representative cases. Thus, teachers use gestures adaptively in micro-interventions in order to foster common ground when instructional communication breaks down.

M. W. Alibali (✉)
Department of Psychology, University of Wisconsin, 1202 W. Johnson St., Madison, WI 53706, USA
e-mail: mwalibali@wisc.edu

M. J. Nathan
Department of Educational Psychology, University of Wisconsin, 1025 W. Johnson St., Madison, WI 53706, USA
e-mail: mjnathan@wisc.edu

R. B. Church
Department of Psychology, Northeastern Illinois University, 5500 North St. Louis Ave., Chicago, Illinois 60625, USA
e-mail: rbchurch@neiu.edu

M. S. Wolfgram
Department of Anthropology, University of Alabama, P.O. Box 870210, Tuscaloosa, AL 35487, USA
e-mail: mwolfgram@bama.ua.edu

S. Kim
Anyang University, Anyang, Korea
e-mail: suyeonster@gmail.com

E. J. Knuth
Department of Curriculum and Instruction, University of Wisconsin, 225 N. Mills St., Madison, WI 53706, USA
e-mail: knuth@education.wisc.edu

Keywords Gesture · Classroom communication · Common ground

1 Introduction

Communication is an integral part of many learning contexts, including tutoring, peer collaboration, and of course, classroom instruction. However, communication sometimes breaks down. Students sometimes fail to comprehend the information their teachers provide, and they sometimes fail to draw appropriate inferences or to construct knowledge, despite teachers' best efforts to create conditions under which students will learn. One way to conceptualize the difficulties that students and teachers have at such "trouble spots" in the classroom discourse is in terms of a lack of *common ground* (Clark and Brennan 1991; Clark 1996), or shared understanding among participants in an interaction. Creating shared understanding between teacher and students is crucial in instructional communication (e.g., Vygotsky 1978; Blake and Pope 2008), and indeed, several recent analyses of discourse in educational settings have focused on how shared understanding is negotiated (Paulus 2009; Evans et al. 2011; Nathan et al. 2007). In this paper, we focus on teachers' efforts to establish and maintain shared understanding with their students during classroom instruction in mathematics.

1.1 Focus on teachers' micro-interventions

Classroom interventions are typically conceptualized at a "macro" level (e.g., a series of lessons extending over days, weeks or years). In contrast, our approach focuses on the "micro" level interventions that teachers implement as an individual lesson unfolds, which we call *micro-interventions*. We define micro-interventions as teachers' "in-the-moment" responses to situations in which students are having difficulty with the instructional material. These micro-interventions occur over small segments of lessons; for example, if a student asks a question that reveals a misunderstanding, the teacher may intervene by clarifying previously presented material, or by using an alternative representation of the material. In doing so, teachers draw on their culturally-embedded beliefs about learning and instruction (see, e.g., Schleppenbach et al. 2007), rather than on a specific pedagogical program (as typically implemented at the "macro" level). Thus, these micro-interventions reflect teachers' spontaneous efforts to intervene when students need assistance.

There is a growing appreciation that teachers adjust their instruction dynamically in response to students' behaviors during classroom interactions (e.g., Cobb et al. 1993; Hatano and Inagaki 1991; Schoenfeld 1998). We suggest that many of these adjustments can be construed as micro-interventions. As one example, Nathan and Kim (2009) found in a case study that, when students produced inaccurate or incomplete answers, the teacher reduced the level of cognitive complexity students needed to respond, whereas when students produced mathematically accurate responses, he increased cognitive complexity. These adjustments scaffolded students' participation, drawing them in and fostering more sophisticated thinking.

In this paper, we investigate teachers' communication around trouble spots during middle school mathematics lessons, focusing in particular on whether teachers adjust their use of *gestures* when trouble spots occur. Toward this goal, we present (1) quantitative analyses of teachers' use of gestures as a means to foster common ground, and (2) qualitative analyses of two representative trouble spots, which offer a view of the moment-to-moment unfolding of trouble spots and consequent micro-interventions. Thus, this work is fine-grained, foundational research focusing on instructional communication. Although it is not immediately connected to improving mathematics education, a better understanding of teachers' micro-interventions holds promise for understanding and perhaps designing interventions in the future.

Gestures are spontaneous movements, usually of the hands and arms, that people produce when they speak (McNeill 1992). In the following sections, we review past research showing that (1) gestures are ubiquitous in teaching and learning settings, including mathematics classrooms, and (2) gestures promote language comprehension. In light of this past research, we hypothesized that gestures are an important tool that teachers draw upon to establish and maintain shared understanding.

1.2 Gestures in learning, teaching, and comprehending language

Ten years ago, Wolff-Michael Roth (2002) noted that there was "very little educational research concerned with the role of gestures in learning and teaching, particularly in the subject areas that have been characterized as dealing with abstract matters such as science and mathematics" (p. 365). In the ensuing years, there has been a burgeoning of interest in the role of gestures in mathematics learning and teaching among researchers with diverse methodological perspectives. Many studies have shown that gestures are integral to *learners'* communication in instructional settings (e.g., Goldin-Meadow and Singer, 2003; Kim et al. 2011; Edwards 2009; Nemirovsky et al. 2012; Radford 2009). Other studies have shown that gestures are crucial in *teachers'* communication, as well (e.g., Flevares and Perry 2001; Richland et al. 2007).

What roles might gesture play in teachers' communication? Some researchers have argued that teachers use gesture to guide students' attention to relevant aspects of the mathematical task at hand (e.g., Alibali, Nathan and Fujimori, 2011). Others have argued that gesture is a key component of the semiotic systems that teachers and students use in mathematics classrooms (Radford et al. 2009; Arzarello 2006). In particular, gesture can be viewed as a "semiotic resource" that teachers (and learners) utilize in developing and refining ideas (e.g., Rasmussen et al. 2004; Arzarello et al. 2009). For example, Arzarello et al. (2009) demonstrated that teachers take up and repeat students' gestures as part of a "semiotic game," in which they correct or revoice student language and guide students toward appropriate mathematical ideas and actions.

Experimental studies on the role of gesture in language comprehension provide compelling evidence that listeners comprehend speech more accurately when it is accompanied by gestures than when it is presented on its own (e.g., Goldin-Meadow et al. 1992; McNeil et al. 2000; Church et al. 2007). Indeed, a recent meta-analysis of 63 studies (Hostetter 2011) revealed a significant beneficial effect of gesture on speech comprehension. Moreover, the average effect size was greater in studies with younger listeners than in studies with adults, suggesting that gesture might be particularly important for school-age children.

A few studies have experimentally tested effects of teachers' gestures on students' learning in instructional contexts. Most of these studies have revealed greater learning from lessons that included gestures than from lessons that did not (e.g., Valenzeno et al. 2003). For example, Church, Ayman-Nolley and Mahootian (2004) presented children in classrooms with one of two videotaped lessons about Piagetian conservation. Children were significantly more likely to learn from the lesson that included gesture. Moreover, this pattern held for both monolingual English-speaking children and English language learners.

Taken together, the literatures on gesture in language comprehension and gesture in learning from instruction suggest that teachers' gestures play an important role in fostering students' comprehension of instructional material and consequently, in students' learning.

1.3 Do teachers take advantage of the "helping hand" offered by gestures?

The literature contains several detailed analyses of teachers' gestures in instructional settings (e.g., Núñez 2005). However, it is not yet known whether teachers use gestures adaptively, in an effort to foster students' comprehension, at moments when intervention is most needed. Flevares and Perry (2001) and Alibali and Nathan (2007) provided some suggestive evidence on this point; however, a more focused investigation of this question is clearly warranted.

Past research has also shown that teachers *can* alter their gestures if they receive instruction about the importance of gesture in instruction (Hostetter et al. 2006; Alibali et al. 2012). For example, Hostetter et al. (2006) asked teachers to give a mathematics lesson twice. After the first lesson, teachers were given a brief tutorial about how to link ideas using gestures. They were then asked to give the lesson a second time, incorporating as many gestures as possible into their lessons. Teachers produced more gestures in the second lesson (after explicit instruction about gestures) than in the first lesson (without any specific instructions about gestures). However, despite this evidence that teachers *can* alter their gestures, to our knowledge, no studies have specifically encouraged teachers to use gesture as a micro-intervention *at trouble spots* in classroom discourse. The fact that teachers can alter their gestures strategically suggests that such an approach would be feasible.

In this paper, we investigate teachers' use of gestures to address trouble spots (see Seedhouse 2004) in instructional discourse. We define *trouble spots* as points in the classroom discourse where students reveal a lack of understanding of the instructional material, e.g., by offering an incorrect response to a teacher's question, or by offering a response characterized by uncertainty or dysfluency. Trouble spots are a regular occurrence in classroom settings; therefore, micro-interventions around trouble spots may contribute to students' learning. However, to date, there has been little systematic analysis of how teachers respond to trouble spots, and no studies have systematically investigated how teachers use gestures to address trouble spots. Thus, trouble spots are a potentially fruitful context in which to examine how teachers adapt their instructional communication when needed.

1.4 Research questions and hypotheses

In this study we investigate the nature of teachers' spontaneous micro-interventions at moments when students reveal lack of understanding of instructional material. We address this question in middle school mathematics instruction. Middle school mathematics includes many new concepts and new symbolic representations; thus, it is an instructional context in which there is likely to be a large number of trouble spots. It is also a context in which gesture may be a particularly effective means of communicating relationships among representations.

In this research, we ask whether teachers alter their use of gestures in response to trouble spots in the classroom discourse. Trouble spots are an indicator that common ground has either not been established, or has been lost. As such, trouble spots are an invitation for a teacher to try other means to establish common ground. Thus, we hypothesized that teachers would produce more gestures in the turns that immediately follow trouble spots, than in the turns that immediately precede trouble spots. In the analyses that follow, we present both quantitative and qualitative evidence in support of this hypothesis.

2 Method

2.1 Source of data

The data were drawn from video recordings of six middle-school mathematics lessons. Five of the lessons took place in public schools in a mid-size Midwestern city; the remaining one took place in a parochial school in the same community. Four of the teachers were female, and two were male. Lessons ranged from 40 to 61 min in length. Lesson topics and grade levels are listed in Table 1.

Participating teachers were aware of the research team's interest in verbal and non-verbal instructional communication. Before the lesson, each teacher completed a brief written survey that focused on the planned lesson content,

Table 1 Grade, number of minutes, and topics for each lesson

Grade 6, 51 min	Developing procedures for multiplying mixed fractions
Grade 7, 43 min	Relationships between scale factor, measurements, and percents
Grade 7, 49 min	Determining growth factors from an exponential population model
Grade 8, 60 min	Using "recursive routines" to solve word problems involving linear rates
Grade 8, 40 min	Connecting divisibility rules with prime and composite numbers, finding the prime factorization of numbers and monomials
Grade 8, 61 min	Definition of polynomial, degree of polynomials, writing polynomials in standard form, how to use algebra tiles

the teacher's expectations for students during the lesson, and the difficulties the teacher expected students to have during the lesson. After the lesson, each teacher participated in a brief oral interview with a member of the research team. The interview included questions about the teacher's view of how the lesson went, whether anything about the lesson or students' reactions was surprising, and which portions of the lesson were new material and which were review.

2.2 Video analysis

The video recordings were transcribed and coded in several passes. In the first pass, we transcribed teachers' and students' speech, and identified students' and teachers' turns at talk.

In a second pass, we identified trouble spots. A trouble spot is a part of a conversational exchange that contains a mistake or that is not sufficiently clear, so that a repair is subsequently initiated (Golab et al. 2009). Given our focus on student learning, we defined trouble spots as points in the discourse where students made errors or otherwise displayed lack of understanding or uncertainty about the lesson material—that is, points where students' understanding of the lesson was compromised or disrupted.

We identified three types of trouble spots: (1) student-initiated *questions* regarding the instruction (see, e.g., Sidnell 2010); (2) *incorrect responses* by student(s) to teachers' questions or statements (called "errors" by Schleppenbach et al., 2007); and (3) *dysfluent utterances* on the part of students, defined as utterances in which students produced incoherent statements or directly expressed lack of certainty (see, e.g., Schegloff et al. 1977). These categories emerged from our analysis of the data, and were also informed by the literature. All of the trouble spots that we identified fell into one of these three broad categories. All three categories indicate a lack of understanding on the part of the student(s), and thus, at face value suggest that common ground had not been established or maintained. Note that all of the trouble spots we identified involved *student-initiated* utterances reflecting lack of understanding; we did not code instances in which teachers expressed uncertainty or lost their train of thought.

After each trouble spot was identified, we identified the teacher turns preceding and following the trouble spot. Turns were typically defined by change of speaker. However, in some cases, students made back-channel responses (e.g., "Oh" or "I get it") or spoke *during* the teacher's turn. If the teacher did not cede the floor, these student responses were not coded as a change of turn. In cases where teachers held the floor for an extended series of utterances preceding or following the trouble spot, we defined the *turn* for our analysis purposes as the teacher's talk on the same idea unit or topic (Brintoon and Fujiki 1989). In the turn preceding a trouble spot, teachers often initiated a new topic explicitly in speech (Hurtig 1977). For example, one teacher introduced a new topic by saying, "Ok, let's take a look at this next one." In the turn following a trouble spot, teachers often finalized the topic. For example, in response to a question about the difference between brackets and braces, one teacher said, "A bracket looks like this," and drew a bracket, answering the question and finalizing the topic.

In our analysis, we compared teachers' use of gestures in turns that preceded and followed trouble spots. This insured that there was some degree of similarity both in the discourse context and in the content of the utterances being compared.

To code gestures, the stream of manual activity was first segmented into individual gestures. Gestures were segmented from one another based on changes in handshape, motion or placement of the hands. Each gesture was then classified into one of the following categories, based on the system developed by McNeill (1992): (1) *pointing* gestures, which indicate objects or locations, typically with an extended finger or hand (e.g., pointing to a number on the board to indicate that number); (2) *representational* gestures, which depict aspects of semantic content via handshape or motion trajectory, either literally (e.g., representing a line by tracing it in the air) or metaphorically (e.g., representing an "idea" as an object held in a cupped hand); (3) *beat* gestures, which are simple, up-and-down rhythmic movements that do not depict semantic content, but instead align with the prosody or discourse structure of speech.[1] We also identified *writing gestures*, defined as writing or drawing actions that had an indexical or pointing function (e.g., underlining or circling

[1] Beat gestures that were *superimposed* on representational gestures were not counted separately, as coders had difficulty identifying these gestures reliably.

something) and that were integrated with speech in the way that hand gestures typically are. Writing gestures could be easily distinguished from writing as a functional act (e.g., writing equations on the board).

We suggest that all of these types of gestures may contribute to students' comprehension of instructional material. *Pointing* and *writing gestures* serve to highlight particular elements of objects or inscriptions within the complex visual field (Goodwin 2007). For example, if a teacher points to or circles the 3 in x^3 while saying the word "exponent", the gesture may facilitate comprehension for students who are uncertain as to what the term "exponent" refers. *Representational gestures* may help listeners to understand unfamiliar terms. For example, if a teacher produces a representational gesture that depicts a line with negative slope while talking about the idea of negative slope, students may be more likely to grasp the meaning of the term. Finally, *beat gestures* may draw listeners' attention to the speaker. If a teacher uses beat gestures, students may be more likely to attend to the teacher, rather than to daydream or attend to something else. Thus, teachers' gestures might foster shared understanding in a variety of ways.

2.3 Reliability of coding

To establish reliability in identifying trouble spots, a second coder reviewed one of the six lessons and identified trouble spots. Agreement between coders was 86 %. To establish reliability in coding gestures, a second coder recoded teachers' gestures in the turns preceding and following eight trouble spots (13 % of the corpus). Agreement was 86 % for identifying individual gestures from the stream of manual behavior, and 79 % ($N = 33$) for classifying gestures as point, representational, beat, or writing gestures.

3 Results

3.1 How frequent were trouble spots in the lessons?

Across the six lessons, we identified a total of 61 trouble spots, yielding an average of 10.2 trouble spots per lesson (range 3–18). Thus, trouble spots were a regular occurrence across teachers and topics. Overall, the corpus included 29 instances of student questions, 28 instances of incorrect responses on the part of students, and 4 instances of dysfluent utterances that reflected student uncertainty.

3.2 Did teachers respond to trouble spots by increasing their use of gestures?

As predicted, in absolute terms, teachers produced more gestures in turns that immediately followed trouble spots than in turns that immediately preceded them ($M = 3.39$ gestures in turns following trouble spots vs. $M = 1.77$ gestures in turns preceding trouble spots), $t(5) = 2.67$, $p = .02$, one-tailed. These turns did not differ significantly in number of words ($M = 26.8$ vs. $M = 21.9$), $t(5) = 0.90$, $p = .41$. Thus, teachers produced more gestures, but not more words, following trouble spots.

Likewise, when the data were cast in terms of rate of gestures per 100 words (summing across all trouble spots for each teacher), teachers gestured at a significantly higher *rate* per 100 words in turns that immediately followed trouble spots than in turns that immediately preceded trouble spots (averages across teachers: $M = 13.08$ vs. $M = 7.37$), $t(5) = 2.19$, $p = .04$, one-tailed. Each of the six teachers fit this overall pattern.

Finally, we examined the types of gesture that teachers produced before and after trouble spots. The majority of teachers' gestures were pointing gestures, replicating past findings (Alibali et al. 2011). The distributions of gesture types were similar before and after trouble spots. In turns preceding trouble spots, teachers produced a total of 118 gestures, and among these, 64 % were points, 25 % representational gestures, 9 % beat gestures, and 2 % writing gestures. In turns following trouble spots, teachers produced a total of 194 gestures; among these, 62 % were points, 29 % representational gestures, 6 % beat gestures, and 3 % writing gestures.

For each teacher, we calculated the average number of gestures of each type produced before and after trouble spots (see Fig. 1). Overall, teachers produced more points following trouble spots than preceding trouble spots, $t(5) = 2.11$, $p = .04$, one-tailed. Teachers also produced more representational gestures after trouble spots, but this difference was not significant, $t(5) = 1.34$, $p = .12$, one-tailed.

Overall, these data suggest that teachers systematically increase their use of gestures, both in absolute number and in rate, following trouble spots. Further, the fact that teachers increased their use of points and representational gestures

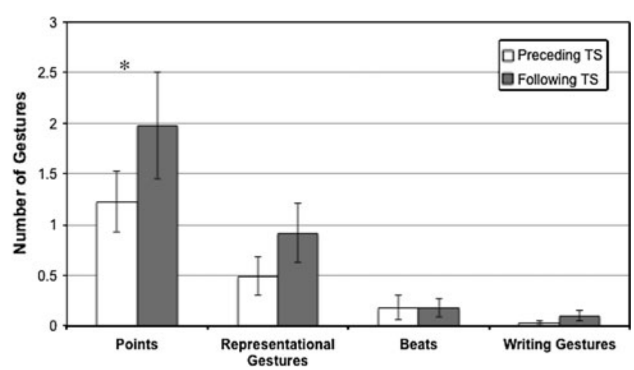


Fig. 1 Average number of gestures of each type across teachers in turns preceding and following trouble spots. *Error bars* represent standard errors. *Note*. * $p < .05$, one tailed

suggests that they use gestures to communicate relevant content following trouble spots. These findings are compatible with the idea that teachers use gestures in an attempt to establish common ground, when it becomes clear that students are having difficulty grasping the material.

3.3 Qualitative analysis of two trouble spot episodes

To illustrate the observed pattern, we discuss two examples, with figures and transcripts that show the teachers' gestures and speech. We selected examples that were representative of the patterns observed at the group level (i.e., examples in which the teacher increased gesture rate following the trouble spot), and we excluded from consideration examples in which the teacher produced no gestures prior to the trouble spot, and those in which one of the teacher's turns (before or after the trouble spot) was very brief (fewer than 10 words).

In the figures, each line of the verbal transcript is numbered (as V1, V2, etc.), and the teacher's and students' words are presented in plain text. Teacher turns are indicated with "T" and student turns with "S". Images of the gestures and writing that occur with each line of transcript are presented below the transcript. Gestures are numbered (as G1, G2, etc.) and each is described in italic text. Relevant motion paths of the hands are overlaid on the images with arrows. Square brackets in the text indicate the particular words during which each gesture or writing act was produced. Writing acts are also numbered (as W1, etc.); writing gestures are numbered with gestures (see "Method" regarding the distinction between writing gestures and writing as a functional act).

The first example was drawn from a lesson about prime factorization. Figure 2 shows the content of the board at the outset of the example.

Left side of the board:

Exponents tell how many times a number is used as a factor.

4^3 → exponent
↘ base

Ex. (4)(4)(4) 4^3
Ex. b^3
(b)(b)(b)

Right side of the board:

$b^2 - c^2 =$ b = 3
(3)(3) – (2)(2) = c = 2
9 – 4 = 5

Fig. 2 Content of board for example 1

Figure 3 presents the transcript of the episode as a whole. Prior to the trouble spot, the teacher indicated the expanded form of a number raised to a power, and labeled it "expanded form". She then presented an expression ($b^2 - c^2$) for which she had provided values for b and c ($b = 3$, $c = 2$), and demonstrated how to evaluate the expression by substituting those values for b and c. Several students expressed confusion (e.g., saying "Wait… Wait. Wh-, what?" or simply, "What?"; lines V9, V10). Thus, this trouble spot was coded as an instance of student-initiated questions. The teacher recognized these questions as a trouble spot, saying, "We'll go over it again" (line V10).

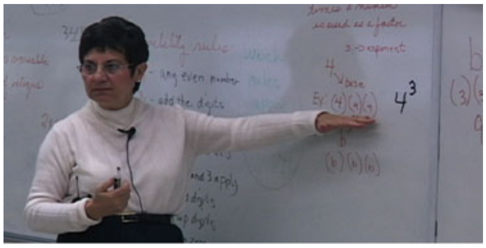
Following the trouble spot, the teacher began to describe the substitution process in a step-by-step fashion. She started by highlighting the link between b in b^2 and the value of b indicated on the board ($b = 3$), using speech and pointing gestures. By this time, students were already expressing that they now understood, saying, "Oh, I got it," (line V13) and "Oh, you're going to tell us what b equals" (line V15). The teacher did not cede the floor, and instead pressed forward with her explanation. She delineated the link between c in c^2 and the value of c indicated on the board ($c = 2$). At this point, it was clear from students' responses that the trouble had been resolved, so she finished her description of the substitution process without much detail—in fact, less than she had provided before the trouble spot.

Prior to the trouble spot, the teacher seemed to assume that students understood that the equations $b = 3$ and $c = 2$ assigned values to the variables in the expression. She stated, "I told you what each one was worth," and simultaneously indicated both equations ($b = 3$ and $c = 2$) using a two-finger point (gesture G3). However, students did not share this understanding—the phrase "what each one was worth" and the two-finger point needed to be "unpacked".

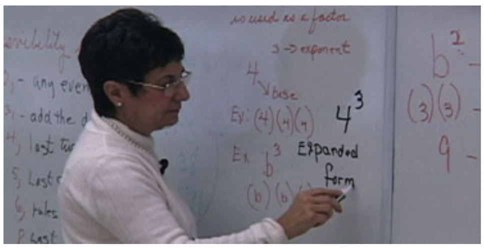
Note that the teacher's verbal phrase is particularly complex. The expression "each one" refers to two variables (b and c) simultaneously, and the phrase as a whole ("what each one was worth") refers to the *function* of the equations (i.e., assigning values to those variables). Thus, with this utterance, the teacher refers, not to a particular element of an equation, nor even to single equation, but to two equations simultaneously. At the same time, she provides information about the function of those equations. This complex information occurs with a gesture that has a double referent: it indicates both equations at the same time, using a two-finger point. Thus, the information students are expected to take in from this utterance is quite complex indeed.

Not surprisingly, the students did not grasp her meaning, and she addressed this lack of shared understanding using speech and gesture to delineate each relation separately.

V1. T: This is [the expanded form when you write it out].
G1. LH palm down under "(4)(4)(4)" on the board



V2. […]
W1. *Writes "Expanded form" on the board*



V3. Then I gave you two examples, I gave you two letters. […]
G2. LH palm down under "$b^2 - c^2 =$"

V4. And then I told you [what each one was worth] and then I substituted [those numbers into here].
G3. LH 2-finger point to "b = 3, c = 2"
G4. LH palm down under "(3)(3) – (2)(2)"

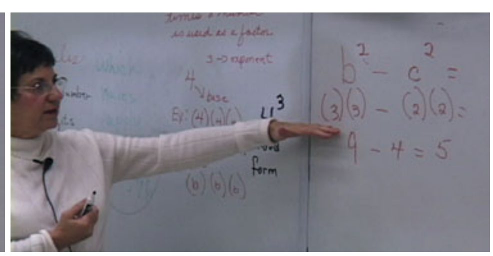
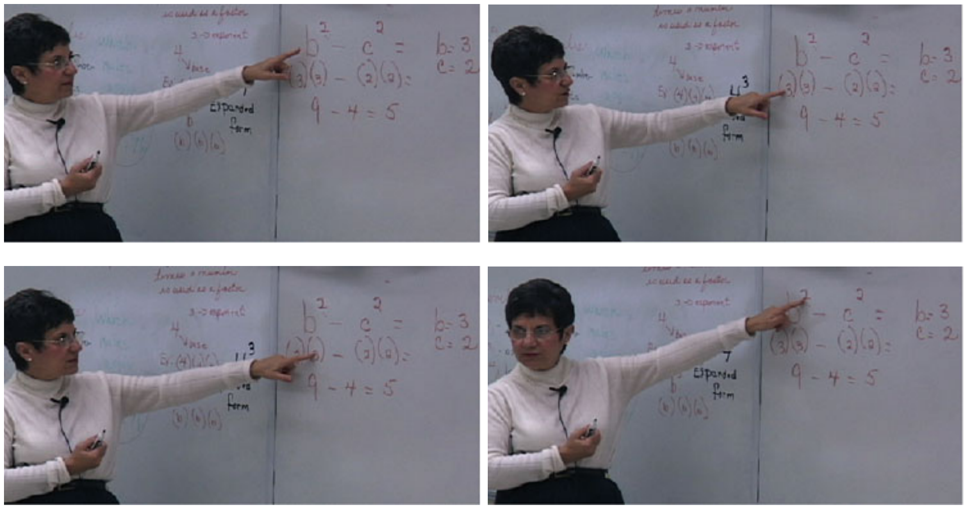


Fig. 3 Example trouble spot. *LH* left hand, *pt* index finger point

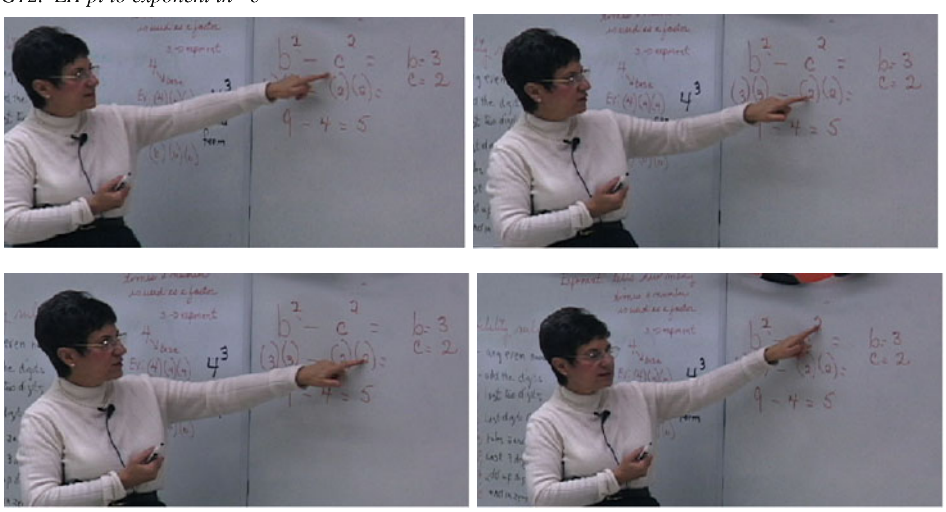
She first linked the b in $b^2 - c^2$ to the b in $b = 3$, saying, "For b, $b = 3$" while pointing first to the b in $b^2 - c^2$ and then to the equation $b = 3$. She then linked the c in $b^2 - c^2$ to the c in $c = 2$, saying, "$c = 2$" while pointing first to the c in $b^2 - c^2$ and then to the equation $c = 2$. Her gestures connected related parts of two symbolic representations (i.e., the variables in the expression $b^2 - c^2$ and the equations that assigned values to those variables), presumably with the goal of helping students link the two representations.

It is also worth noting that the teacher repeatedly used a palm-down handshape in delineating the corresponding

V5. So if I did [b] was three, it's [three] times [three] 'cuz there are [two].
G5. LH pt to b in "b^2"
G6. LH pt to first "(3)"
G7. LH pt to second "(3)"
G8. LH pt to exponent in "b^2"



V6. T: And [c], [two] times [two] because there are [two].
G9. LH pt to c in "c^2"
G10. LH pt to first "(2)"
G11. LH pt to second "(2)"
G12. LH pt to exponent in "c^2"



V7. And then I [multiplied] which got [nine]
G13. LH pt to "(3)(3)"
G14. LH pt to "9" in third line

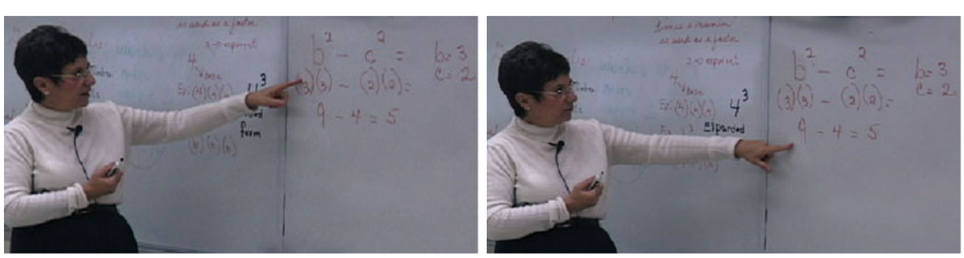
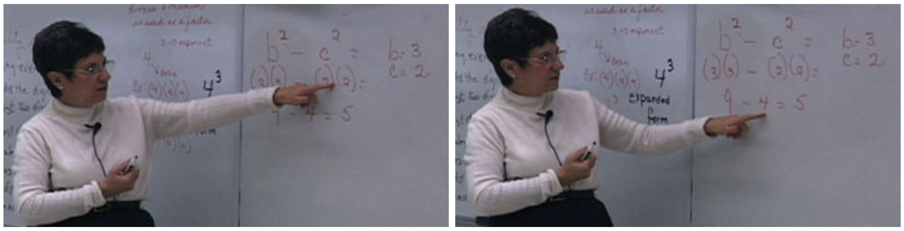
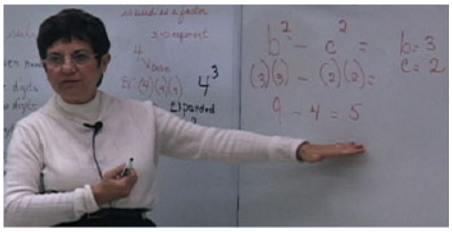


Fig. 3 continued

V8. and I [multiplied] which is [four] and I got [five] through substitution.
G15. LH pt to "(2)(2)"
G16. LH pt to "4" in third line
G17. LH palm down under "5" in third line



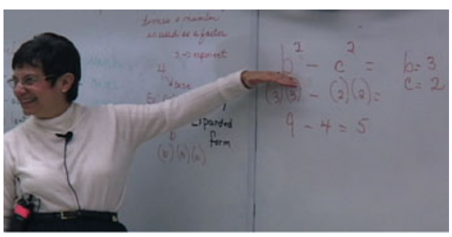


V9. S1: Wait… Wait. Wh-, what? (TROUBLE SPOT)

V10. S2: (At the same time) What? (TROUBLE SPOT)

V11. T: (At the same time) Alright. We'll go over it again.

V12. T: [For b]
G18. LH palm down under b in "b^2"



V13. S1: Oh, I got it, I got it, I got it.

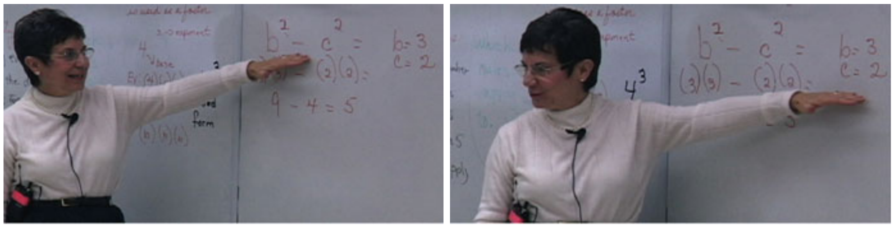
V14. T: [b] equals [three].
G18. Held from previous line
G19. LH palm down under "b = 3"

Fig. 3 continued

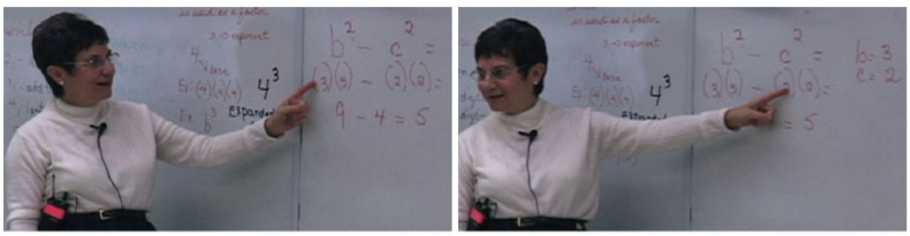
parts of the two representations. This repetition of gesture form across a series of gestures is called a *catchment* (McNeill and Duncan 2000), and such catchments serve to promote cohesion in discourse (McNeill 2010). In this context, the catchment serves to highlight, or perhaps even forge, conceptual connections across representations (see Nathan and Alibali 2011). The teacher altered her hand-shape, ending the catchment, as she went on to describe substituting and multiplying.

In quantitative terms, in the turn prior to the trouble spot, the teacher produced 17 gestures with 82 words, for a rate of 20.7 gestures per 100 words. It is worth noting that

V15. S1: Oh. You're gonna tell us what *b* equals?
V16. T: [*c*] equals [two].
G20. LH palm down under c in "c^2"
G21. LH palm down under "c = 2"



V17. S1: Oh, okay.
V18. T: Alright. And [then I substituted] into [there].
G22. LH pt to "(3)(3)"
G23. LH pt to "(2)(2)"



V19. S3: (At the same time) Oh, got it.
V20. T: [And] multiplied.
G24. LH pt to "9 – 4 = 5"

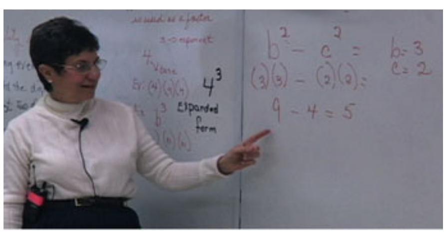


Fig. 3 continued

this pre-trouble-spot rate is even higher than the high end of the ranges reported in other studies of non-instructional settings (e.g., Alibali et al. 2001; Hostetter and Alibali 2010). However, despite this high baseline, she substantially increased her gesture rate after the trouble spot, producing 7 gestures with 17 words, for an extraordinarily high rate of 41.2 gestures per 100 words.

In this example, this teacher realizes that she did not share common ground with her students, as manifested in their questions and expressions of lack of understanding. In response, she sought to re-establish common ground by more carefully delineating the relationships between the equations used to assign variables and the expression being evaluated, and she did so using a very high rate of gestures.

The second example was drawn from a lesson focusing on the patterns of growth exhibited by cubes with different side lengths. In this 8th-grade lesson, the teacher sought to demonstrate that the growth patterns of different constituent parts of a cube—total number of blocks, number of corner blocks (3 faces showing), number of edge blocks (2 faces showing), number of face blocks (1 face showing), and number of internal cubes (0 faces showing)—follow different mathematical functions. For example, the number of corner blocks is a constant function; the number of edge blocks is a linear function of side length. The teacher summarized values for each variable for cubes of different side lengths in a table on an overhead transparency.

Before the trouble spot, the teacher and class had generated table entries for cubes with side lengths 2, 3, 4, and 5, with the exception of the entry for the number of blocks with one face showing for a cube of side length 5. One student suggested that, to find the missing value, one could start with the total number of blocks in the $5 \times 5 \times 5$ cube, and subtract the number of corner blocks, the number of edge blocks, and the number of internal cubes. The teacher acknowledged that this would be an accurate way to determine the number of blocks with one face showing, but encouraged students to consider another way, namely, finding the number of blocks with one face showing on each side of the cube, and multiplying by the number of sides.

While holding a Rubik's cube with side length 4, the teacher asked students how many blocks would be "in the middle of the face" on a $5 \times 5 \times 5$ cube. In posing the question, he gestured to the four blocks "in the middle of the face" on the $4 \times 4 \times 4$ cube, using a circling gesture to represent the "middle" of the face. A student answered uncertainly, saying "4? or no, never mind." This utterance was coded as a trouble spot in which the student offered a dysfluent response. It is noteworthy that the student's uncertain answer, 4, is in fact the *actual* number of blocks with one face showing on each face of the $4 \times 4 \times 4$ cube that the teacher was holding—however, the teacher had asked about a 5×5 cube face, not a 4×4 one. By gesturing to the "middle" blocks on the 4×4 cube face, he intended for students to think about "middle" blocks of the 5×5 cube face.

Following the student's response, the teacher made more specific representational gestures on the $4 \times 4 \times 4$ cube, depicting a hypothetical 5×5 cube face and highlighting the "middle" 3×3 square within it, saying "If we have a five by five cube, it would be kind of a little cube here in the middle (Fig. 4)." Using representational gestures, the teacher "created" a hypothetical 5×5 cube face in gesture space, with the actual 4×4 cube face as the bottom left portion of the 5×5 face (see Fig. 5). He then traced part of the outer ring of blocks on the hypothetical 5×5 cube face to highlight the referent of "middle" in this context, and then delineated the "middle" 3×3 section of the hypothetical 5×5 face by pointing in a circular motion over the relevant 3×3 ("middle") section of the hypothetical 5×5 face (the upper right 3×3 section of the actual 4×4 face).

In this example, the teacher seemed to realize that his original, pre-trouble-spot gesture—indicating the "middle" of the 4×4 cube face to refer to the "middle" of a 5×5 cube face—was confusing for students. Teacher and student did not share common ground, as the student was focusing on the $4 \times 4 \times 4$ cube and teacher was focusing on the hypothetical $5 \times 5 \times 5$ cube. After the trouble spot the teacher gesturally "created" a 5×5 cube face that incorporated the actual 4×4 cube face. In this way, he sought to re-establish common ground, by depicting specific content in greater detail. This effort was successful, as in the student's subsequent turn, he stated "3×3"—the actual size of the "middle" section of a 5×5 cube face.

It is also worth noting that the teacher used gestures with circular motion repeatedly when speaking about the "middles" of cube faces. This catchment may have served to highlight the connections between the middle sections of the actual 4×4 cube face and the hypothetical 5×5 cube face.

In quantitative terms, in the turn prior to the trouble spot, the teacher produced 8 gestures with 53 words, for a rate of 15.1 gestures per 100 words. Following the trouble spot, he increased his gesture rate, producing 5 gestures with 20 words, for a rate of 25 gestures per 100 words. As the qualitative analysis reveals, the nature of his gestures also changed. After the trouble spot, he represented the 5×5 cube face that he wished students to imagine using more specific, detailed representational gestures than he had prior to the trouble spot.

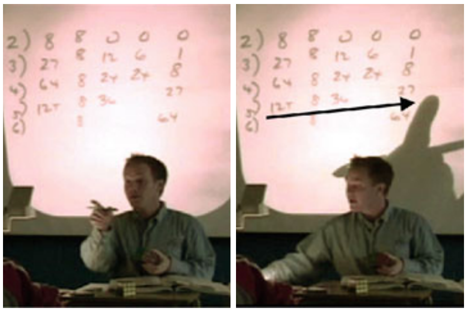
4 Discussion

At trouble spots in instructional discourse, when it becomes clear that teachers and students do not have shared understanding, teachers increase their use of gestures, presumably in an effort to aid students' understanding. Thus, even without instruction or training in how to use gestures effectively, teachers spontaneously draw on multiple modalities in micro-interventions at moments when their students need assistance. Teachers seem to implicitly understand that gestures are a tool that they can use to foster students' comprehension and learning (see, e.g., Valenzeno et al. 2003; Goldin-Meadow et al. 1999).

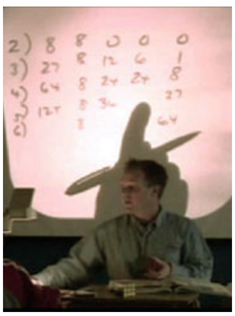
Because we did not gather information about students' learning, we cannot make strong claims about the effectiveness of teachers' micro-interventions for student learning. However, future studies could address this issue using an experimental approach. For example, one could set up a lesson designed to provoke student misconceptions, and then vary experimentally whether teachers use gesture in micro-interventions to address those misconceptions. We hypothesize that students' learning would vary as a function of the quality of teachers' micro-interventions. If this were the case, it would provide strong support for the idea that such micro-interventions contribute in important ways to student learning.

To our knowledge, this study is the first systematic analysis of teachers' gestures in micro-interventions. We studied this issue in the domain of middle-school mathematics, because it is a rich domain that involves abstract representations and conceptual connections that are often difficult for students. However, we suspect that the adaptive use of gesture to promote others' comprehension is a general feature of instructional communication. In our own work, we have observed gestures during micro-interventions in other content domains (geometry and pre-engineering lessons) and other age groups (elementary and high school students). In addition, Marrongelle (2007) describes

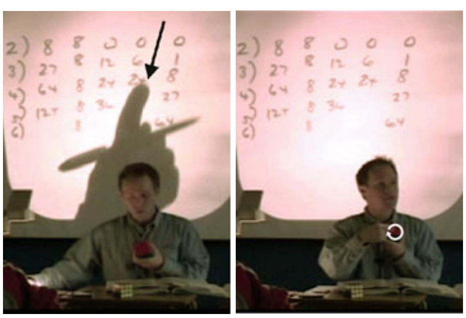
V1. T: [You could subtract] everything [right]
G1. RH pt to student
G2. RH pt traces across row for cube with side length 5



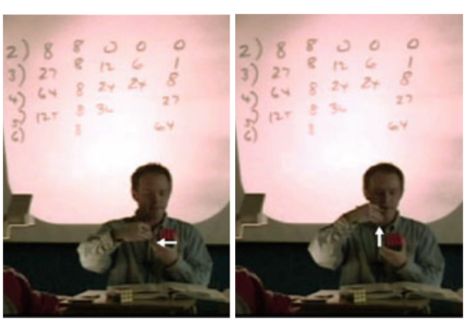
V2. and that's a [great way to check it],
G3. RH pt to empty space in table



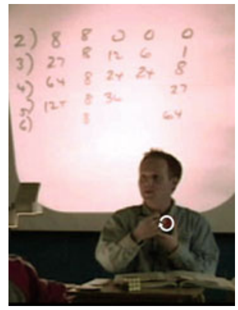
V3. but let's do it another way, too.
V4. The [one sticker ones] are [again the ones in the middle of the face],
G4. RH pt traces one-face-showing column
G5. RH pt circles over middle of cube face 3x clockwise



V5. if we have a [five] [by five cube],
G6. RH claw traces base of cube face
G7. RH claw traces height of (hypothetical) cube face

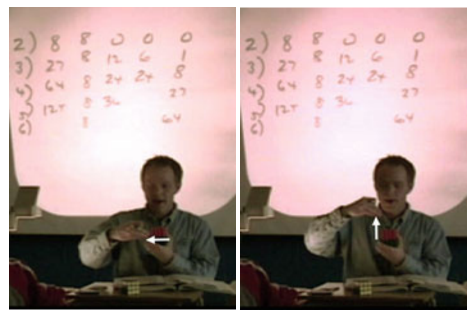


V6. [how many would there be in the middle of the face]?
G8. RH claw circles over middle of face 3x clockwise



V7. S: Four? Or no, never mind. (TROUBLE SPOT)

V8. T: If we have a [five] [by five] cube,
G9. RH claw traces base of face
G10. RH claw traces height of (hypothetical) face



V9. [it would be kind of] [a little cube] [here in the middle].
G11. RH pt traces over left column of blocks on face with finger
G12. RH pt traces outer edge of top and right sides of cube face
G13. RH pt circles over middle of face 1x counterclockwise

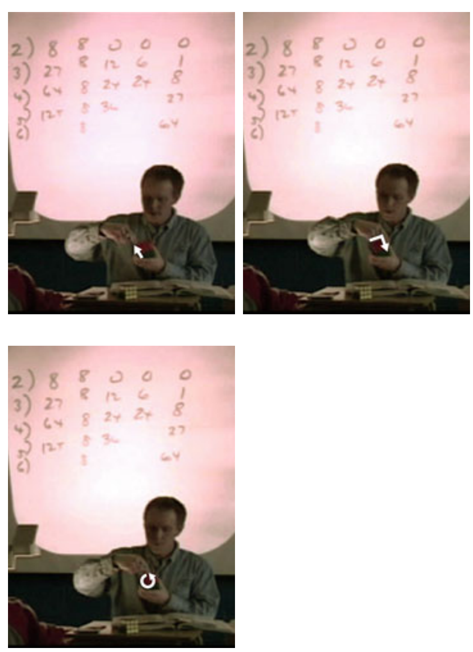


Fig. 4 Example trouble spot. *RH* right hand, *pt* index finger point (colour figure online)

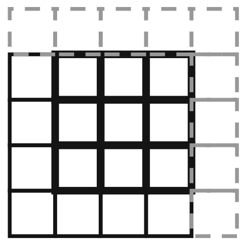


Fig. 5 Schematic of Rubik's cube face. *Solid lines* indicate the face of the actual 4 × 4 cube that the teacher held during the example. *Dotted lines* indicate the additional rows of blocks that he depicted in gesture when talking about the hypothetical 5 × 5 cube. *Bold lines* indicate blocks that would have one face showing in the 5 × 5 cube

a case in which a college student in a course on differential equations alters his gesture to help a fellow student through a trouble spot. Thus, we expect that teachers and students use gestures adaptively in micro-interventions across a range of content areas and grade levels. However, this prediction must be tested in future research.

In the following sections, we consider theoretical explanations of teachers' spontaneous gestural micro-interventions around classroom trouble spots, focusing on two core issues: (1) the role of gesture in establishing common ground, and (2) how gesture manifests embodied knowledge. We conclude by considering educational implications of our findings.

4.1 Gestures are used to establish and maintain common ground

We argue that gestures promote comprehension and learning because they contribute to establishing and maintaining common ground. Several investigators have claimed that the form of speakers' gestures is influenced by knowledge that they share with their interlocutors (e.g., Holler and Stevens 2007; Singer et al. 2008). Taking this idea a step further, Nathan and Alibali (2011) argued that gesture is a tool that speakers use to *establish* common ground, for example, by delineating shared referents or connecting novel representations to more familiar ones.

Thus, by using gestures adaptively to establish and maintain common ground, teachers can create the conditions to promote student learning. Establishing common ground is particularly crucial in instructional communication in mathematics, which often involves references to new concepts and representations. It may be particularly challenging to establish and maintain common ground in mathematics classrooms, where multiple representations of abstract ideas are commonplace. Thus, gesture may play a particularly important role in mathematics instruction. As Sfard (2009) noted, "gestures are an invaluable means for ensuring that all the interlocutors 'speak about the same mathematical object'" (p. 197). Teachers' increased use of pointing gestures after trouble spots could be construed as an effort to insure a common focus on specific mathematical objects and relationships.

But just how might gesture serve to guide attention or foster understanding? Theories of embodied cognition offer some answers to this question.

4.2 Embodied accounts of gesture in thinking and communication

Theories of embodied cognition hold that human cognitive processes are rooted in the interactions of the human body with the physical world (Barsalou 2008; Wilson 2002; Glenberg 2010). From an embodied perspective, human cognition is shaped by the capabilities and limitations of human perceptual systems and human bodies. With respect to mathematical cognition specifically, theorists have argued that cognition is embodied in (at least) two senses: mathematical cognition is based in perception and action, and it is grounded in the physical environment (see, e.g., Lakoff and Núñez 2001).

Many theorists have argued that gesture is a source of evidence for the embodiment of cognitive processes (e.g., Shapiro 2011; Núñez 2005). In a recent paper, Alibali and Nathan (2012) argued that spontaneous gestures provide several types of evidence for the embodiment of mathematical cognition. Two types of evidence are particularly relevant to the present study: (1) pointing gestures ground mathematical thinking in the physical environment, and (2) representational gestures reflect simulations of action and perceptual states. We suggest that teachers' gestures around trouble spots in the classroom discourse manifest these two mechanisms, and that both of these mechanisms help to establish common ground between teachers and students.

Pointing gestures were ubiquitous in our data, and they appeared to focus teachers' and students' attention jointly on common referents, such as elements of inscriptions or physical objects. For example, if a teacher points to the 3 in x^3 while saying the word "exponent", a student who is not certain what an exponent is may be more likely to understand the teacher's utterance. By grounding mathematical terms and ideas in the shared physical environment, teachers and students can successfully achieve shared reference.

Representational gestures were also common in our data, and many of those representational gestures reflected simulations of actions or perceptual states (see Hostetter and Alibali 2008). For example, the teacher in the second trouble spot example used gestures to depict features of the 5 × 5 × 5 cube that he had in mind, and presumably helped learners to envision this 5 × 5 × 5 cube as well.

As another example, one teacher said, "what are the two factors or numbers you multiply together…", while producing a gesture in which she represented two numbers with her index and middle fingers in a V shape, and represented multiplication by bringing the fingers together and crossing them. The V-shaped gesture simulated an inscription (specifically, an equation) by "pointing" to two (imaginary) numbers in space, and also simulated the abstract action of multiplying numbers via the physical action of bringing the fingers together and crossing them. Such a gesture might help learners to ground the abstract operation of multiplication in a familiar physical action, and in so doing, might help to bolster shared understanding of the multiplication operation. By representing an abstract action via a familiar physical action, the teacher's gesture might help the student grasp the instructional material, thereby promoting common ground.

In sum, we suggest that teachers' gestures foster shared understanding via two mechanisms: (1) by grounding talk in the physical environment, thereby insuring joint attention and shared reference, and (2) by manifesting simulated perceptual states and actions, which in turn bring to mind familiar perceptual states and common physical actions that are readily grasped by learners.

4.3 Implications for educational practice

This study focused on teachers' spontaneous micro-interventions around trouble spots in classroom discourse. Given the frequency of trouble spots, it is important for teachers to have tools to effectively address them when they occur. One such tool is enriching their gestural communication. Gestures are readily available at all times, and they can be tailored to the specific communication failures that have occurred, as we have seen in the qualitative analyses presented in this paper.

Teachers in this study frequently used gestures to highlight important mathematical relationships that were challenging for students to understand. Understanding connections is one of the overarching process standards described in the *Principles and Standards for School Mathematics* (NCTM 2000, p. 402), and knowledge of connections is a critical element of conceptual understanding in mathematics. However, connections are often difficult for students, so trouble spots may be likely when connections are the focus of instruction. Based on our findings, we believe that teachers' gestures play a key role in fostering shared understanding of important connections, especially when trouble spots occur and micro-interventions are needed.

Our findings may also have implications for more formal sorts of interventions to help teachers communicate effectively. Growing evidence suggests that teachers can successfully alter their gestures in response to professional development experiences that focus on how to use gestures to communicate effectively (Hostetter et al. 2006). Specifically, teachers increased their gesture rates, and increased their use of gestures to make connections between representations, after receiving instruction about the importance of gesture in instruction. Moreover, such experiences on the part of teachers can lead to greater learning for their students (Alibali et al. 2012). These findings pave the way for the possibility that teachers could learn to use gestures effectively as one approach to improve students' comprehension and learning.

Moreover, there is growing evidence that certain types of gestures are more effective at fostering comprehension and learning than others (Hostetter 2011). In particular, gestures that convey task-relevant information that is not expressed in the accompanying speech seem to be particularly beneficial for student learning (Singer and Goldin-Meadow 2005). Furthermore, there is evidence that memory for information learned via gesture is less likely to fade over time than information learned solely via speech (Church et al. 2007)—a possible reason why instruction with gesture leads to greater retention of instructional material than instruction without gesture (e.g., Cook et al. 2007). As knowledge about the effectiveness of different sorts of gestures grows, we will become better able to make empirically-based recommendations about the types of gestures to encourage in teachers, both in planned instructional language (e.g., lectures), and in spontaneous micro-interventions.

4.4 Conclusion

In this study, we analyzed teachers' micro-interventions in response to trouble spots in classroom interactions, and we found that teachers increased their use of gestures when students displayed lack of understanding. Teachers appear to use gestures adaptively to support students' learning, particularly when students' comprehension falters. We argue that teachers' gestures connect speech to the physical environment, insuring joint attention and shared reference. Further, gestures manifest simulated perceptual states and actions, thus presenting an embodied view of concepts expressed verbally. These gestural mechanisms help teachers to establish and maintain common ground with their students as lessons unfold.

By examining teachers' practices in naturalistic instruction, and considering those practices in light of empirical work on comprehension and learning, we can generate new approaches to improving instructional communication on a broader scale. Furthermore, because teachers naturally engage in gesturing during instruction, evidence-based prescriptions for more optimal timing and use of gestures have the potential to scale up with relatively little cost in additional resources. We suggest that helping teachers learn to effectively use gesture in response to trouble spots will yield benefits for students' learning.

Acknowledgments This research was supported by Grant # R305H060097 from the U. S. Department of Education, Institute of Education Sciences (Alibali, PI). All opinions expressed herein are those of the authors and not the U. S. Department of Education. We thank Maia Ledesma, Kristen Bieda, and Elise Lockwood for their contributions to this research. Most of all, we thank the teachers and students who opened their classrooms to us and allowed us to videotape their instruction.

References

Alibali, M. W., Heath, D. C., & Myers, H. J. (2001). Effects of visibility between speaker and listener on gesture production: Some gestures are meant to be seen. *Journal of Memory and Language, 44*, 169–188.

Alibali, M. W., & Nathan, M. J. (2007). Teachers' gestures as a means of scaffolding students' understanding: Evidence from an early algebra lesson. In R. Goldman, R. Pea, B. Barron, & S. J. Derry (Eds.), *Video research in the learning sciences* (pp. 349–365). Mahwah: Erlbaum.

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from students' and teachers' gestures. *Journal of the Learning Sciences, 21*(2), 247–286.

Alibali, M. W., Nathan, M. J., & Fujimori, Y. (2011). Gestures in the mathematics classroom: What's the point? In N. Stein & S. Raudenbush (Eds.), *Developmental Cognitive Science Goes To School* (pp. 219–234). New York: Routledge, Taylor and Francis.

Alibali, M. W., Young, A. G., Crooks, N. M., Yeo, A., Ledesma, I. M., Nathan, M. J., et al. (2012). Students learn more when their teacher has learned to gesture effectively (Submitted).

Arzarello, F. (2006). Semiosis as a multimodal process. In *Revista latinoamericana de investigacion en mathematic educativa, numero especial*, pp. 267–299.

Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics, 70*(2), 97–109.

Barsalou, L. W. (2008). Grounded cognition. *Annual Review of Psychology, 59*, 617–645.

Blake, B., & Pope, T. (2008). Developmental psychology: Incorporating Piaget's and Vygotsky's theories in classrooms. *Journal of Cross-Disciplinary Perspectives in Education, 1*(1), 59–67.

Brintoon, B., & Fujiki, M. (1989). *Conversational management with language-impaired children*. Rockville: Aspen.

Church, R. B., Ayman-Nolley, S., & Mahootian, S. (2004). The role of gesture in bilingual education: Does gesture enhance learning? *International Journal of Bilingual Education and Bilingualism, 7*, 303–319.

Church, R. B., Garber, P., & Rogalski, K. (2007). The role of gesture in memory and social communication. *Gesture, 7*, 137–158.

Clark, H. H. (1996). *Using language*. Cambridge: Cambridge University Press.

Clark, H. H., & Brennan, S. A. (1991). Grounding in communication. In L. B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), *Perspectives on socially shared cognition*. Washington, DC: APA.

Cook, S. W., Mitchell, Z., & Goldin-Meadow, S. (2007). Gesturing makes learning last. *Cognition, 106*, 1047–1058.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking and classroom practice. In E. Forman, N. Minick, & C. Stone (Eds.), Contexts for learning: *Sociocultural dynamics in children's development* (pp. 91–119). New York: Oxford University Press.

Edwards, L. D. (2009). Gestures and conceptual integration in mathematical talk. *Educational Studies in Mathematics, 70*, 127–141.

Evans, M. A., Feenstra, E., Ryon, E., & McNeill, D. (2011). A multimodal approach to coding discourse: Collaboration, distributed cognition, and geometric reasoning. *Computer-Supported Collaborative Learning, 6*, 253–278.

Flevares, L. M., & Perry, M. (2001). How many do you see? The use of nonspoken representations in first-grade mathematics lessons. *Journal of Educational Psychology, 93*, 330–345.

Glenberg, A. M. (2010). Embodiment as a unifying perspective for psychology. *Wiley Interdisciplinary Reviews: Cognitive Science, 1*, 586–596.

Golab, J., Lind, C., & Okell, E. (2009). Conversation analysis of repair in interaction with adults who have acquired hearing impairment. In *Hearing Care for Adults 2009*. Chicago: Phonak.

Goldin-Meadow, S., & Singer, M. A. (2003). From children's hands to adults' ears: Gesture's role in the learning process. *Developmental Psychology, 39*, 509–520.

Goldin-Meadow, S., Kim, S., & Singer, M. (1999). What the teachers' hands tell the students' minds about math. *Journal of Educational Psychology, 91*, 720–730.

Goldin-Meadow, S., Wein, D., & Chang, C. (1992). Assessing knowledge through gesture: Using children's hands to read their minds. *Cognition and Instruction, 9*, 201–219.

Goodwin, C. (2007). Environmentally coupled gestures. In S. Duncan, J. Cassell, & E. Levy (Eds.), *Gesture and the dynamic dimensions of language* (pp. 195–212). Amsterdam: John Benjamins.

Hatano, G., & Inagaki, K. (1991). Sharing cognition through collective comprehension activity. In L. B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 331–348). Washington: APA.

Holler, J., & Stevens, R. (2007). The effect of common ground on how speakers use gesture and speech to represent size information. *Journal of Language & Social Psychology, 26*, 4–27.

Hostetter, A. B. (2011). When do gestures communicate? A meta-analysis. *Psychological Bulletin, 137*(2), 297–315.

Hostetter, A. B., & Alibali, M. W. (2008). Visible embodiment: Gestures as simulated action. *Psychonomic Bulletin & Review, 15*, 495–514.

Hostetter, A. B., & Alibali, M. W. (2010). Language, gesture, action! A test of the gesture as simulated action framework. *Journal of Memory and Language, 63*, 245–257.

Hostetter, A. B., Bieda, K., Alibali, M. W., Nathan, M. J., & Knuth, E. J. (2006). Don't just tell them, show them! Teachers can intentionally alter their instructional gestures. In R. Sun (Ed.), *Proceedings of the 28th Annual Conference of the Cognitive Science Society* (pp. 1523–1528). Mahwah: Erlbaum.

Hurtig, R. (1977). Toward a functional theory of discourse. In R. O. Freedle (Ed.), *Discourse production and comprehension*. Norwood: Ablex.

Kim, M., Roth, W.-M.,& Thom, J. (2011). Children's gestures and the embodied knowledge of geometry. *International Journal of Science & Mathematics Education*, 9(1), 207–238.

Lakoff, G., & Núñez, R. (2001). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.

Marrongelle, K. (2007). The function of graphs and gestures in algorithmatization. *Journal of Mathematical Behavior, 26*(3), 211–229.

McNeil, N. M., Alibali, M. W., & Evans, J. L. (2000). The role of gesture in children's comprehension of spoken language: Now they need it, now they don't. *Journal of Nonverbal Behavior, 24*, 131–150.

McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. Chicago: University of Chicago Press.

McNeill, D. (2010). Gesten der Macht und die Macht der Gesten (gestures of power and the power of gestures). In E. Fischer-Lichte & C. Wulf (Eds.), *Gesten, Inszenierung, Aufführung und Praxis (Gesture, Staging, Performance, and Practice)*. Munich: Wilhelm Fink.

McNeill, D., & Duncan, S. (2000). Growth points in thinking-for-speaking. In D. McNeill (Ed.), *Language and gesture* (pp. 141–161). Cambridge: Cambridge University Press.

Nathan, M. J., & Alibali, M. W. (2011). How gesture use enables intersubjectivity in the classroom. In G. Stam & M. Ishino (Eds.), *Integrating gestures: The interdisciplinary nature of gesture* (pp. 257–266). Amsterdam: John Benjamins.

Nathan, M. J., Eilam, B., & Kim, S. (2007). To disagree, we must also agree: How intersubjectivity structures and perpetuates discourse in a mathematics classroom. *Journal of the Learning Sciences, 16*(4), 525–565.

Nathan, M. J., & Kim, S. (2009). Regulation of teacher elicitations in the mathematics classroom. *Cognition and Instruction, 27*(2), 91–120.

NCTM. (2000). *Principles and standards for school mathematics*. Reston: Author.

Nemirovsky, R., Rasmussen, C., Sweeney, G., & Wawro, M. (2012). When the classroom floor becomes the complex plane: Addition and multiplication as ways of bodily navigation. *Journal of the Learning Sciences, 21*(2), 287–323.

Núñez, R. (2005). Do real numbers really move? Language, thought, and gesture: The embodied cognitive foundations of mathematics. In F. Iida, R. Pfeifer, L. Steels, & Y. Kuniyoshi (Eds.), *Embodied artificial intelligence* (pp. 54–73). Berlin: Springer.

Paulus, T. M. (2009). Online but off-topic: Negotiating common ground in small learning groups. *Instructional Science, 37*, 227–245.

Radford, L., Edwards, L., & Arzarello, F. (2009). Introduction: beyond words. *Educational Studies in Mathematics, 70*, 91–95.

Rasmussen, C., Stephan, M., & Allen, K. (2004). Classroom mathematical practices and gesturing. *Journal of Mathematical Behavior, 23*, 301–324.

Richland, L. E., Zur, O., & Holyoak, K. J. (2007). Cognitive supports for analogies in the mathematics classroom. *Science, 316*, 1128–1129.

Roth, W.-M. (2002). Gestures: Their role in teaching and learning. *Review of Educational Research, 71*, 365–392.

Schegloff, E., Jefferson, G., & Sacks, H. (1977). The preference for self-correction in the organisation of repair in conversation. *Language, 53*, 361–382.

Schleppenbach, M., Flevares, L. M., Sims, L. M., & Perry, M. (2007). Teachers' responses to student mistakes in Chinese and U.S. mathematics classrooms. *The Elementary School Journal, 108*(2), 131–147.

Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education: Contributions from Cognitive Psychology, 4*, 1–94.

Seedhouse, P. (2004). *The interactional architecture of the language classroom: A conversation analysis perspective*. Malden: Blackwell.

Sfard, A. (2009). What's all the fuss about gestures? A commentary. *Educational Studies in Mathematics, 70*(2), 191–200.

Shapiro, L. (2011). *Embodied cognition*. New York: Routledge.

Sidnell, J. (2010). *Conversation analysis: An introduction*. West Sussex: Wiley-Blackwell.

Singer, M. A., & Goldin-Meadow, S. (2005). Children learn when their teacher's gestures and speech differ. *Psychological Science, 16*, 85–89.

Singer, M. A., Radinsky, J., & Goldman, S. R. (2008). The role of gesture in meaning construction. *Discourse Processes, 45*, 365–386.

Valenzeno, L., Alibali, M. W., & Klatzky, R. L. (2003). Teachers' gestures facilitate students' learning: A lesson in symmetry. *Contemporary Educational Psychology, 28*, 187–204.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Trans.). Cambridge: Harvard University Press.

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review, 9*, 625–636.