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The Symbol Precedence View of Mathematical Development: A Corpus Analysis of the Rhetorical Structure of Textbooks

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This study examined a corpus of 10 widely used prealgebra and algebra textbooks, with the goal of investigating whether they exhibited a symbol precedence view of mathematical development as is found among high school teachers. The textbook analysis focused on the sequence in which problem-solving activities were presented to students. As predicted, textbooks showed the symbol precedence view, presenting symbolic problems prior to verbal problems. Algebra textbooks showed this pattern more strongly than prealgebra textbooks aimed at middle school, paralleling grade-level differences found among teachers. Finally, textbooks published after 1990 contained far fewer symbol-only sections, as expected from recent mathematics education reform documents. We interpret these findings in light of research on learning from texts and on the role of textbooks in shaping teachers' instructional practices and students' learning.

Textbooks serve as a major resource for learning, but analyses of their composition and organization are often neglected in research on learning from text. Learning from text depends on several interacting factors, including the nature

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of the learning context, the prior knowledge and processing strategies available to the learner, and the design of the learning materials (Britton & Gugoz, 1991; Kintsch, Franzke, & Kintsch, 1996; McNamara, Kintsch, Songer, & Kintsch, 1996). Structural aspects of texts are particularly important, such as coherence at the microstructural and macrostructural levels (e.g., Kintsch, 1990; van Dijk, 1980; van Dijk & Kintsch, 1983). Text coherence has a powerful influence on problem-solving performance, inference making, and later retention (Kintsch, 1998). For example, when text coherence is improved by adding bridging inferences (Britton & Gugoz, 1991) or causal connections and background information (Beck, McKeown, Sinatra, & Loxterman, 1991), recall, understanding, and inference making also improve.

McNamara and her colleagues (McNamara & Kintsch, 1996; McNamara et al., 1996) showed that text coherence also interacts with readers' prior knowledge to produce differences in reading comprehension and inference making. They found that improvements in local and global coherence for a passage on the human heart showed comprehension gains for low-knowledge readers. Unlike low-knowledge readers, high-knowledge readers could spontaneously make the necessary inferences that were missing in the low-coherence text. The low-coherence texts led to better performance in problem-solving and inference-making tasks for high-knowledge readers (see also Mannes & Kintsch, 1987). Evidence from reading times suggested that the deeper processing needed to comprehend the impoverished texts helped high-knowledge readers formulate richer situation models (McNamara & Kintsch, 1996).

In addition to text coherence, rhetorical structure has also been shown to influence text comprehension processes. *Rhetorical structure* refers to the differences in hierarchical organization found in different types of passages, such as compare and contrast texts, that help link to readers' common schemas and guide their expectations (e.g., Mannes & Kintsch, 1987; Meyer, 1977; van Dijk & Kintsch, 1983) and their preferences (Nolen, Johnson-Crowley, & Wineburg, 1994). We seek to add to this literature with an analysis of rhetorical structure of a set of texts designed for algebra and prealgebra instruction. For this study, we apply the term *rhetorical structure* rather narrowly, confining ourselves to the organizational sequence of problem-solving activities in mathematics textbooks.

We address the following three specific objectives: (a) examine aspects of the rhetorical structure of algebra textbooks that govern the ordering of problem-solving activities, (b) investigate whether this structure differs for prealgebra and algebra textbooks, and (c) examine whether this structure has changed over a 10-year span that includes a major educational reform initiative. We first briefly review research on the influences of textbooks on learning and teaching practices. Then, we present two competing views of mathematical development suggested by teacher and student data. These two views guided our analyses of the rhetorical structure of a sample of algebra and prealgebra textbooks. Specifically, we examined textbooks for the presence of implicit organizational patterns commensurate with views of mathematical development that are commonly found among teachers but at odds with student data. We interpret these findings in light of research on learning from texts and on the role of textbooks in shaping teachers' instructional practices and students' learning. Because the views shown within textbooks are also found among mathematics teachers, we speculate on the causal role that textbooks may play in the development of teachers' beliefs about learning and development.

INFLUENCES OF TEXTBOOKS

This study involved a corpus analysis of algebra and prealgebra textbooks. Corpus analyses have become an increasing priority in the study of discourse processes for models of text comprehension (e.g., Landauer & Dumais, 1997) and, more commonly, in studies of spoken language (Biber, Conrad, & Reppen, 1998; Jurafsky, Shriberg, & Biasca, 1997; MacWhinney, 2000). Yet systematic analyses of textbook corpora are lacking despite the potential influence of textbook structure on student learning. For example, Mayer (1981, 1982) found that students' memories of algebra story problems were distorted, such that problem types that occurred with low frequency in textbooks were often misrecalled as high-frequency items. Raman (1998) attributed differences in beliefs about mathematics held by students enrolled in high school and in college calculus courses to the disparate treatments of mathematical definitions in high school and college level mathematics textbooks, "which, for better or for worse, tend to have a strong influence on the way mathematics is taught and learned" (p. 1). Brenner, Herman, Ho, and Zimmer (1999) found that achievement differences in Asian and American students' uses of mathematical representations were paralleled in differences between Asian and American textbooks.

Investigators have also identified ways that textbooks shape teachers' instructional practices and curriculum planning (Borko & Livingston, 1989; Borko & Shavelson, 1990; Cooney, 1985; Ornstein, 1994). For example, Flanders (1994) reported that teachers had higher performance expectations for students on items covered by textbooks than items taught solely on the basis of teachers' own content knowledge. In a review of the literature, Johnsen (1993) found that "all available investigations from the 1980s indicate that teachers largely follow the teaching plans incorporated into the textbooks" (p. 287).

LINKS BETWEEN TEACHERS' VIEWS AND STUDENT PERFORMANCE

Our focus in this study is on algebra and prealgebra textbooks. The time is ripe for such an analysis because of recent advances in our knowledge base both about students' algebraic thinking (Hegarty, Mayer, & Monk, 1995; Koedinger, Alibali, &

Nathan, 2000; Koedinger & Nathan, 1999; Reed, 1999), and about teachers' views of students' algebraic thinking (Nathan & Koedinger, 2000a, 2000b).

Not surprisingly, teachers sometimes hold inaccurate views about students' preferences and students' thinking. For example, in reading, Orange and Horowitz (1999) found that high school teachers misperceived African American and Mexican American students' preferences for literature and language arts activities. In the domain of mathematics, first-grade teachers underestimated the frequency with which students used more basic problem-solving strategies (e.g., counting) and overestimated their use of more advanced strategies (e.g., derived facts and direct modeling methods; Carpenter, Fennema, Peterson, & Carey, 1988).

Our investigation was guided by recent research on educators' beliefs about students' developmental trajectories from arithmetic to algebra. Nathan and Koedinger (2000b) asked high school mathematics teachers ($n = 67$) to rank order a set of mathematics problems from easiest to hardest to reflect their expectations about the problem-solving difficulties of their students. The participants were volunteers recruited from a summer workshop for dedicated mathematics teachers. These teachers taught in the Southeastern United States in a wide range of settings, including predominantly minority-based, inner-city schools, rural communities, and middle-income suburban areas. The majority of teachers (70%) ranked verbally presented problems (e.g., story problems and word equations) as more difficult than symbolic equations. Rarely did teachers offer the reverse ordering when making their predictions.

This finding was replicated with another sample of teachers from the western portion of the United States (Nathan & Koedinger, 2000a). As part of that replication, mathematics teachers from elementary, middle, and high school levels ($n = 105$) participated in the difficulty ranking task. They also responded to a 42-item survey that assessed teachers' views about student learning and development, mathematics teaching, and the effectiveness of students' solution strategies.

The high school teachers in this second sample ($n = 39$) replicated the earlier finding. They ranked verbal problems as harder for their students than symbolic problems. In the survey, high school teachers also tended to dismiss the effectiveness of students' invented, nonalgebraic solution methods (e.g., guess and test) and tended to agree with statements such as "Using algebra for story problem solving is the most effective approach there is," and "Solving math problems presented in words should be taught only after students master solving the same problems presented as equations." These ranking data and survey responses led Nathan and Koedinger (2000a) to conclude that high school teachers tended toward a *symbol precedence view* of algebra development, whereby students must first master symbolic representations and procedures before moving on to verbally presented problems. Teachers' level of agreement with the symbol precedence items was highly correlated with their difficulty rankings of symbolic and verbal problems, $p < .0001$.

High school students' performance data, however, did not follow high school teachers' expectations. Ninth-grade students in two different samples ($n_1 = 76$, $n_2 = 171$; Koedinger & Nathan, 1999), each with a year or more of formal algebra instruction, correctly solved fewer than 30% of the symbolic equations. In contrast, they successfully solved approximately 50% of the mathematically matched verbal problems. Analyses of solution strategies revealed that students tended to successfully apply informal strategies when solving verbally presented story and word-equation problems but they unsuccessfully applied symbol manipulation methods when solving equations, and they showed high no-response rates for equation items. Koedinger and Nathan compared verbal problems with a situational context (story problems) and verbal problems without a context (word equations) to symbolic equations, and showed that it was in fact the verbal aspects of the items rather than the problem context that led to strategy differences and the resulting performance advantage.

In addition, students who could solve verbal problems could not necessarily solve matched symbolic problems, whereas students who accurately solved symbolic problems were very likely to solve the matched verbal problems (Nathan & Koedinger, 2000b). This led the investigators to suggest that, in contrast to high school teachers' beliefs, algebra students follow a *verbal precedence model* of mathematical development. According to this view, verbally based reasoning about quantitative relations (e.g., reversing the events of a story problem, which suggests inverting the mathematical operations) precedes symbolic reasoning. This pattern of results is consistent with findings by Case and his colleagues (Case, 1991; Case & Okamoto, 1996) on the primary role of verbal representations in early number development, rational number processing (Moss & Case, 1999), and reasoning about functions (Kalchman & Case, 1998). More recent research on algebra problem solving has shown the existence of a symbolic advantage on more complex problems (e.g., problems in which the unknown occurs twice; Koedinger et al., 2000). However, in this study we limit our investigation to prealgebra and basic algebra textbooks that consist primarily of simpler problems that do not typically show the symbol advantage.

In contrast with high school teachers, middle school teachers ($n = 30$) were more accurate at predicting the order of students' problem-solving performance difficulties. Middle school teachers' predictions were significantly correlated with the problem-solving performance of students, $p < .05$. Surprisingly, the rank ordering provided by high school teachers was not significantly related to student performance at all, despite the high school teachers' more extensive mathematics education. Responses to the belief survey also showed that middle school teachers held students' intuitions in higher regard than did high school teachers, and tended to believe more strongly that students could invent effective problem-solving methods that were not symbol based (Nathan & Koedinger, 2000a). Nathan and Koedinger (2000a) suggested that the differences between middle school and high school teachers' beliefs may be based on their classroom experiences. Many students at the middle school level have not had formal algebra training and so they may be more likely than high school students to use invented methods freely in classrooms. Thus, middle school mathematics teachers may actually have more opportunities than high school teachers to observe the power of these informal methods and the role these strategies play during the transition from arithmetic to algebraic reasoning.

HYPOTHESES

In this study, we examine the rhetorical structure of algebra textbooks, and in particular the sequence of activities presented in individual textbook sections. We believe that sequencing is an essential aspect of the rhetorical structure of mathematics texts, just as sequencing is important in other complex reading activities (e.g., Perfetti, Britt, & Georgi, 1995; Wineburg, 1991). Our specific focus here is on the organization of categories of problem-solving activities in the written exercises portion of each textbook section.

Because of the strong relation between classroom curriculum and textbook content reported in the literature, our analysis was guided by teachers' beliefs about student learning. Specifically, we wished to determine whether problem-solving activities presented in textbooks are organized in accordance with the symbol precedence view of algebra development. We hypothesized that textbooks will tend to reflect the symbol precedence view exhibited by high school mathematics teachers. This is the belief that symbolic problems are easier for students to solve than verbally presented problems and should therefore be presented first. Greeno, Collins, and Resnick (1996) suggested a similar claim in their review of behaviorist and empiricist theories of learning.

> Typical sequences of instruction begin with training in a procedure, facts, or vocabulary in a simplified context, followed by presentation of the material in somewhat more complicated settings. Standard mathematics textbooks are examples, in that procedures for calculating are presented and practiced, followed by word problems. (p. 33)

If the claim is true and symbolic reasoning is thought to developmentally precede verbal reasoning, then we would expect to find that symbolically presented problems such as equations should be presented before verbal reasoning tasks such as story problems within the same textbook sections. We term this the *symbol precedence hypothesis.*

As a second question, we asked whether the tendency for the symbol precedence organization varies between prealgebra- and algebra-level textbooks. Based on the documented differences in middle school and high school teachers' beliefs (Nathan & Koedinger, 2000a), we hypothesized that prealgebra-level textbooks will show the symbol precedence organization less strongly than algebra textbooks produced by the same publishers.

Finally, we asked whether the prevalence of symbol precedence organization in textbooks has changed over time. To address this question, we examined differences between textbooks that preceded and followed reform-oriented reports such as the National Council of Teachers of Mathematics (NCTM) *Curriculum Standards* (1989) and *Everybody Counts* (Nation Research Council [NRC], 1989). In the terminology of current educational reform, mathematics is to be presented as a multifaceted tool for solving problems and reasoning, as well as a medium for communication (NCTM, 1989). Contemporary mathematics instruction seeks to instill in students a conceptual understanding of numbers, symbols, diagrams, and procedures that is robust enough to promote mathematical and scientific learning and reasoning in novel settings. For algebra, this implies a curriculum that encourages uses of a variety of representational forms beyond just the symbolic and emphasizes verbal reasoning about unknown quantities, generalized quantitative relations and procedures, and modeling of situations. In light of these recent mathematical reform ideas (e.g., NCTM, 1989; NRC, 1989, 1990; Ohio Mathematics Education Leadership Council, 1989), we hypothesized that the symbol precedence organization will be less common among the textbooks published after 1990.

METHOD

Materials

The textbooks used in this study were chosen because they were used by the teachers who participated in Nathan and Koedinger's (2000a) algebra problem ranking study. This set of textbooks was initially determined by examining the district textbook adoptions list and cross-referencing that list with teachers (n = 25) who attended a university workshop on mathematics instruction. The final corpus of textbooks contained 10 volumes that spanned a decade (1986–1995) and that were produced by five publishing companies: Glencoe; Harcourt Brace Jovanovich, Inc. (HBJ); Houghton Mifflin (HM); McDougal Littell (ML); and The University of Chicago School Mathematics Project (UCSMP). As shown in Table 1, two textbooks were chosen from each publisher: a prealgebra edition, designed for the middle grades, and an algebra edition, designed for the early high school grades.

The sample contained a total of 1,083 sections across the 10 textbooks. There was a range of 77 to 149 sections for each textbook and a range of 3 to 5 pages per

TABLE 1
Titles and Publication Information for Each Textbook

Publisher	*Prealgebra Text*	*Algebra Text*
Harcourt Brace Jovanovich	*Pre-Algebra: Skills/Problem Solving/Applications* (Brumfiel, Golden, & Heins, 1986)	*Introductory Algebra 1* (Jacobs, 1988)
Houghton/Mifflin	*Pre-Algebra: An Accelerated Course* (Dolciani, Sorgenfrey, & Graham, 1988)	*Algebra: Structure and Method, Book 1* (Brown, Dolciani, Sorgenfrey, & Cole, 1990)
UCSMP	*University of Chicago School Mathematics Project: Transition Mathematics* (Usiskin et al., 1995).	*University of Chicago School Mathematics Project: Algebra* (McConnell et al., 1990)
McDougal, Littell	*Gateways to Algebra and Geometry: An Integrated Approach* (Benson et al., 1994)	*Algebra I: An Integrated Approach* (Benson et al., 1991)
Glencoe	*Merrill Pre-Algebra: A Transition to Algebra* (Price, Rath, & Leschensky, 1992)	*Merrill Algebra 1: Applications and Connections* (Foster, Winters, Gell, Rath, & Gordon, 1995)

Note. UCSMP = University of Chicago School Mathematics Project.

section. Prealgebra textbooks included topics such as arithmetic, introducing the vocabulary of algebra, integers, solving one-step equations, factors and fractions, operations on rational numbers, solving and graphing equations and inequalities, proportion and percentage, probability and statistics, introductory analytic geometry, area and volume, and polynomials. Algebra textbooks additionally covered the following areas: symbol manipulation, two-step problem solving, general problem-solving strategies and reasoning skills, using technology (e.g., calculators and computers) for computation, data analysis, graphing systems of linear equations, discrete mathematics, geometry, trigonometry, and applications of algebra in authentic settings.

Procedure

We analyzed the organizational structure of the prealgebra and algebra textbooks to address our three research questions. Textbooks were coded for the frequencies of certain patterns of presentation by examining the "written exercises" portions of each section of a chapter. Written exercises contain the problem-solving activities for the new topics introduced in each section. They are often assigned to students as homework or classroom seatwork, and teachers often draw from them when constructing lessons and course examinations. Pattern codes were determined by looking at the pattern of problems within each section. Sections devoted to the review of prior content were excluded from the analyses so the pattern that emerged from the coding process represented only those curricular items that were newly introduced.

Pattern codes. Codes were assigned in the following manner. First, the presentation format of the first written exercise in each section was coded as either symbolic (e.g., an algebraic equation) or verbal (e.g., an algebra story problem). If a written exercise was presented only in Arabic numbers or algebraic notation, it was coded as *symbolic.* If a written exercise contained words and phrases, it was coded as *verbal.* Table 2 shows the problem codes that were administered along with example problems taken from the textbooks under study.

A written exercise that addressed an activity outside of solving an arithmetic or algebraic problem was coded as a symbolic or verbal "other." Examples of these problems are naming arithmetic operations, expressing numbers in scientific notation, questions about the history of mathematics, reading graphs, using a number line, doing mathematical operations on polynomials without solving for the unknown, and identifying polygons.

After the first written exercise of a section was coded (symbolic or verbal), the remaining exercises were coded to determine the pattern code for the entire section. If the first written exercise for a section was coded as symbolic, then the section was coded as symbol-to-symbol (SS) only if all of the remaining problems in that section were also symbolic. If a verbal problem followed the initial symbolic exercise, the section was coded as symbol-to-verbal (SV). The same method was used for sections that began with verbal problems. Four mutually exclusive pattern designations were possible for the sequence of problems in a section:

- Symbolic presentation only (SS).
- Symbolic presentation followed by verbal presentation (SV).

TABLE 2
Examples of Codes Issued and Representative Problems

Code Issued	*Example Problems (With Textbook Source)*
Symbolic	56 – 49 = ? (Brumfiel et al., 1986)
	$X - 7 = 13$ (Brown et al., 1990)
Verbal	The product of a number and 15 is 105. Find the number. (Price et al., 1992)
	The price of a 4-fluid ounce bottle of perfume is $29.95. Find the unit price of the perfume to the nearest tenth of a cent. (Jacobs, 1988)
Symbolic other	Graph the ordered pair (4, 2) on a coordinate plane. (Brumfiel et al., 1986)
Verbal other	$17 = n - 4$ is called an _______. (Foster et al., 1995)

- Verbal presentation only (verbal-to-verbal, or VV).
- Verbal presentation followed by symbolic presentation (verbal-to-symbol, or VS).

All sections were initially coded by a single coder. A second person recoded 20% of the sections and achieved complete agreement with the first set of codes.

Textbook scoring. Based on the coding of individual textbook sections, we derived two variables that characterized textbooks as a whole. These were then used in the subsequent statistical analyses. To test the symbol precedence hypothesis, a measure of the SV pattern preference displayed in each textbook was created. This SV preference measure was defined as the number of SV sections divided by the total number of sections that included both verbal and symbolic problems:

$$\text{SV\_Preference} = \text{SV} / (\text{VS} + \text{SV}) \qquad \text{Equation 1}$$

A measure of SS pattern preference was calculated for each textbook based on the overall proportion of SS patterns among all sections:

$$\text{SS\_Preference} = \text{SS} / (\text{VS} + \text{SV} + \text{VV} + \text{SS}) \qquad \text{Equation 2}$$

Note that Equation 1 depends on sections with both SV and VS patterns, whereas Equation 2 considers all of the coded textbook sections. Because of this, the two measures are mathematically independent from each other, and each can vary from 0 to 1, irrespective of the behavior of the other variable.

RESULTS

The data analyses were driven by three hypotheses. The first hypothesis is that problem sequencing in the textbooks under investigation adheres to the symbol precedence view. The second hypothesis is that high school level algebra textbooks exhibit the symbol precedence pattern more strongly than prealgebra textbooks do. The third hypothesis is that prereform textbooks in the sample exhibit the symbol precedence pattern more than those published after the release of national reform mandates in mathematics education. Table 3 shows the frequency of each of the four presentation patterns for each textbook, along with publication year and number of sections.

Test of the Symbol Precedence Hypothesis

The symbol precedence hypothesis holds that mathematics problem-solving activities will first be presented in a symbolic form followed by activities in verbal form. Presumably, this follows the belief that at first students reason best with for-

TABLE 3
Summary of Textbook Data Including Publication Year, Percentages, and Frequencies of Pattern Codes

					Symbol-First Patterns				Verbal-First Patterns				SV Measure		SS Measure
					SS		SV		VS		VV				
Textbook	Year	Curriculum	Age	No. Sections	%	Freq	%	Freq	%	Freq	%	Freq	SV/(SV+VS)	χ^2 (SV)	SS/(SS + SV + VS + VV)
HBJ prealgebra	1986	prealgebra	older	107	35	37	35	37	7	8	23	25	.82	18.69*	.35
HBJ algebra	1988	algebra	older	112	28	31	39	44	5	6	28	31	.88	28.88*	.28
Houghton prealgebra	1988	prealgebra	older	98	20	20	37	36	14	14	29	28	.72	9.68*	.20
Houghton algebra	1990	algebra	older	109	23	25	51	56	6	6	20	22	.90	40.32*	.23
UCSMP algebra	1990	algebra	older	113	8	9	38	43	34	38	20	23	.53	0.31	.08
McDougal algebra	1991	algebra	newer	84	0	0	69	58	24	20	7	6	.74	18.51*	0
Glencoe prealgebra	1992	prealgebra	newer	149	0	0	54	80	9	13	38	56	.86	48.27*	0
McDougal prealgebra	1994	prealgebra	newer	77	0	0	60	46	34	26	6	5	.64	5.56*	0
Glencoe algebra	1995	algebra	newer	121	0	0	54	65	12	15	34	41	.81	31.25*	0
UCSMP prealgebra	1995	prealgebra	newer	113	1	1	25	28	37	42	37	42	.40	2.80	.01
Total				1,083	11	123	45	492	17.5	189	25.8	279	.73		.12

Note. SV = symbol-to-verbal; SS = symbol-to-symbol; VS = verbal-to-symbol; VV = verbal-to-verbal; freq = frequency; HBJ = Harcourt Brace Jovanovich; UCSMP = University of Chicago School Mathematics Project.

*$p < .05$

mal representations and later advance to verbally presented tasks. Operationally, this hypothesis would be supported if SV patterns were found to occur more frequently among sections with verbal and symbol problems than would be expected by chance.

As a test of the symbol precedence hypothesis, the measure of SV preference was compared to its expected value of 0.5 in a one-group *t* test (Glass & Hopkins, 1996). Table 3 displays the SV preference measure (Equation 1) for each of the 10 books. The average measure of SV preference was 0.73 ($SE = .052$), which is significantly greater than the expected value of 0.5, $t(9) = 4.6$, $p < .001$. As shown in Figure 1, this pattern also held for the five prealgebra textbooks, $t(4) = 2.30$, $p < .05$, and the five algebra textbooks, $t(4) = 4.09$, $p < .01$, considered separately.

This result shows that across the corpus there is a preponderance of symbol precedence patterns. As a further test of the primacy of the SV pattern, we looked at this tendency within each individual textbook. Identifying this pattern both across and within textbooks would help to characterize the level of acceptance of the symbol precedence view within our sample.

For this analysis, each textbook was analyzed using a one-group chi-square test (Glass & Hopkins, 1996). This analysis allowed us to determine specifically which books use the SV pattern more often than the VS pattern, where each pattern has an expected frequency of occurrence of 50%. As the chi-square column of Table 3 shows, 8 out of the 10 textbooks showed a reliable preference for SV patterns in structuring their written exercises. Only the UCSMP textbooks deviate from this

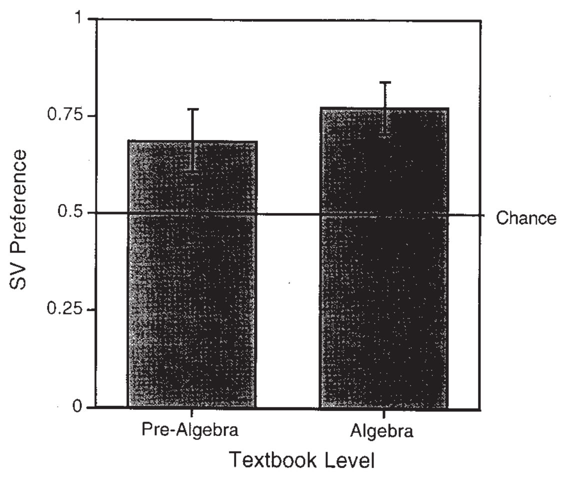

FIGURE 1 The symbol-to-verbal (SV) preference measure (Equation 1) for prealgebra and algebra books compared against chance. (Note that the error bars are for the one-sample comparisons of each type of textbook as compared to chance, and not the paired comparison of prealgebra vs. algebra textbooks.)

tendency. These textbooks differ from the others in that they are produced by an academic institution rather than a commercial textbook publisher and so may have different demands placed on their organization.

Taken together, these results establish the strong preferences of the publishers of textbooks in the sample to introduce algebraic activities for new learners in a symbolic form and then move learners on to verbal problems as applications and extensions. The curricular approach exhibited here exactly parallels the one presented by high school teachers when they inaccurately judged that their algebra students would solve symbolic problems more readily than verbal problems.

As reported, middle school teachers did not support the symbol precedence view as strongly, however, and they were more likely than high school teachers to rank algebra story problems as easier than symbolically presented algebra equations (Nathan & Koedinger, 2000a). Based on this difference, we next explored a second hypothesis that middle school level textbooks would display the symbol precedence pattern less strongly than algebra textbooks.

Comparing prealgebra and algebra textbooks. The second analysis tests for differences in SV preference between prealgebra and algebra textbooks. Because pairs of prealgebra and algebra textbooks in this corpus are matched by publisher, a paired t test (Glass & Hopkins, 1996) was used. Because the hypothesis is a directional one, namely that algebra-level textbooks will show a higher frequency of SV patterns, a one-tailed test was used.

As expected, the SV preference measure was higher in algebra textbooks than in prealgebra textbooks ($M = .77$ vs. .69, respectively; M of paired differences = .086), $t(4) = 2.2$, $p < .05$ (also see Figure 1). Because high school level textbooks are more likely to exhibit the symbol precedence pattern than are middle school textbooks, it is also the case that high school teachers are more likely to work with curricular materials on a daily basis that strongly exhibit the symbol precedence view. Previous research (Nathan & Koedinger, 2000a) established that high school teachers are more likely than their middle school counterparts to assume that facility with formal symbolic representations and procedures precedes the development of students' verbal reasoning abilities. Our finding parallels that result and suggests that the views of teachers and textbook authors are linked, and that the nature of this relationship deserves further study. We return to this issue in the Discussion section.

Comparing prereform and postreform textbooks. In the final set of analyses we tested the hypothesis that the symbol precedence pattern is more common in older textbooks that came out during or before 1990. This prediction stems from the expectation that the reform documents of 1989 and 1990 reconceptualize mathematics as a verbal activity and state that instruction should draw on many alternative representations including verbal descriptions and situations, pictures, table, and graphs, as well as equations (e.g., NCTM, 1989; NRC, 1989, 1990). For this analysis, SV preference (Equation 1) is compared for “older” (1990 and earlier) and “newer” textbooks (after 1990), based on their year of publication[1] (see Table 3). An analysis of variance (ANOVA) did not reach significance, $F(1, 8) = 1.85$, *ns,* although there was a trend suggesting that older algebra textbooks tended to exhibit the SV pattern more frequently ($M = .77$) than newer ones ($M = .69$).

We next examined the relative presence of symbol-only patterns over time using SS preference measure (Equation 2). An ANOVA revealed that use of symbol-only sections changed considerably over time, $F(1, 8) = 26.1$, $MSE = .005$, $p < .001$. The means confirm that textbooks published after 1990 were far less likely to use SS patterns in their written exercises ($M = .002$) than were textbooks published before 1990 ($M = .23$). It seems that algebra textbook developers did not strictly follow the word of education reformers’ call for greater emphasis on verbal reasoning in algebra. However, publishers did seem to be influenced by the reform charge, and apparently interpreted this as a need to reduce the dependence on symbol-only (SS) sections. As is evident in the data of Table 3, SS patterns essentially went to zero about 1 year after the major standards documents were made public. Even so, newer textbooks still showed a strong reliance on early exposure to symbolic forms of reasoning and representation (SV patterns).

DISCUSSION

This investigation illustrates a theoretically motivated analysis of the rhetorical structure of a corpus of mathematics textbooks. Prior studies have revealed that high school teachers widely believed that competence with symbols precedes verbal reasoning. We set out to see if textbook structure likewise was consistent with a symbol precedence view of algebra development. Even with a sample size of only 10 textbooks, the statistical analyses reached significance, indicating a large and robust effect for the organization of problem-solving activities. New problem-solving activities tended to be introduced first in symbolic formats, such as algebraic equations (the symbol precedence hypothesis). As the new material was elaborated or applied, the activities tended to shift to a verbal format such as an algebra story problem. Rarely were activities introduced with verbally presented problems and developed toward symbolic reasoning, even though there is evidence that suggests students’ verbal reasoning abilities precede their symbolic skills (e.g., Case & Okamoto, 1996; Kalchman & Case, 1998; Nathan & Koedinger, 2000b). Algebra textbooks

[1]It is worth noting that there is some sampling bias in aggregating the two chronological sets of older and newer textbooks. Through happenstance, the newer set had more prealgebra textbooks, whereas the older set had more algebra sections. However, if the prealgebra influence was dominant, then as a first-order prediction, one would expect newer books to have fewer SV sections than older texts, but that difference was not statistically reliable.

were more disposed toward this pattern than prealgebra textbooks (Hypothesis 2), as suggested by the differences among high school and middle school teachers' beliefs. Prereform textbooks were slightly, but not reliably, more inclined than newer ones to show the symbol precedence pattern (Hypothesis 3). However, newer textbooks were far more likely than older editions to abandon symbol-only patterns, suggesting that the reform mandates had some influence on textbook design. We next interpret these results in light of current research on learning and teaching from texts and the role beliefs play in shaping teaching practices.

Research on Learning From Instructional Texts and the Role of Rhetorical Structure

A great deal of past research on learning from text has focused on text coherence (e.g., Beck et al., 1991; Britton & Gulgoz, 1991; Kintsch, 1998; McNamara & Kintsch, 1996). Research has also shown that these coherence effects depend strongly on the reader's prior knowledge and the match between that knowledge base and the coherence structure of the text (Beck et al., 1991; Britton & Gulgoz, 1991; Kintsch, 1990; Mannes & Kintsch, 1987; McNamara et al., 1996). In this study, we suggest that the organizational sequence of problem-solving activities—the rhetorical structure—of a textbook is also an important consideration for analyzing learning materials and determining their compatibility with learners. Methodologically, studies of text content and structure are complementary. Their use, both separately and in combination, should further advance our understanding of discourse processes and comprehension.

Our investigation shows that problem-solving activities within algebra textbooks are sequenced in accordance with the symbol precedence view of mathematical development. This organization is embedded within the sequencing of problems, and thus would not be immediately apparent from a content-level analysis. However, current findings from content analyses of texts are also relevant here. They have indicated that the match between a text and learner is essential for predicting success in learning (e.g., McNamara et al., 1996). Thus, as a first-order approximation, we surmise that textbooks organized around the principle of symbol precedence are not optimally tailored to the many students who appear to follow a verbal precedence trajectory of algebra development. Although much more research needs to be conducted to fine tune our understanding of the compatibility between learners and textbooks, establishing parallel structures between developmental processes and curricular organization appears to be a good starting point.

Rhetorical structure seems to be underutilized as a component to guide the design of mathematical textbooks, but it shows promise here and elsewhere. For example, Koedinger and Anderson (1998) compared the curricular sequence used by a popular algebra textbook (i.e., Forester, 1984) to an alternative sequence of their own design. The approach used in the Forester textbook assumed that students would best learn to generate algebraic expressions from patterns and relations through a deductive process. As Forester (1984) described, there are exercises in which "students are forced to write an expression representing a variable quantity. Then they evaluate the expression for several values of the variable, and write and solve equations involving the expression" (p. xi). The intention is for the student to create a symbolic representation of the varying quantities that fit a given situation, and then instantiate that general equation for instances that satisfy specific input or output constraints. Following this rationale for many of the written exercises, the book instructs students in a step-by-step manner to first write a mathematical expression in terms of the unknown of a story problem; second, compute the output (Y) value for two different input (X) values; and finally, compute an input (X) value of the expression evaluated to a specific output (Y) value.

Koedinger and Anderson (1998) compared learning with Forester's (1984) curricular sequence to an inductive approach of their own design. Here, the learner induces the formal expression (Forester's Step 1) at the conclusion of a series of specific input–output relations. Koedinger and Anderson (1998) based their instructional approach on general learning principles:

> [E]ven for mathematical experts in a decidedly *deductive* domain, [such as] geometry theorem proving, problem-solving knowledge has a fundamentally *inductive* character. While much of mathematical reasoning in its externalized written form is the deductive manipulation of symbols, *the underlying cognitive processes that support effective reasoning* draw on indications from prior perceptual experience (cf. Cheng & Holyoak, 1985). If expert mathematical knowledge is fundamentally organized as inductive abstractions, not deductive rules, then perhaps instruction that supports and encourages such inductive reasoning would more effectively lead to expertise. (p. 164, italics added)

In an instructional experiment, students ($n = 30$) who learned using Koedinger and Anderson's (1998) inductive support procedure gained significantly more than students who used Forester's (1984) deductive procedure (26% gains vs. 5% gains, respectively). Furthermore, students who received inductive support were over 40% faster at solving the most difficult set of symbolization problems ($M = 28$ s vs. 48 s) than those in the deductive condition. This study of the impact of problem organization shows how intuitively appealing certain beliefs about learning may be and how important it is to test these beliefs in learning settings.

Textbooks and Influences on Teachers' Beliefs

To understand teaching and learning, it is essential to understand beliefs (Garner & Alexander, 1994). Indeed, understanding the beliefs held by educators is central to the improvement of instruction (Fenstermacher, 1979). However, the nature, impact, and origins of educators' beliefs are still poorly understood. Beliefs are described as "mental constructions of experience," which are taken as true regardless of the actual evidence (Sigel, 1985, p. 351; also see Thompson, 1992). It is generally accepted that such beliefs arise through enculturation and the social construction of experiential knowledge (Calderhead & Robson, 1989; Pajares, 1992). To Dewey (1933), beliefs accounted for "the matters that we now accept as certainly true, as knowledge, but which nevertheless may be questioned in the future" (p. 6). Although beliefs may be inaccurate reflections of the world, they can be an invaluable aid for the teacher tackling ill-structured tasks, such as curricular planning and classroom instruction. Once formed, beliefs guide us gracefully through areas of uncertainty, and help us to interpret novel actions, events, and information.

Given the important role that textbooks play in teachers' planning and instructional decisions, it should not be surprising that they are considered by some to be a major influence on teaching and teachers' views of learning and instruction. Flanders (1994) reported an empirical investigation of the relations between intended, implemented, and tested curricula of eighth-grade mathematics classes. This study showed that teachers' expectations for student success were highest for test items (taken from the Second International Mathematics Study; Garden, 1987) covered by the classroom textbooks, even though students also practiced solving items not covered in the textbooks.

Teachers' choices for topic sequencing also rely strongly on textbook organization. Borko and her colleagues, among others (e.g., Borko & Livingston, 1989; Borko & Shavelson, 1990; Cooney, 1985; Johnsen, 1993; Ornstein, 1994), found mathematics textbooks to be a primary resource for lesson planning by both expert and novice high school teachers. The influence of textbooks on teacher instruction and subsequent student achievement is even considered as a major factor in explaining international differences among American and Asian students (e.g., Brenner et al., 1999; Mayer, Sims, & Tajika, 1995; Stevenson et al., 1990).

Views of mathematical instruction can linger for long periods of time. In our study, verbal problems were found to be rare as a means to introduce new topics. Similar findings were documented in the first yearbook of the NCTM. In it, Smith (1926) described a popular mathematics textbook sampled from the previous quarter century. Smith found that "out of nearly 1600 exercises in the first 147 pages only 111 were verbal problems … [and] another text of that period gave about 1800 exercises in the first 128 pages; of these only 109 were of the verbal variety" (p. 21). Overman (1923), in his writings about algebra learning and instruction earlier this century, stated that "the learning of the algebraic language is, without doubt, the greatest difficulty presented to the beginner by the subject. The success of our textbook writers … has not been very great" (p. 216). His advice was to present the symbolic form of equations prior to verbal ones, because, in his opinion, "in algebra it is easier to translate from algebra to English than from English into algebra, and such practice should be given first as a preparation for the more difficult work to follow " (Overman, 1923, p. 217). When we encounter well-ingrained beliefs, we should not expect them to change easily, even in the face of overwhelming evidence (e.g., Dole & Sinatra, 1994).

This study did not set out to establish a casual link between teachers' views and textbook structure. Indeed, we doubt that such a link could ever be established empirically given the myriad influences on both. However, there is reason to reflect on the parallels and even speculate a bit on how such a link might be manifested.

Educators' beliefs about learning and instruction are likely to come from many sources, such as teacher education and professional development programs, curricular guidelines and instructional materials, and even from one's early student experiences (e.g., Ball, 1988). Views of learning mathematics may even stem from views of learning to read and the early role of alphabetic and phonetic decoding. Additionally, textbooks may serve as a source of enculturation for high school teachers that shapes their beliefs and educational practices. It is possible that by working so closely with textbook curricula and their sequencing choices, teachers come to internalize these views and ultimately believe that symbolic reasoning is fundamentally easier for students than verbal reasoning and that verbal reasoning abilities develop after, and are dependent on, symbolic reasoning skills.

This raises the specter about the role that textbooks play in influencing curricula, enculturating teachers, and perpetuating inaccuracies about students' mathematical reasoning and development. The concern, of course, is not that textbooks influence teachers' views or school curricula. Rather, it is that textbooks may shape them in ways that are counterproductive for student learning. As this study suggests, textbooks can be designed with questionable assumptions about student learning. Without analyses of textbook content and structure, these assumptions can go unchecked and can be implemented throughout the educational system, to the potential detriment of students and teachers.

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