Complementary effects of sense-making and fluency-building support for connection making: A matter of sequence?

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Abstract. Multiple graphical representations can significantly improve students' learning. To acquire robust knowledge of the domain, students need to make connections between the different graphical representations. In doing so, students need to engage in two crucial learning processes: sense-making processes to build up conceptual understanding of the connections, and fluency-building processes to fast and effortlessly make use of perceptual properties in making connections. We present an experimental study which contrasts two hypotheses on how these learning processes interact. Does understanding facilitate fluency-building processes, or does fluency enhance sense-making processes? And consequently, which learning process should intelligent tutoring systems support first? Our results based on test data and tutor logs show an advantage for providing support for sense-making processes before fluency-building processes. To enhance students' robust learning of domain knowledge, ITSs should ensure that students have adequate conceptual understanding of connections between graphical representations before providing fluency-building support for connection making.

Keywords: Multiple graphical representations, connection making, learning processes, intelligent tutoring system

1 Introduction

Instructional materials almost universally use multiple graphical representations: flow diagrams are used in programming, schemas and tree diagrams in biology, charts and diagrams in math - to mention only a few examples. Intelligent tutoring systems (ITSs) across domains include graphical representations and provide adaptive support on students' interactions with them [e.g., 1, 2]. Fractions are one domain in which multiple graphical representations are used extensively [3], because different graphical representations emphasize complementary conceptual aspects of fractions [4]. To benefit from multiple representations, however, students need to make connections between them [5]. Connection making allows students to integrate different conceptual aspects into one coherent mental model of the domain. Therefore, connection making between representations is key to students' ability to acquire robust knowledge of the domain: knowledge that transfers to novel tasks and lasts over time [6].

Critical processes in acquiring robust knowledge are sense-making processes and fluency-building processes [6]. Prior research on connection making has mostly focused on supporting students in making sense of connections between representations [e.g., 7, 8]. Sense-making processes in connection making lead to conceptual understanding about how different graphical representations relate to one another by explicitly and verbally reasoning about corresponding components [7] (e.g., how do circle and number line depict the components of numerator and denominator?).

Although support for fluency in retrieving math facts has recently received attention in the ITS literature [9], little research has investigated support for perceptual fluency-building processes in connection making. Fluency-building processes lead to perceptual knowledge about which representations correspond to one another, which can be retrieved fast and effortlessly [10] (e.g., by "just seeing" that a circle and a number line show the same fraction). Being fluent in relating different representations of fractions is recognized as an important foundation for later Algebra learning [3]. Kellman et al. [10] demonstrate the effectiveness of a training for students to gain perceptual experience in finding corresponding math representations.

In prior work, we developed activities for an ITS for fractions that specifically support sense-making processes and fluency-building processes for connection making between multiple graphical representations [11]. In an experiment with the Fractions Tutor, we demonstrate that both types of support for connection making are *necessary* in order for students to benefit from multiple graphical representations [11]: only students who received support for both types of learning processes significantly outperformed a single-representation control condition.

Although we know that sense-making processes and fluency-building processes in making connections between multiple graphical representations interact, we do not know *how* they interact. Does sense-making support enable students to benefit from fluency-building support, or vice versa? The answer to this question has significant implications for the sequence in which instructional support for these learning processes should be provided. We investigate this question in an experiment with the Fractions Tutor.

An analysis of errors that students made during practice with the Fractions Tutor in our earlier experiment [11] yields hypotheses for this question. In this prior study, sense-making support was always provided before fluency-building support. Students who received a combination of sense-making and fluency-building support made fewer errors on fluency-building problems than students who received only fluency-building support. This finding supports the *understanding-first hypothesis* that conceptual understanding of connections equips students with knowledge about the structural correspondences between graphical representations. Such knowledge enables them to attend to relevant aspects of the graphical representations while developing fluency in making connections. According to a contrasting, alternative hypothesis, the *fluency-first hypothesis*, having fluency in making connections frees up cognitive resources that students need in order to engage in sense-making processes [10].

Both hypotheses make different predictions which sequence of support for sense-making processes and fluency-building processes is most effective. According to the understanding-first hypothesis, students should learn better when sense-making support for connection making is provided *before* fluency-building support. By contrast, the fluency-first hypothesis predicts that students should learn better when fluency-building support for connection making is provided *before* sense-making support. Knowing which sequence is most effective will enable designers of ITSs to develop adaptive support for connection making that takes advantage of the complementary effects of sense-making and fluency-building processes.

We contrast these hypotheses in an experiment with the Fractions Tutor, using activities we developed for sense-making support and fluency-building support in connection making between different graphical representations of fractions.

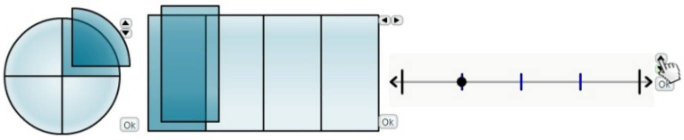

Fig. 1. Interactive representations used in Fractions Tutor: circle, rectangle, number line.

2 Methods

2.1 Fractions Tutor

The Fractions Tutor uses three interactive graphical representations of fractions: circles, rectangles, and number lines (see Fig. 1). Each graphical representation emphasizes complementary aspects of fractions as an abstract concept [4]. Circle and rectangle are both area models which depict fractions as parts of a whole. The whole is inherent to the shape of the circle, but not to the rectangle. The number line depicts fractions as measures of parts of a length and can depict fractions larger than 1.

The design of the Fractions Tutor is based on iterative development through a number of classroom experiments with over 3,000 students. Our recent classroom experiment with 599 4^{th}- and 5^{th}-graders provides empirical evidence that it leads to robust learning gains [11]. The entire curriculum of the Fractions Tutor encompasses a range of topics and activities. For the purpose of the present study, we selected a subset of activities which focus on key aspects of students' conceptual understanding of fractions: equivalent fractions and fraction comparison. Specifically, we use activities designed to help students make sense of connections between different graphical representations and to become fluent in making connections.

The design of the *sense-making support* problems makes use of the worked-example principle [12]. Students are first presented with a worked example that uses one of the area models (i.e., circle or rectangle) to demonstrate how to solve a fractions problem. Students complete the last step of the problem and are then presented with an equivalent problem in which they have to use the number line to complete the problem themselves. At the end of the problem, students are prompted to relate the two graphical representations to one another. On all steps, the Fractions Tutor provides adaptive error feedback and hints on demand. Fig. 2 shows an example of a sense-making support problem for equivalent fractions.

The *fluency-building support* problems are based on Kellman et al.'s fluency training for perceptual expertise in connection making [10]. Students are presented with a variety of graphical representations and have to sort them into sets of equivalent fractions (see Fig. 3), or order them from smallest to largest, using drag-and-drop. Students are encouraged to solve the problems by visually estimating the relative size of the fractions, rather than by counting or computationally solving the problems.

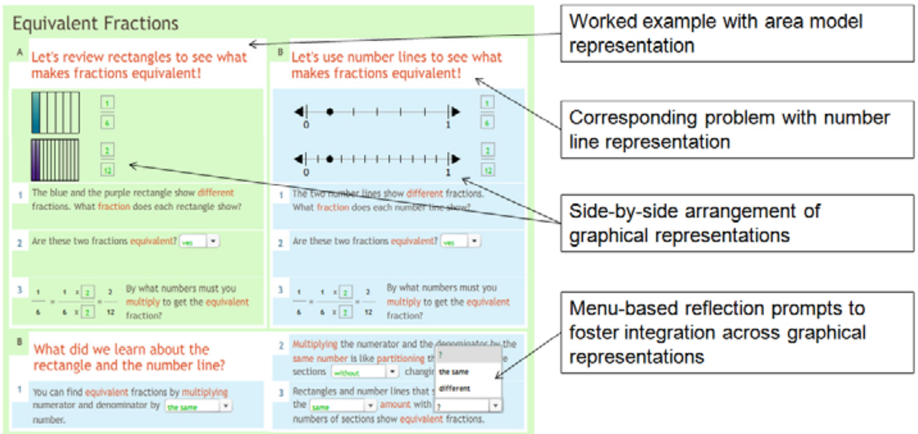

Fig. 2. Sense-making support for connection making.

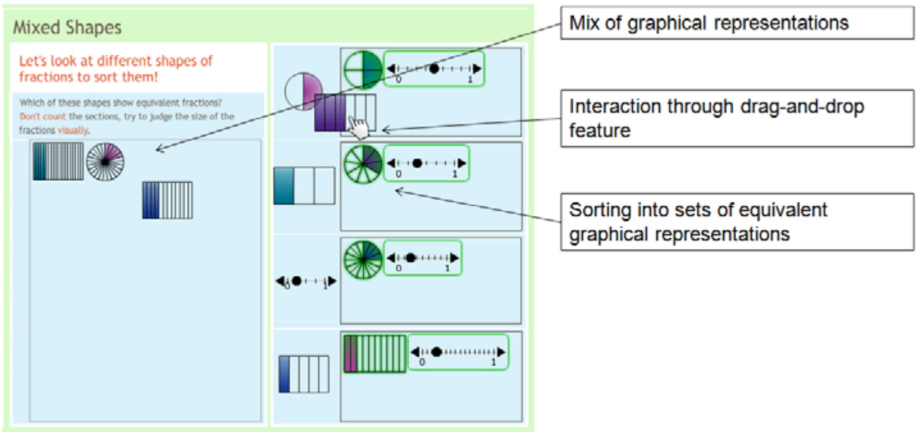

Fig. 3. Fluency-building support for connection making.

2.2 Assessments

We assessed *reproduction of fractions knowledge* based on quiz items with circles, rectangles, and number lines, presented in a format identical to the problems in the Fractions Tutor. Specifically, reproduction-understanding items assessed students' conceptual understanding of connections between graphical representations with regard to equivalent fractions and fraction comparison. Reproduction-fluency items assessed students' fluency in making connections with regard to equivalent fractions and fraction comparison. Students' performance on reproduction-understanding items was computed as the proportion of correct responses to the maximum number correct responses. For reproduction-fluency items, we computed efficiency scores to take into account the speed with which students solved the quiz items, following [13]:

$$\text{reproduction-fluency} = \frac{Z\ (\text{proportion correct}) - Z\ (\text{time on quiz items})}{\sqrt{2}}.$$

Higher reproduction-fluency scores indicate higher efficiency at solving reproduction-fluency items correctly.

We assessed students' transfer of fractions knowledge based on equivalent pretests and posttests. A *near transfer* scale assesses students' ability to solve fractions problems with circles, rectangles, and number lines similar to those in the Fractions Tutor, presented in a different format. *Far transfer* items included test items on equivalence and comparison without graphical representations. Students' scores on both transfer scales were computed as the proportion of correct responses to the maximum number correct responses.

2.3 Experimental Design and Procedure

Table 1. Sequence of activities by experimental condition.

Activity Type	**Understanding-first condition**	**Fluency-first condition**
Test	Pretest: near / far transfer	Pretest: near / far transfer
Tutor: equivalence	***Sense-making support:*** 4 tutor problems	***Fluency-building support:*** 4 tutor problems
Quiz 1: equivalence	Reproduction-understanding, reproduction-fluency	Reproduction-understanding, reproduction-fluency
Tutor: equivalence	***Fluency-building support:*** 4 tutor problems	***Sense-making support:*** 4 tutor problems
Quiz 2: equivalence	Reproduction-understanding, reproduction-fluency	Reproduction-understanding, reproduction-fluency
Tutor: comparison	***Sense-making support:*** 4 tutor problems	***Fluency-building support:*** 4 tutor problems
Quiz 1: comparison	Reproduction-understanding, reproduction-fluency	Reproduction-understanding, reproduction-fluency
Tutor: comparison	***Fluency-building support:*** 4 tutor problems	***Sense-making support:*** 4 tutor problems
Quiz 2: comparison	Reproduction-understanding, reproduction-fluency	Reproduction-understanding, reproduction-fluency
Test	Posttest: near / far transfer	Posttest: near / far transfer

Thirty-nine students from grades 4 and 5 participated in the experiment. Sessions were conducted individually in the lab. Students were randomly assigned to different sequences of sense-making problems and fluency-building problems. In other words, all students worked on the same tutor problems, but in different orders. Students in the *understanding-first condition* received sense-making support before fluency-building support, for each topic (i.e., equivalence and comparison). Specifically, students in the understanding-first condition first worked on four sense-making problems for equivalent fractions. Next, they worked on four fluency-building problems for equivalent fractions. They then worked on four sense-making problems for fraction comparison, followed by four fluency-building problems for fraction comparison.

By contrast, students in the *fluency-first condition* received fluency-building support before sense-making support, again for each topic. Specifically, students in the fluency-first condition first worked on four fluency-building problems for equivalent fractions, then on four sense-making problems for equivalent fractions. Next, they worked on four fluency-building problems for fraction comparison, followed by four sense-making problems for fraction comparison.

Table 1 details the sequence of assessment problems and tutor problems for each experimental condition. Students first completed a pretest. They then worked on the Fractions Tutor. After every four tutor problems, students completed two quiz items (i.e., reproduction-understanding and reproduction-fluency for the given topic). After completing all tutor problems as well as the last set of quiz items, students were given an immediate posttest.

3 Results

One student was excluded from the analysis because he did not complete both topics of the Fractions Tutor, resulting in $N = 38$ students ($n = 20$ in the understanding-first condition, $n = 18$ in the fluency-first condition). We report partial eta-squared, a standard measure of effect size in the educational psychology literature, with η^2 of .01 corresponding to a small effect, .06 to a medium effect, and .14 to a large effect [14].

3.1 Quiz: Reproduction-Understanding and Reproduction-Fluency

To analyze differences between conditions on the quiz items, which assess reproduction of fractions knowledge, we conducted repeated measures MANCOVAs. We used condition as the independent factor, performance on the near and far transfer pretests as covariates, and quiz time (i.e., first and second quiz for the given topic) as repeated factor. Reproduction-understanding and reproduction-fluency were dependent measures.

Fig. 4 shows students' reproduction-fluency scores per quiz assessment. Results show a significant main effect of quiz time on reproduction-understanding, F (1,34) = 4.26, $p < .05$, $\eta^2 = .11$, but not for reproduction-fluency ($F < 1$). There was no significant main effect of condition on reproduction-understanding, $F(1,34) = 1.12$, $p = .30$, nor quiz-fluency ($F < 1$). Yet, there was a significant interaction between quiz time and condition on reproduction-fluency, $F(1,34) = 4.75$, $p < .05$, $\eta^2 = .12$. Pairwise comparisons on reproduction-fluency show that the fluency-first condition marginally significantly outperforms the understanding-first condition at the first assessment of reproduction-fluency, $t(34) = 1.68$, $p = .10$, $\eta^2 = .07$, whereas the understanding-first condition marginally significantly outperforms the fluency-first condition at the second assessment of reproduction-fluency, $t(34) = 1.71$, $p = .10$, $\eta^2 = .08$. This result indicates that the fluency-first condition outperforms the understanding-first condition on the fluency-reproduction quiz *only until* students in the understanding-first condition receive fluency-building support. After having received fluency-building support (at quiz time 2), the understanding-first condition outperforms students in the fluency-first condition on the fluency-reproduction quiz.

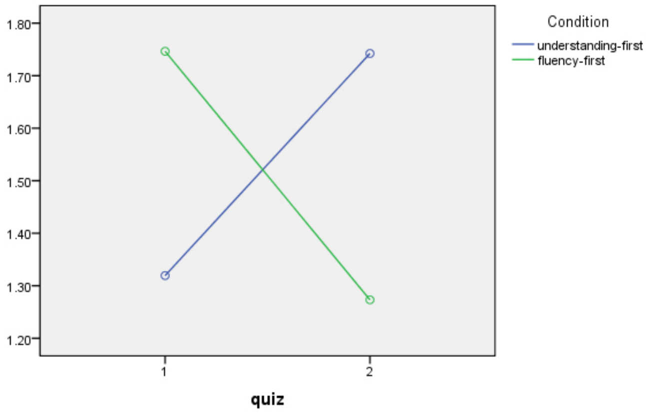

Fig. 4. Reproduction-fluency scores by condition by quiz time.

3.2 Posttest: Transfer of Knowledge

To analyze differences between conditions on the posttests, which assess transfer of fractions knowledge, we conducted repeated measures MANOVAs with test time (pretest and posttest) the repeated factor, and near and far transfer performance as dependent measures.

Results demonstrate a significant main effect of test time on near transfer, $F(1,36) = 5.96$, $p < .05$, $\eta^2 = .14$, but not far transfer, $F(1,36) = 2.66$, $p = .11$. There was no significant main effect of condition on near transfer ($F < 1$) nor far transfer, $F(1,36) = 1.18$, $p = .28$, nor a significant interaction between test time and condition (Fs < 1). These findings indicate that both conditions significantly improved their ability to transfer fractions knowledge to novel test items equally.

3.3 Learning Curves: Differences in Rates of Learning

We examined “learning curves” using the DataShop web service [15] which depict the average error rate (across students and knowledge components) as a function of the amount of prior practice (i.e., the number of opportunities a student has had to apply a given knowledge component). Following standard practice in Cognitive Tutors research [6], we viewed each step in a tutor problem as a learning opportunity for the particular knowledge component involved in the step. We used a set of 19 knowledge components as a basis for this analysis. We considered a step in a tutor problem to be correct if the student solved it without hints and errors (i.e., if the student’s first action on the step was a correct attempt at solving, as opposed to an error or a hint request). We expect that, if learning occurs, error rates will decrease with the number of learning opportunities students have encountered.

Fig. 5 shows the aggregate learning curves based on error rates across knowledge components for the understanding-first condition and the fluency-first condition. The error rates decrease for both conditions, but the curves diverge: the understanding-first condition demonstrates a faster decrease in error rates than students in the fluency-first condition. As the standard errors in Fig. 5 indicate, this difference is reliable after the third attempt per knowledge component. These results show that students in the understanding-first condition learn more efficiently than students in the fluency-first condition.

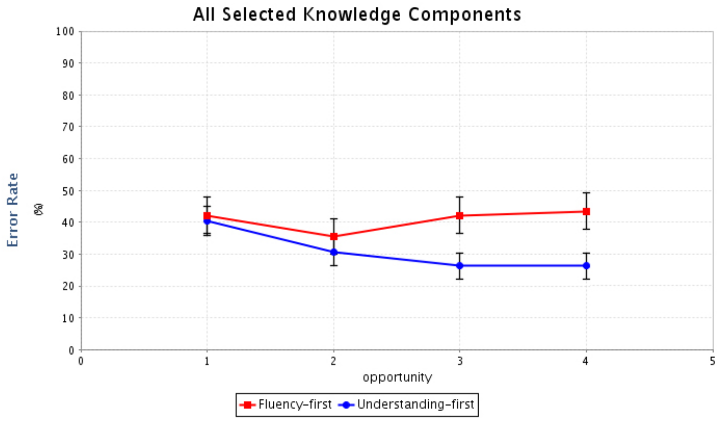

Fig. 5. Learning curves by condition across knowledge components. Bars show standard errors.

4 Discussion and Conclusion

Prior research shows that both sense-making processes and fluency-building processes play an important role in connection making: both learning processes need to be supported in order for students' robust learning of domain knowledge to benefit from multiple graphical representations [11]. Our results shed light into the question of how these learning processes interact. We contrasted two competing hypotheses. On the one hand, the *understanding-first hypothesis* posits that conceptual understanding of connections between graphical representations enables students to acquire fluency by helping them focus on conceptually relevant aspects of graphical representations. According to the *fluency-first hypothesis*, on the other hand, fluency in making connections between representations frees cognitive resources so that students can invest in sense-making processes to develop conceptual understanding of connections between graphical representations.

Our results support the understanding-first hypothesis which predicts that students learn better when sense-making processes are supported *before* fluency-building processes. Students in the understanding-first condition outperformed students in the fluency-first condition on fluency in reproduction of fractions knowledge, with medium effect sizes. Furthermore, an analysis of students' learning rates based on the tutor log data demonstrates that across all knowledge components, students in the understanding-first condition learn more efficiently than students in the fluency-first condition. In addition, students in the understanding-first condition end with a lower error-rate than students in the fluency-first condition. This result is in line with the advantage of the understanding-first condition on the reproduction-fluency quiz. By contrast, our results do not support the fluency-first hypothesis, that perceptual expertise in making connections between graphical representations frees cognitive resources [10] which are needed to make sense of how and why different graphical representations relate to one another. In particular, our findings indicate that students are more likely to acquire fluency in making connections purely based on visual cues, if they have previously acquired conceptual understanding of the connections.

Our results do not show differences between conditions on understanding-reproduction items. This finding indicates that the advantage of the understanding-first condition lies mainly in helping students benefit from fluency-building support, rather than helping students benefit from sense-making support. This interpretation is consistent with the understanding-first hypothesis that conceptual understanding of connections between graphical representations enables students to acquire fluency-building support.

Our results do not show an advantage of the understanding-first condition on near or far transfer assessments. Instead, both conditions improve their ability to transfer knowledge of fractions equally, with medium to large effect sizes. A possible explanation for this finding is that the items on the near and far transfer tests relied more on students' understanding of connections between graphical representations than on their ability to fluently make connections between representations. According to Kellman et al. [10], fluency training promotes students' ability to extract information more efficiently from representations. Future learning of novel graphical representations might benefit from fluency in making connections. However, such test items were not part of the near and far transfer assessments used in the present study. In future research, we will investigate whether there is an advantage of the understanding-first condition over the fluency-first condition in students' ability to learn how to use a novel graphical representation of fractions, such as a set representation.

Taken together, our results indicate that conceptual understanding of connections between multiple graphical representations enhances students' ability to acquire fluency in making connections, rather than vice versa. Consequently, ITSs should provide instructional support for making sense of connections between graphical representations *before* instructional support for fluency-building processes in making connections. Adaptive versions of connection-making support should ensure that students have acquired conceptual understanding of connections between graphical representations before providing fluency-building support. As multiple graphical representations are ubiquitously used across many science and math domains, our results have the potential to impact students' learning across a wide range of settings. We are currently planning a classroom experiment to investigate the extrinsic validity of these findings.

Acknowledgements

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References

1. Schwonke, R., Ertelt, A. Renkl, A.: Fostering the translation between external representations. Does it enhance learning with an intelligent tutoring program? In: J. Zumbach, et al. (eds) Beyond knowledge: The legacy of competence, pp. 117-119. Springer Netherlands (2008)
2. Koedinger, K.R.: Toward evidence for instructional design principles: Examples from Cognitive Tutor Math 6. In: 24th Annual Meeting of the North American Chapter of the International Group of the Psychology of Mathematics Education. ERIC/CSMEE Publications, Athens, GA (2002)
3. NMAP: Foundations for Success: Report of the National Mathematics Advisory Board Panel. U.S. Government Printing Office (2008)
4. Charalambous, C.Y., Pitta-Pantazi, D.: Drawing on a Theoretical Model to Study Students' Understandings of Fractions. Educational Studies in Mathematics, 64, 293-316 (2007)
5. Ainsworth, S.: DeFT: A conceptual framework for considering learning with multiple representations. Learning and Instruction, 16, 183-198 (2006)
6. Koedinger, K.R., Corbett, A.T., Perfetti, C.: Knowledge-Learning-Instruction Framework: Bridging the Science-Practice Chasm to Enhance Robust Student Learning. Cognitive Science, 36, 757–798 (2012)
7. Seufert, T.: Supporting Coherence Formation in Learning from Multiple Representations. Learning and Instruction, 13, 227-37 (2003)
8. Bodemer, D., Ploetzner, R., Feuerlein, I., Spada, H.: The Active Integration of Information during Learning with Dynamic and Interactive Visualisations. Learning and Instruction, 14, 325-341 (2004)
9. Arroyo, I., Royer, J. M., & Woolf, B. P.: Using an intelligent tutor and math fluency training to improve math performance. I. J. of AIED, 21, 135-152 (2011)
10. Kellman, P.J., Massey, C.M., Roth, Z., Burke, T., Zucker, J., Saw, A., Aguero, K. Wise, J.: Perceptual learning and the technology of expertise: Studies in fraction learning and algebra. Pragmatics & Cognition, 16, 356-405 (2008)
11. Rau, M.A., Aleven, V., Rummel, N.: Sense Making Alone Doesn't Do It: Fluency Matters Too! ITS Support for Robust Learning with Multiple Representations. In: Cerri, S. et al. (eds) Intelligent Tutoring Systems. Springer Berlin / Heidelberg. pp. 174-184 (2012)
12. Renkl, A.: The worked-out example principle in multimedia learning In: Mayer, R. (ed) Cambridge Handbook of Multimedia Learning, R. Mayer, Editor. Cambridge University Press, Cambridge, UK. pp. 229-246 (2005)
13. Van Gog, T. Paas, F.: Instructional efficiency: revisiting the original construct in educational research. Educational Psychologist, 43, 1-11 (2008)
14. Cohen, J.: Statistical Power Analysis for the Behavioral Sciences. Lawrence Erlbaum Associates, Hillsdale, NJ (1988)
15. Koedinger, K.R., Baker, R., Cunningham, K., Skogsholm, A., Leber, B., Stamper, J.: A data repository for the EDM community: The PSLC Data-Shop. In: Romero, C. et al (eds), Handbook of educational data mining. CRC Press, Boca Raton, FL. pp. 10-12 (2010)