COGNITION AND INSTRUCTION, *21*(2), 175–207

A Study of Whole Classroom Mathematical Discourse and Teacher Change

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This article presents a comparison of the first 2 years of an experienced middle school mathematics teacher's efforts to change her classroom practice as a result of an intervening professional development program. The teacher's intention was for her teaching to better reflect her vision of reform-based mathematics instruction. We compared events from the 1st and 2nd year's whole class discussions within a multilevel framework that considered the flow of information and the nature of peer- and teacher-directed scaffolding. Discourse analyses of classroom videos served both as an analytic tool for our study of whole classroom interactions, as well as a resource for promoting discussion and reflection during professional development meetings. The results show that there was little change in the teacher's specific goals and beliefs in light of a self-evaluation of her Year 1 practices, but substantial changes in how she set out to enact those goals. In Year 2, the teacher maintained a central, social scaffolding role, but removed herself as the analytic center to invite greater student participation. Consequently, student-led discussion increased manifold, but lacked the mathematical precision offered previously by the teacher. The analyses lead to insights about how classroom interactions can be shaped by a teacher's beliefs and interpretations of educational reform recommendations.

Instruction designed to promote meaningful learning in mathematics has been receiving increased attention due to recent reform initiatives in mathematics education (National Council of Teachers of Mathematics [NCTM], 1989, 1991, 1995,

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2000). Many proponents of educational reform view the process of coming to know mathematics as a social endeavor taking place during the interactions within a classroom community—interactions which provide an opportunity for students to learn through thinking, talking, agreeing, and disagreeing about mathematics (e.g., Ball, 1993; Bauersfeld, 1995; Cobb, Yackel, & Wood, 1993; Lampert, 1990). Accordingly, the nature of classroom discourse plays a critical role in promoting the kind of learning and thinking that is valued (Ball, 1991). Researchers have identified certain principles to characterize desirable forms of classroom discourse. Scardamalia, Bereiter, McLean, Swallow, and Woodruff (1989), for example, described as "constructive" the discourse that emerges in fourth- and fifth-grade science classes that use an electronic bulletin board. These students collaboratively build a body of knowledge about scientific phenomena, and support and defend publicly made claims—forming a kind of "community of scholarship" in the classroom (Bruer, 1993, p. 252). Similarly, Lampert (1989) intended for new knowledge in her mathematics classroom to be "constructed as a joint venture in the class rather than as a communication from teacher to student" (p. 257). Ball (1993) believed that mathematics education must use classroom discourse to develop in students the idea of "doing" mathematics, of conjecturing, scrutinizing, and defending one's ideas, as well as learning about it.

In the NCTM *Professional Standards for Teaching Mathematics* (1991), students' participation in the discourse-based classroom is central as they are called on to initiate lines of inquiry and challenge the ideas presented by peers, the teacher, and textbooks. The *Professional Standards* describe the teacher's role in initiating and guiding interactions of this nature as one of eliciting and engaging children's thinking; listening carefully; monitoring classroom conversations and deciding when to step in and when to step aside. This "guide on the side" role is far from a passive one, yet it is intended to give students the space they need to direct discussion and allow students the opportunity to meaningfully explore their own mathematical ideas, articulate them, and to explore the thinking offered by others. Rather than primarily explaining and demonstrating, the teacher is asked to craft instruction in a nontraditional way, at times leading from behind, at times stepping in as a mathematical authority, and at other times carefully guiding the discussion and activities and seeding ideas. Enacting classroom practices that support discourse-based mathematical activity, however, poses difficult challenges for many teachers, as such practices often bear little resemblance to teachers' current practices, or to the practices in which teachers participated as students themselves. Consequently, as mathematics teachers begin to respond to the challenges posed by reform, investigations into their interpretations of reform-based instruction are needed to fully understand how educational reform will be implemented, and how it will achieve its goals. It is particularly important to examine the interpretations of reform instruction and guidelines made by "[ordinary] teachers who teach in ordinary schools, under ordinary conditions," for these insights are valuable in the refinement of widespread reform, and the design of future professional and curricular standards (Sowder & Schappelle, 1995, p. 280).

The central focus of this article is one teacher's interpretation of reform-based instruction and its manifestation in her classroom practice. In particular, we analyze the nature of whole group discourse that occurred in the classroom of a sixth-grade mathematics teacher teaching in ordinary circumstances. We examined changes in the nature of the discourse over 2 successive years during the teacher's efforts to change her classroom practice so that it better reflected her vision of reform-based mathematics instruction. In particular, we investigated how the teacher's beliefs about learning and instruction, and her interpretations of mathematics education reform, influenced her teaching practices and the nature of classroom interactions. In so doing, we see this article contributing to the body of research literature on teacher cases and the influence of teachers' beliefs on instruction (e.g., Ball, 1993; Eisenhart et al., 1993; Putnam, Heaton, Prawat, & Remillard, 1992; Schifter & Simon, 1992; Shulman, 1987). We also set out to identify characteristics of the classroom that support mathematical discourse to better articulate the conditions that promote one major aspect of reform-based mathematics instruction.

A critical methodological concern in this investigation is how to describe the nature of classroom discourse in an interpretable manner and still capture the complexities inherent in large group interactions. We present a multilevel analysis that allows us to examine several levels of interaction that occur simultaneously during whole class instruction and discussions, and to consider the teacher's beliefs, instructional actions, and the changes in her thinking and practices that shape the classroom discourse.

THEORETICAL PERSPECTIVE

Our multiyear study[1] of one sixth-grade (middle school) mathematics teacher suggests that three major forces are at work in the classroom and in the decision-making space of the teacher as she plans future lessons and reflects on prior classroom events. First, the teacher and the students in the class provide varying degrees of analytic and social scaffolding for one another (Williams & Baxter, 1996). Simultaneously, the members of the class regulate the flow of information

[1]This study is part of a larger multiyear project exploring the development of middle school students' mathematical ideas, and teachers' beliefs about students' knowledge and reasoning processes (Koedinger & Nathan, in press; Koedinger, Nathan, & Tabachneck, 1995; Nathan & Koedinger, 2000a, 2000b, Tabachneck, Koedinger, & Nathan, 1995).

(Hatano & Inagaki, 1991) between the teacher and students, and among the students themselves. Finally, we acknowledge the pivotal role that the teacher's goals and beliefs play in shaping her classroom practices (e.g., Borko & Putnam, 1996).

Analytic and Social Scaffolding in the Classroom

Williams and Baxter (1996), building upon the work of Edwards and Mercer (1987), described two kinds of instructional scaffolding: analytic and social. Analytic scaffolding is "the scaffolding of mathematical ideas for students" (Williams & Baxter, 1996, p. 24) and is intended to support students' learning of mathematical content during classroom interactions. Examples of analytic scaffolding include a teacher redescribing student contributions to a discussion in more precise mathematical terms or a teacher highlighting particular aspects of student contributions in light of their potential utility for introducing more advanced mathematical ideas. Social scaffolding is "the scaffolding of norms for social behavior and expectations regarding discourse" (p. 24) and is intended to facilitate students' participation in classroom interactions. Examples of social scaffolding include asking students to provide explanations for solutions to problems or eliciting contributions to whole class conversations from all students. It is important to note that scaffolding of this latter nature is not particular to mathematics, that is, such scaffolding could apply to the instruction of any subject matter. Accordingly, we characterize scaffolding grounded in normative aspects of mathematical discussions (e.g., what counts as an acceptable mathematical explanation) as analytic scaffolding (cf. Yackel & Cobb, 1996).

Each type of scaffolding plays an integral part in the establishment of instructional practices consonant with recent reform initiatives; yet, achieving an appropriate balance is often difficult for teachers facing change. Ball (1996) underscored this difficulty by acknowledging the conflicting demands of teaching the curricular content with integrity while still honoring children's ideas and ways of thinking. McClain and Cobb (2001) observed these conflicting demands firsthand; the classroom teacher with whom they worked felt it was important during whole class interactions to accept all student contributions equally (an example of social scaffolding) and did not attempt to indicate those contributions that were particularly valued mathematically (an example of analytic scaffolding). Consequently, student contributions often did not advance the mathematical agenda of the class. Williams and Baxter (1996) similarly noted the tensions that arise in this reform-based climate as teachers simultaneously work to "inculcate knowledge while apparently eliciting it" (p. 24). For example, a teacher may explicitly elicit knowledge from a student even as she provides subtle feedback regarding the mathematical relevance and accuracy of the student's response. Ideally, balance between the social and analytic demands is reached when students' own social constructions of mathematical ideas are also connected to the ideas and conventions of the mathematical community.

The Flow of Information in the Classroom

In addition to the types of scaffolding that occur in classrooms, Hatano and Inagaki (1991) considered differences in the relative hierarchical status of the participants in an interaction. They discuss how the flow of information during interactions can occur vertically, from a more knowledgeable other to a learner, or pass horizontally among peers at a comparable level of expertise. Attention to horizontal information flow has increased a great deal as educators have embraced the important role that socially constructed knowledge plays in learning (Bauersfeld, 1995; Cobb et al., 1993; Resnick, 1987; Vygotsky, 1978). Hatano and Inagaki (1991) provided support for the use of classroom discourse as a medium for learning, arguing that in a vertical interaction, the "less mature member" is often less motivated to expend the effort to construct knowledge because of the presumption that the more knowledgeable member can easily reproduce or construct that knowledge. Their findings echo those of Williams and Baxter (1996) that show the tendency for the student in a scaffolding interaction to try to provide a response that gives the teacher what she wants to hear. Because of this tendency, and the premium now placed on socially mediated construction of knowledge as a means to promote deep and sustained learning (e.g., Greeno, Collins, & Resnick, 1996; Resnick, 1987), a central charge in contemporary educational reform calls for instructional practices that encourage student-led and student-to-student discourse about curricular content. Although many teachers recognize the potential for such interactions, they also wrestle with how to provide this while ensuring that the ideas presented are commensurate with the mathematical community and the curricular agenda.

Teacher Goals and Beliefs

In addition to general characteristics of the classroom discourse, we consider the ways this particular teacher perceives her role as a mathematics instructor. To understand and inform practice, it is necessary to also understand the teacher's curricular goals and her beliefs about student learning and development as they are crucial to the enactment of curricular goals (Eisenhart, Shrum, Harding, & Cuthbert, 1988; Fenstermacher, 1979; Thompson, 1984, 1992). Thompson (1992) cautioned us, however, to treat teachers' beliefs critically: "Teachers' often unexamined assumptions or beliefs about what children are capable or not capable of learning can render them impervious to matters of children's cognitions" (pp. 142–143). For example, many high school teachers believe that mathematics instruction should emphasize student mastery of symbols and procedures (Nathan & Koedinger, 2000b). The instructional approaches that follow from this view of mathematics, in practice, tend to focus on the mechanics of symbol manipulation, rarely addressing the conceptual underpinnings of those symbols and procedures. Furthermore, teachers' beliefs are not always congruent with their actions in the classroom, as has been documented in a number of studies (e.g., Borko et al., 1992; Cooney, 1985; Raymond, 1997; Thompson, 1992).

The teacher's classroom practices can then be seen, in part, as attempts to strike an appropriate balance between regulating information and providing and monitoring scaffolding, which are governed by her perceptions of the goals of contemporary mathematics education reform as well as her own goals and beliefs (e.g., Borko & Putnam, 1996). Thus, our analyses of scaffolding and information flow in the classroom are necessarily informed by an understanding of the teacher's instructional goals and her beliefs of student thinking and learning.[2] Our approach for analyzing the teacher's curricular and professional goals and beliefs with respect to her practices employs an extended version of the practical argument (PA) method (Fenstermacher, 1994; Richardson, 1994), which is presented in the Method section.

METHOD

Participants

We observed a sixth-grade mathematics teacher in a public middle school over a 2-year period. The school operated as a neighborhood school with a middle school philosophy. The mathematical performance of the students in Years 1 and 2 on the California Achievement Test ranged from the 5th to the 99th percentile. There were five students in the Year 1 class and four students in the Year 2 class who received special education support for physical and cognitive disabilities. A paraprofessional served the classroom once a week to help the teacher meet these students' special needs.

The teacher, Ann (a pseudonym), taught sixth-grade mathematics for 17 years. She was, in many respects, a typical middle school mathematics teacher. Her

[2]Although we recognize the important role teachers' subject matter conceptions play in shaping their classroom practices, the particular focus of this article is on teacher beliefs about student thinking and learning.

training was in elementary education, and at various times (including the duration of this study) she balanced her mathematics preparations with classes in sixth-grade science and language arts. Ann's mathematical education, by several accounts, including her own, was lacking in some major areas. As a result, Ann regularly voiced concerns about her mathematics knowledge during the 2-year study.

Data Sources

Data sources included videotapes of weekly classroom observations, written field note accounts of the classroom events, audiotapes of biweekly debriefing sessions with the classroom teacher, and audiotapes of summer meetings with the teacher and professional development team.

We selected eight 90-min lessons from the 2-year data set to obtain a sampling of typical classroom interactions. Lessons were selected from the 4 months (October, November, February, and March) each year that were most representative of the teachers' normal instructional practice (i.e., time periods with the fewest distractions due to school-year breaks, the start-up and ending of school, and holidays).

Lessons were composed of distinct *episodes* such as warm-up activities, review of previously learned mathematical content, homework review, break time, individual or small-group work, whole class discussion, and so on. This study focuses on episodes with whole group interactions, in particular, on the interactions of students and the teacher as they discussed mathematical content and the relevant social aspects of the class. Other types of interactions (such as class administration) and groupings (small group and individual work) were excluded from the analyses reported in this study. We have chosen to focus exclusively on whole class discussion, as this is a feature of classroom practice that has received extensive attention in recent reform documents (NCTM, 1991, 2000).

The audiotaped data from the Year 1 and Year 2 summer sessions focused on the teacher's efforts to examine her teaching practice. A four-person professional development team composed of the teacher along with mathematics educators and researchers was assembled to support this process. During the summer following the Year 1 classroom observations, we reviewed typical classroom episodes and used an interview protocol (the practical argument method, described later in this section) to coconstruct with the teacher the rationale for her actions. Accounts of the teacher's curricular goals and her beliefs about student learning and mathematics instruction emerged from these sessions. The emphasis during the first summer session was on reviewing videotaped events from the teacher's classroom and comparing them to the goals laid out by the teacher. During the following academic year, we used the teacher's stated beliefs and goals from that first summer to interpret the classroom events of Year 2. Note that it was not the intention of the team to try to change Ann's beliefs or goals. Instead, the justifications for Ann's classroom actions, as elicited by the interview protocol, were used by the teacher and the team in subsequent sessions to assess the effectiveness of the teacher's actions with respect to her stated goals and beliefs.

Our framework for analyzing teachers' curricular and professional goals and practices began with an interview protocol termed the *practical argument* method as articulated by Fenstermacher (1994), Richardson (1994), and their colleagues. The PA method is intended to produce the rationale for piecing together the justifications and supporting ideas for taking some kind of action, though its governing rules differ from those of formal logical argumentation (Vasquez-Levy, 1998). As Fenstermacher (1994) stated, "Practical reasoning…is the thinking we do about our actions. When we are engaged in practical reasoning, we are connecting the objectives of our actions (what we are trying to accomplish) with the ways and means of obtaining these objectives" (p. 24). Richardson's (1994) edited volume describes the use and impact of the PA method for staff development for teachers participating in her Reading Instruction Study. In the course of our work, the PA method was focused and extended in several ways (see Nathan, Elliott, Knuth, & French, 1997, for a detailed account). In the first extension, the teacher's own pedagogical goals for her mathematics classroom were placed in the central role of the practical argument structure. Second, the assessment of the teacher's actions with respect to her goals (i.e., her intended actions) served as the critical avenue for reassessment and change of her actions. Third, to address the inherent limitations of the accuracy of retrospective recall (a basis for the PA method), teaching and planning tasks were developed which allowed the teacher to provide concurrent verbal reports and cued retrospective reports of her actions and their justifications (cf. Ericsson & Simon, 1984).

Data Analyses

We chose to perform analyses of the classroom discourse that examined student and teacher interactions from multiple, complementary levels. By having complementary levels, we believe we are able to document the classroom interactions in a more complete manner, and potentially identify more subtle forms of interaction. For example, it is one thing to notice that students are speaking up more in the classroom. But there is also a need to examine the nature of student and teacher speech, and what it may say about student and teacher participation and mathematical practices. During the intervening summers of our classroom observations, teacher professional development activities were designed around some of these classroom discourse analyses. In this way, discourse analyses of classroom videos served both as an analytic tool for our study of whole classroom interactions and as a resource for promoting discussion and reflection during the professional development sessions.

The classroom discourse analyses were derived from a research methodology developed to examine whole classroom interactions from several mutual levels (Nathan, Knuth, & Elliott, 1998). At one level, we examined the moment-to-moment flow of information among the members of the classroom. Data for this analysis came from classroom videotapes and field notes, and included coding the speech acts according to who speaks to whom. The focus of this analysis was on measuring the relative amount of information that flowed from teacher to student (vertically), student to teacher (vertically), or student to student (horizontally). At the next level, we analyzed the content of the various speech acts, looking at the nature of scaffolding, whether the scaffolding was provided by the students or teacher, and whether it served analytic or social purposes. At the global level, we captured general patterns of interaction that occurred over an entire classroom interaction. In addition, we examined how these interaction patterns related to the social and curricular goals promoted by the teacher (as determined using the PA method) in her efforts to comply with her interpretations of the charges of reform-based mathematics instruction.

Coding scheme. A coding scheme was devised to characterize the nature of separate whole-class interactions. The coding scheme, coding table, and coding tabulation forms are shown in Table 1. Two coders reached consensus on all of the codes assigned to the speech acts in the sample.

When analyzing the flow of information, the directionality of each speech act was designated with a From/To label (e.g., *TS* coding speech from the teacher to a student). For the analysis of the nature of scaffolding, each utterance was assigned a code labeling the content of the speech act (e.g., *dm* coded a declaration of a mathematics concept or principle). In addition, each content category was assigned a code for the nature of scaffolding it represented, if any (e.g., *dm* is a form of analytic scaffolding). Some examples showing the application of the coding scheme are presented below:

Teacher: "In a proper fraction, the numerator is smaller than the denominator." *TS dm* (Teacher to a student, declaration of mathematics, analytic scaffolding)

Teacher: "Turn your desks to face the front." *TC mg* (Teacher to class, management statement, social scaffolding)

Teacher: "Anyone else?" *TC mg* (Teacher to class, management statement, social scaffolding)

Student B: "Factor is for multiplying only." *SS dm* (Student to student, declaration of mathematics, analytic scaffolding)

Taken together, the analyses of information flow and scaffolding describe the specific information exchanged, its hierarchical patterns, the nature of the scaffolding in

TABLE 1
Video Codes for Analytic and Social Scaffolding

Code	*Description*	*Scaffolding*
TS	Teacher to student (Vertical flow of information)	
qm	Ask question—Math	A
rm	Response to math question/comment from student	A
qn	Ask question—Nonmath	S
mg	Management—discipline, studenting, admin, homework	S
x	Other	A & S
TC	Teacher to class (Vertical flow of information)	
om	Open invitation—Math question, challenge, yes-no	A
dm	Declaration of math principle, fact, rule	A
mg	Management—discipline, studenting, admin, homework	S
x	Other	A & S
ST	Student to teacher (Vertical flow of information)	
rm	Response to open invitation—Math	A
dm	Declaration of math principle, fact, rule	A
rg	Response to management—Math	A
qm	Ask question—Math	A
qn	Ask question—Other (break time, homework)	S
x	Other	A & S
SS	Student to student/class (Horizontal flow of information)	
pm	Make presentation to the class—Math	A
qs	Ask question to student—Math	A
qn	Ask question to student—Other	S
dm	Declaration of math principle, fact, rule	A
rm	Response to question/comment from student—Math	A
om	Open invitation—Math	A
mg	Management—discipline, studenting	S
x	Other	A & S

Note. A = Analytic scaffolding; S = Social scaffolding.

that information exchange, and the role the teacher and student participants played in the interaction. The frequency of occurrence of each of the codes was tabulated to allow for quantitative comparisons across episodes and between Years 1 and 2 of the study.

Patterns of interaction at the global level. At the global level, we collapsed across the many types of speech acts of an episode to show topologically the patterns of interactions that emerged within the classroom. This allowed us to see how the members of the classroom fit into the larger discourse structure that emerged during whole class mathematics instruction. It also provided a means to abstract away some of the minutiae of the prior levels and qualitatively compare through visual means the nature of classroom discourse with the general goals voiced by the teacher during the interviews (cf. Strom, Kemenya, Lehrer, & Forman, 2001). In constructing the global level, a directed graph identified pathways between discussants, with the whole class identified as a node in the graph. Year-to-year comparisons of select episodes further allowed us to see how reform-driven changes in the teacher's views and practices resulted in changes in the nature of the classroom discourse, and provided a window into the teacher's interpretations of reform-based instruction.

RESULTS AND DISCUSSION

We first present a synopsis of the teacher's goals and beliefs as they were elicited during the first summer session. We then consider, in turn, one episode from each year that is representative of the whole class interactions. For the Year 1 episode, we present a summary of the events, a description of the professional development activities based upon the episode, our multilevel analysis of the classroom interactions, followed by a brief summary, and finally, the teacher's self-evaluation of the year. We follow a similar structure with the Year 2 episode, but emphasize its similarities to and differences from the Year 1 episode. Comparative analyses are presented for the entire corpus of data (eight sample episodes) obtained during Years 1 and 2 to show changes in the classroom resulting from the inquiry into Ann's beliefs, interpretations of reform instruction, and curricular goals. The two example episodes serve to ground the larger sample of data, and help in the interpretation of the comparative analyses that follow. Our focus in these comparative analyses is not on the content of the two mathematics lessons per se, but on the changes in whole classroom interactions that took place.

Connecting Teacher Goals, Beliefs, and Actions

During the summer following the Year 1 classroom observations, the professional development team, as a group, reviewed videotapes and student work from that year. The videotapes and student work provided a grounded discussion about Ann's beliefs and goals, rather than talking about them in the abstract.[3] The revised

[3]It is worth noting at this point that the summer professional development team was careful to not prescribe to Ann some version of proper, reform-based practice. Ann was already familiar with reform-based mathematics education, having attended many district-wide in-services on the subject. Although we do not claim to have been completely uninfluential, as a research team, we focused on her goals and beliefs, and how they influenced her practice. As a professional development team, we let Ann evaluate her practice with respect to her goals, and provided a means—via our multilevel framework for analyzing

version of the PA method (Nathan et al., 1997) was employed to help document the ways that Ann's actions in her class related to her goals and beliefs about teaching and learning.

As the team probed Ann for her goals and beliefs during Year 1, a rich picture emerged of Ann's actions and her interpretations of the goals of mathematics reform. Ann, we learned, held several significant goals for her mathematics instruction, two of which are particularly germane for this study. (For a more detailed analysis of the teacher's goals and beliefs, as well as the process of elicitation and refinement, see Nathan et al., 1997.) First, Ann wanted her students to value all of the opinions expressed in class. This was evident in Ann's statements that students needed to listen to each other's ideas and respect them. She spent a great deal of time each class asking students to contribute their ideas ("Anyone else?"), and, barring severe time limits, would continue doing so until all interested students had the opportunity to speak. Ann also reminded students often to listen when people spoke, occasionally asking other students to restate a speaker's ideas. During the summer professional development program, Ann revealed that listening to others was an idea she remembered from her teacher training and the in-service workshops on mathematics reform. It was also a characteristic that extended into her language arts instruction, and had ultimately been instilled in her from her family when she was growing up (Nathan et al., 1997). Valuing the ideas and opinions of everyone in class was such a central goal for Ann that during the professional development program it came to be referred to as Ann's *general tenet*.

As a second goal, Ann wanted all of her students to learn that mathematics problems could be solved by many methods. Ann was convinced that many students inappropriately looked for a solitary method when solving mathematics problems. She referred to this view as "the way." Ann expressed to the team that any mathematics teaching that emphasized "the way" did not accurately reflect her understanding of current mathematics reform. She was particularly sensitive to this because her own early mathematics education did not acknowledge the range and diversity of solution methods possible in mathematics. Ann felt high-achieving math students were often myopic in their approaches. She also described how "the way" systematically excluded low-achieving students who struggled in mathematics class. Ann stated to the team that since she was not a strong mathematics student herself as a child, this struck a deep chord with her. When she became exposed to more pluralistic views of mathematical problem solving in her (reform-oriented) teacher education program, Ann decided this would be a central goal of her own teaching practice.

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classroom discourse—to reflect and scrutinize the success of those goals. Seeing a desire to revise her actions, we made resources available to her such as classroom videotapes, tapes of other teachers, and readings about teaching and learning.

These two goals of mathematics instruction were complementary for Ann. She felt students should learn to value diverse solution methods presented by either the teacher or one's peers, and that these various methods should be publicly expressed to expand students' mathematical understandings.

In addition to eliciting Ann's curricular goals, we came to see how certain beliefs she held shaped her practices. Ann's beliefs about student learning and development can be summarized by two statements: (a) students learn best from other students, and (b) students learn best through class participation. In the former case, Ann believed that middle school students, as part of their separation from adults, regard the thoughts and opinions of their peers as more important than those of adults. As a teacher and a parent, she had seen students rely on each other more in middle school than in fifth grade, developing stronger social bonds with one another, and defending each other. Ann also found in her own learning experiences that peer explanations were more accessible and helpful to her than those from experts. She projected this view onto her students. In the latter case, Ann expressed the belief that class participation was key to successful learning. She stated that students learned more when they were engaged. Toward this end, she sought out activities that invited students to participate. For example, she solicited a wide range of opinions on mathematical ideas, and asked students to state their agreement and disagreement with ideas presented by her and other classmates. Similar to the relationship between Ann's two instructional goals, these two beliefs about learning were complementary. She believed the involvement of all students in class activities (e.g., engaging activities and student-led presentations and discussions) enabled students to learn from one another, learn more actively, and thus, learn more effectively.

Year 1 Classroom Excerpt: The Painting Problem

In an effort to connect Ann's goals and beliefs with her actions in the classroom, and to demonstrate how her thinking and practices changed over a 2-year period, we present two multilevel analyses of the classroom discourse that occurred in sample episodes drawn from Year 1 and Year 2. The analyses of these examples will focus on the nature of scaffolding and the flow of information as the teacher and students engage in whole class discussions.

In this first excerpt (about 7 min long), Mary, a student, was invited up to the overhead projector at the front of the classroom (a common practice) to present her fraction-based solution to the painting problem.

> Suppose Ms. Jones, an experienced painter, can paint a wall in 3 hours, while rookie painter Mr. King paints the same wall in 7 hours. How long will it take them if they work together? Choose the most sensible answer from those given below. Explain your decision.

Answer Choices Given: (a) 21 hr (b) 10 (c) between 5 and 10 hr (d) 5 hr (e) 4 hr (f) 3 hr (g) 2 hr (h) 1/2 hr.

During her presentation, Mary wrote the equation "1/3 + 1/7 = 10/21" on the overhead transparency. Ann then asked Mary to explain how she determined this equation. As Mary began her explanation, Ann interrupted with a mathematical declaration directed at Mary: "You have to find the common denominator…." Mary successfully addressed Ann's directive. Shortly after, Mary was asked (again by Ann), "What does 10 over 21 mean?"

In summary, this whole class episode contained 82 speech acts, 14 of which were teacher-initiated interruptions of students (usually of the student-presenter). Of these interruptions, 12 (86%) were mathematical statements made by the teacher to individual students, or to the class as a whole. As seen by the number of mathematical interruptions, the teacher was invested in addressing the fraction interpretation error and in making its conceptual underpinnings clear to students through Mary's demonstration.

Professional development activity based on the painting problem episode. As described earlier, the sessions from the first summer professional development produced a climate where the research team (including Ann) reviewed the teacher's goals, beliefs, and practices using the PA method. To make sense of the painting problem episode from Ann's perspective, we reviewed the videotapes with Ann during the first summer.

While viewing the video, Ann told the professional development team that she was probing Mary's conceptual understanding of the fractions in her solution, while simultaneously assisting her in the mechanics of problem solving and organizing Mary's presentation so it served Ann's pedagogical goals. Ann recalled why she decided to have a student present a solution, and in particular, why she asked Mary:

> I remember specifically asking Mary to come up because about six students did [the same error] she did. And I knew that she would be willing to come up and that she wouldn't die of embarrassment…to…know that there was a little catch there in how she was thinking.
>
> I also called her up because she did very clearly explain the concept of the one-third plus one-seventh. So I wanted to…to…give credit to all the people who thought about that in a fractional way.
>
> I remember specifically that I thought that there were three ways you're gonna solve this problem. You could use [process of elimination from the choices given]. You could have done it with fractions. And I had a feeling you could draw a model, but I had not come up with any model.
>
> I guess I felt like the three methods that we discussed could possibly satisfy everybody's meth…way of solving it. You hit on everybody that way.

Ann revealed that she wanted to use Mary's presentation to expose a common conceptual error in interpreting the fractions of the problem. The fraction 10/21 could be interpreted by students as either the number of hours each person worked, or the fractions of the wall that was painted by the two actors in one hour. Mary erroneously interpreted it as the time worked. During the presentation, Ann walked Mary through this conceptual error and also highlighted a common long division error in Mary's solution.

As the discussion of this episode continued, a research team member (RM1) asked Ann for further elaboration.

RM1: Can I probe just a little more on…

Ann: Yeah.

RM1: So, there's this, you said there's this idea that getting out all these different ways of thinking is a really important goal, a perfect kind of way to do that, um…but you could have done that, too.

Ann: Uh-huh

RM1: So, there's maybe more thinking # about why students…

Ann: (Overlaps #): Why kids as opposed to me?

RM1: Yeah.

Ann: OK. Boy! (2-second pause). It's just sort of ingrained in me that you ask kids to do it. But I think if I really were to analyze that I just think that especially middle level kids…uh…pay attention to their peers. They tend to tune out a teacher after a while. … There's an identification.

Commensurate with her stated beliefs, Ann chose a student presenter to foster peer learning, and to provide a means for demonstrating alternative problem-solving methods to the whole class. In support of her goal of sharing multiple ways to reason about problems, Ann invited a student to present one of several solution methods. Rather than demonstrating the method herself, however, Ann selected Mary to serve as her proxy, in hopes that this would further engage students.

As part of the professional development activity designed around the painting problem, Ann participated in a self-evaluation of her instruction. The emphasis in the self-evaluation was to assess Ann's success of meeting her Year 1 goals during various class lessons, to reflect on what led to success or failure and, if necessary, to revise her actions, plans, and curricular goals when planning for Year 2. Initially, Ann evaluated the painting problem and the other sample lessons reviewed from the first year using the classroom videotapes. In the painting problem, 18 out of 24 students participated in the class discussion, with 2 of the students leading presentations of their problem solutions. In this particular lesson, as in others, Ann was very pleased to see such a large number of students participating, and concluded that because of the high participation level there should have been a lot of learning occurring. Not surprisingly, Ann labeled this a very good lesson, and commented that "sharing ideas really helps." After Ann provided her own evaluation of the instruction, the team, along with Ann, made an evaluation based on the multilevel analysis of classroom interaction information flow and scaffolding behavior targeted in this study. First, however, we present the results of our analyses of the Year 1 classroom interactions; Ann's self-evaluation, in light of these analyses, then follows.

Analyses of Year 1 Classroom Interactions

With Ann's self-evaluation in mind, we next present the results of the classroom interaction analyses of the sample of Year 1 data. This examination considers the flow of information, the nature of scaffolding, and the global patterns of interaction evident in the classroom. As part of the professional development team, Ann participated in a small number of these analyses, and these undoubtedly affected her perceptions of the class interactions with respect to her goals and beliefs. The remaining analyses were conducted by the authors without Ann's participation.

Analysis of the flow of information. The first analysis of the classroom discourse focused on the information flow among students and the teacher. A total of 392 speech acts were recorded in the Year 1 sample (Table 2). Although students spoke a moderate amount (111 speech acts out of 392, or 28.3% of the time) overall, they addressed each other directly only five times (1.2% of the whole class utterances, see Table 2), with the remaining 106 utterances directed toward the teacher.

Despite the teacher's belief that students prefer to learn from their peers, and despite her favorable evaluation of the level of participation in many of her lessons, very little student-to-student talk was evident in the eight classroom

TABLE 2
Percentage and Number of Speech Acts for Whole Classroom Discourse from the Year 1 Sample

	Horizontal (Student-to-Student)		*Vertical (Teacher-to-Student)*		*Vertical (Student-to-Teacher)*		*Total*	
	%	*Number*	*%*	*Number*	*%*	*Number*	*%*	*Number*
Analytic	1.2	5	52.3	205	26.3	103	80	313
Social	0		19.4	76	0.6	3	20	79
Total	1.2	5	71.7	281	27	106	100	392

episodes from the Year 1 sample.[4] The vast majority of analytic information flowed vertically during Year 1, either from the teacher to the class (71.4% of the time), or, less frequently, from students to the teacher (27.1%).

Analysis of scaffolding. The vast majority of information flowed vertically during Year 1, either as scaffolding from the teacher to the class, or as student questions and responses directed to the teacher. The level of information flow, however, offers no analysis of the content of the discourse. Results from the scaffolding analysis reveal something of the substance of the verbal exchanges, particularly the nature of the scaffolding offered to one another.

When the teacher provided scaffolding, it was twice as likely to be analytic in nature, addressing mathematics content, rather than the social aspects of the classroom climate. Students contributed some scaffolding to one another, however, and even provided some for the teacher (such as instructing her in novel solution methods, or addressing arithmetic slips). Student-initiated scaffolding during the 1st year of study was exclusively analytic in nature (Table 2), emulating the analytic quality of the teacher's talk.

Analysis of patterns of interaction at the global level. The analysis at the global level shows how classroom participants interacted during any given whole class episode. For this analysis, each class member, as well as the teacher and the whole class as a unit, was represented as a node, and directed links were made between nodes identifying discourse exchanges. The level of interaction for each member of the classroom was measured by a simple count of the number of utterances initiated by and received by that node, with line thickness representing relative frequency (Figure 1). We use this global depiction to show the relative amount of information exchanged among class members and to provide a concise illustration of the nature of the discourse. Strom et al. (2001) used a similar, though more abstract, representational form that focuses on the order and content of the discourse as well.

In Year 1, the teacher clearly served as the hub of these whole class conversations. Nearly all of the exchanges flowed through her, and she talked primarily to the class as a whole. Consistent with the information flow and content analyses, and despite her stated goals, Ann's Year 1 actions did not successfully promote much horizontal information flow among the students. The overall pattern of interaction is illustrated by the hub and spokes pattern shown in Figure 1, which visually depicts the central role the teacher played in whole class discussions, and highlights the vertical nature of information flow.

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[4]This pattern of infrequent student-to-student discourse for whole class discussion is undoubtedly different than data from small groups and student pairs.

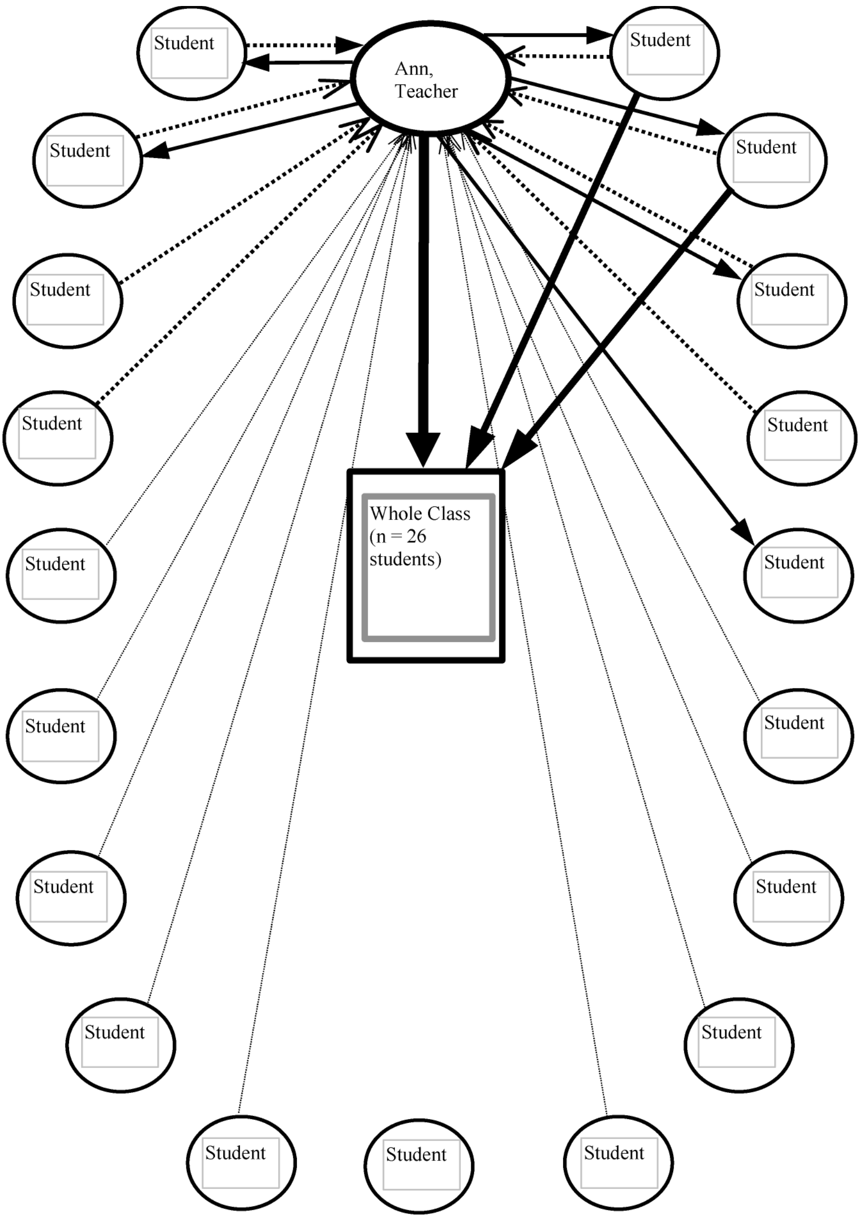

FIGURE 1 A hub and spoke diagram showing who speaks to whom in Ann's class during Year 1.

Year 1 Summary

Together, the information flow and scaffolding analyses gave the professional development team an opportunity to evaluate the classroom discourse that occurred during the first year of observation. The interactions during the painting problem episode provide a good perspective from which to consider the character of much of the classroom discourse that year. During the painting problem lesson, Mary, the presenting student, started out sharing her information through student-to-class and student-to-student exchanges. Students asked Mary questions directly, which she fielded, providing analytic scaffolding to the querant. After the fourth teacher interruption, however, Mary shifted modes, engaging Ann directly in each subsequent utterance in a vertical flow of information, which dominated the remainder of the painting episode.

Analysis of this episode helps identify some factors that can foster and impede student discourse. By interrupting Mary repeatedly, Ann did two things. First, she delegated a qualified surrogate to a teaching role, but reclaimed the central role as the mathematical and social authority of the classroom. Second, she redirected the flow of information from a horizontal to a vertical dimension, thereby placing a boundary between Mary as a peer-presenter, and Mary as a student in need of instruction from an informed other.

It appears that students in this class were less inclined to address one another directly when there was a strong centralized teacher present during classroom exchanges. Classroom talk tended to be directed primarily at the teacher, who then addressed a particular student or the class as a whole. The teacher's role was a familiar one. She served as the mathematical authority and, to a lesser extent, the authority on social conduct. From the data, it is clear that Ann was the central player in all of the verbal interactions concerning the whole class. As a consequence, students rarely participated in exchanges unless called upon, relied predominantly on the teacher for information rather than other students, and rarely used discourse as a means to construct their own conceptions, test out hypotheses, or question other students' ideas. Despite Ann's regard for students' ideas, her frequent invitations for students to contribute their ideas, and her goals for a highly participatory classroom environment, we still see a highly teacher-directed form of discourse.

Teacher Evaluation of Year 1: Reflection and Revision of Classroom Practices

Participation in the multilevel analysis of the classroom discourse served as a resource for promoting reflection and revision of Ann's original goals, beliefs, and classroom practices. In particular, review during the summer professional development sessions of the information flow, scaffolding, and global level analyses of the Year 1 episodes led Ann to reconsider some of her classroom tactics as she realized that some of her most important goals around peer interaction and learning through participation were not being met. Ann planned to remove herself from the hub role more often in Year 2, staying out of class discussions so that there would be more room for students to think, ask questions, and publicly express their own ideas. To this end, during the first summer, Ann planned to be physically less central (remaining outside the ring of desks, often near her own desk) during student presentations and discussions, and she resolved to speak less often.

Ann also observed that in the Year 1 videotapes students did not really listen well to each other or to the presenting student. She made a commitment to provide training in active listening and effective presentations early in Year 2. In fact, Ann brought in a professional facilitator/team builder to work with the students four times during the school year on effective communication and group problem solving. She explored alternative desk configurations to better facilitate student-to-student discussion. Ann also helped students establish classroom norms that gave students more permission to speak and ask questions of one another. These norms were publicly posted and referred to throughout the subsequent year by both the teacher and the students.

Year 2 Classroom Excerpt: The Factors and Multiples Lesson

The second year's observations occurred after the summer professional development. To illustrate the changes, a new example episode, factors and multiples, is presented to focus interpretation of the greater patterns that occurred in the lesson sample. As background, the terms *factor* and *multiple* had been used extensively over the prior 2 weeks, and students had done class work involving factor trees, played the Factor Game (a game in which opponents receive points equaling the sum of the factors of the chosen numbers), and completed Venn diagrams of overlapping and mutually exclusive groups of multiples and factors. The factors and multiples episode (11.5 min long) focuses on the students' attempts to arrive at a working definition for the terms factor and multiple. Although the mathematical content addressed in this episode is clearly different from that in the painting problem episode—mathematical conventions versus mathematical problem solving—the tasks presented to students in each episode do provide contexts in which analytic scaffolding could occur. In this episode, for example, the teacher might recast student responses in more precise mathematical language—a form of analytic scaffolding—as a means to ecculturate students into the mathematical realm. Moreover, this episode also reveals quite explicitly Ann's new approach to facilitating classroom discussion and drawing students more fully into peer-directed discourse. The purpose of this Year 1–Year 2 comparative analysis is not to compare content, but to compare the nature of the classroom discourse as the role and actions of the teacher changed.

Several marked differences emerged in comparing Year 2 to Year 1. Ann's interruptions of students in Year 2 were principally to keep the student-led discussion going and make certain that students shared their ideas and listened to one another. This pattern contrasts with Year 1 episodes like the painting problem in which Ann's interventions tended to be mathematical rather than social. The following two comments made by Ann are representative of the Year 2 tactics to encourage student participation:

Ann: Other opinions?
Ann: If you don't have an opinion will you try and get one so we can keep this [discussion] going a little longer.

Ann structurally removed herself from the central role in mathematical discussions, but frequently invited students to speak. In so doing, Ann based her actions on her beliefs about student learning, and her interpretation of the prescriptions of the mathematics reform movement that called on teachers to promote student-to-student discourse. Students responded positively to Ann's overtures, and held up the mathematical end of the discussion.

Professional development activity based on the "factors and multiples" episode. In response to this shift in practice, Ann saw tremendous growth in student-to-student mathematical talk. This was consistent with beliefs Ann had to promote learning among her students, particularly that students learn best from their peers and through classroom participation, as determined in Year 1 interviews. Ann interpreted these events favorably, although she was also aware of the greater amount of class time these interactions took. Still, from Ann's perspective, students were clearly learning that mathematics included forming and expressing one's ideas, not simply asking the teacher or doing calculations. Overall, Ann regarded the shift positively, emulating the classroom behaviors she had marveled at in videos of mathematics educator Deborah Ball ("Shea Numbers"; Ball, 1993) and science educator Jim Minstrell (Minstrell & Matteson, 1993). She felt she was well on her way to implementing some of the essential notions of reform-based pedagogy. Although this new role was demanding, Ann generally felt energized after reviewing episodes from the second year, and optimistic about her new instructional practice.

Analyses of Year 2 Classroom Interactions

The intervening summer professional development program influenced Ann's thinking and practices. Her new role was evident, and it demonstrably influenced the classroom climate. However, the multilevel analysis of classroom discourse provides for a more complete and rigorous examination of classroom events, and leads to a perspective different from Ann's. Here we present the results of the classroom interaction analyses for Year 2 as compared to the findings from Year 1, in terms of the flow of information, classroom scaffolding, and the global interaction patterns.

Analysis of flow of information. There was a large rise in the proportion of peer-to-peer discourse from Year 1 to Year 2. Throughout the sample episodes of Year 2, there were 191 speech acts. The increase in student-initiated discourse was dramatic, shifting from just over 1% for all whole classroom talk in Year 1 (both analytic and social scaffolding, Table 2), to nearly 33% in Year 2 (Table 3). Nearly 1 out of every 3 statements in Year 2 (62 out of 191 utterances) was made by a student to a peer (Table 3). The increase in student-to-student discourse was traded off against teacher-led discourse as Ann had expected. Ann's contribution to classroom discourse in the sample dropped from over 70% to less than half of the statements. A content analysis of this data set revealed the nature of the changes in these verbal exchanges between the teacher and students as well as among the students themselves.

Analysis of scaffolding. The analysis of the types of scaffolding that occurred during whole classroom discourse show that the changes in vertical discourse were primarily in the analytic realm. By reducing the amount of analytic scaffolding she provided in Year 2, Ann strategically set out to increase the level of student-to-student discourse, to allow for greater student participation, and to make the exchanges more student-centered, and more engaging. That Ann succeeded in abrogating her role as the mathematical authority is evident in the changes in vertical information flow data shown in Years 1 and 2 (compare Tables 2 and 3). As Table 2 shows, the proportion of whole classroom discourse devoted to analytic scaffolding in the 1st year of study was mostly vertical in nature, and occupied

TABLE 3
Percentage and Number of Speech Acts for Whole Classroom Discourse from the Year 2 Sample

	Horizontal (Student-to-Student)		*Vertical (Teacher-to-Student)*		*Vertical (Student-to-Teacher)*		*Total*	
	%	*Number*	*%*	*Number*	*%*	*Number*	*%*	*Number*
Analytic	20.4	39	31	59	19.4	37	70.8	135
Social	12	23	15.7	30	1.6	3	29.3	56
Total	32.4	62	46.6	89	21	40	100	191

78.6% of all talk in our sample. In the 2nd year (Table 3), the analytic scaffolding from the teacher dropped to about 50% of all whole class exchanges. Social scaffolding remained relatively unchanged as a proportion of all discourse over the 2 years, however, staying at around 20% (Table 3). Thus, Ann maintained a strong involvement in guiding student talk and managing the classroom, while stepping aside to let the math talk happen around her. Ann relied on students to compensate for her relative analytic absence. As Tables 2 and 3 show, the proportion of analytic scaffolding students provided each other during classroom exchanges increased more than 15-fold over the 2-year period, from 1.2% of the speech acts, to over 20%.

The proportion of social scaffolding the students provided each other increased as well during the 2nd year, even though Ann's role hardly diminished. Recall that social scaffolding was coded for utterances that helped establish the classroom norms for interacting. The documented increase in students' social role suggests that as students took over the discussion in the mathematical realm, they also recognized a need to actively direct the social interactions of the classroom to conduct these conversations. Students participating more frequently in discussions also realized the need to reinforce accepted rules and establish new norms.

These analyses provide a view on the conditions that appear to engender thoughtful and sustained discourse. The factors and multiples episode provides a good example. Ann did not occupy the central role as the mathematical authority as she did during Year 1, and there was a greater amount of information flowing among the students than in the past. (Ann did, however, tutor students frequently in small group and individual work settings during class and study periods.) In her role as the facilitator of class discussions, Ann managed to keep discussions going, getting students involved, soliciting views, and reminding students of the social norms of the classroom. This shift was very successful in stimulating student-to-student talk.

Ann set out to make the discourse more accessible and more productive for her students, and, in accord with the mathematical reforms, to use the power of these content-centered verbal exchanges to enhance students' understandings. Ann's strategy to remove herself from these interactions so that more student-to-student exchanges would occur seems to have been successful. The hope was that, ultimately, an increase in horizontal information flow would lead students to contribute more, expand the range of mathematical methods they encountered, increase their involvement with other classmates, and support greater learning. As will be discussed shortly, this is consistent with Ann's goal to involve the students more in the classroom interactions.

However, without the teacher participating as a mathematical authority, these young students did not always have the resources to construct or verify correct mathematical ideas or conventions. Throughout the factor and multiples episode, there were frequent misstatements, and several competing hypotheses about the nature of factors and multiples that went unaddressed, with no apparent way for students to resolve their differences and address their confusion. At one point in the class, students were trying to decide what exactly factors and multiples were, and how widely the notions applied.

Brad: [A factor is] a number multiplied by another number to get the answer.
Darias: A number that goes evenly into another number.
Kenny: I think it's for multiplication and subtraction.
Bob: I think it's for multiplication. No, I think it's for everything.

Finally, with no convergence reached during this discussion, one student proposed a social resolution to the mathematical question:

Anthony: Let's vote!

The need to resolve the discussion was apparent. However, since Ann essentially removed herself from the analytic aspects of the classroom discourse and gave her attention primarily to the social aspects, there was no clear authority for students to turn to in the face of their uncertainty. Voting, to the students, seemed as reasonable as any other method for establishing the definitions of these mathematical terms.

Patterns of year 2 interactions at the global level. In this final analysis, it is evident that the data from the Year 2 episodes illustrate substantial changes in the patterns of whole classroom interactions. As the analyses of information flow and scaffolding suggest, students frequently addressed one another directly, or spoke to the class as a whole. Ann's statements to the class during the 2nd year were more often social in nature than in the 1st year, emphasizing management issues and the social facilitation of student-driven discussions. Tracking the patterns of exchanges over the 2nd year revealed a star pattern with a less evident central mathematical authority (Figure 2). Students frequently shared information horizontally with one another as they worked together to determine the definitions of terms (e.g., factors and multiples), specifics of procedures, and interpretations of problem solutions.

Year 2 Summary

The research team noted that students were interacting more frequently with each other, but, as illustrated in the factors and multiples episode, saw there was often a lack of rigorous argumentation and evidence in the discussions, and a lack of convergence toward acceptable mathematical ideas and conventions. With no clear mathematical authority participating, student ideas were offered publicly for others to pick up, refute, or ignore, often with no basis for evaluation other than

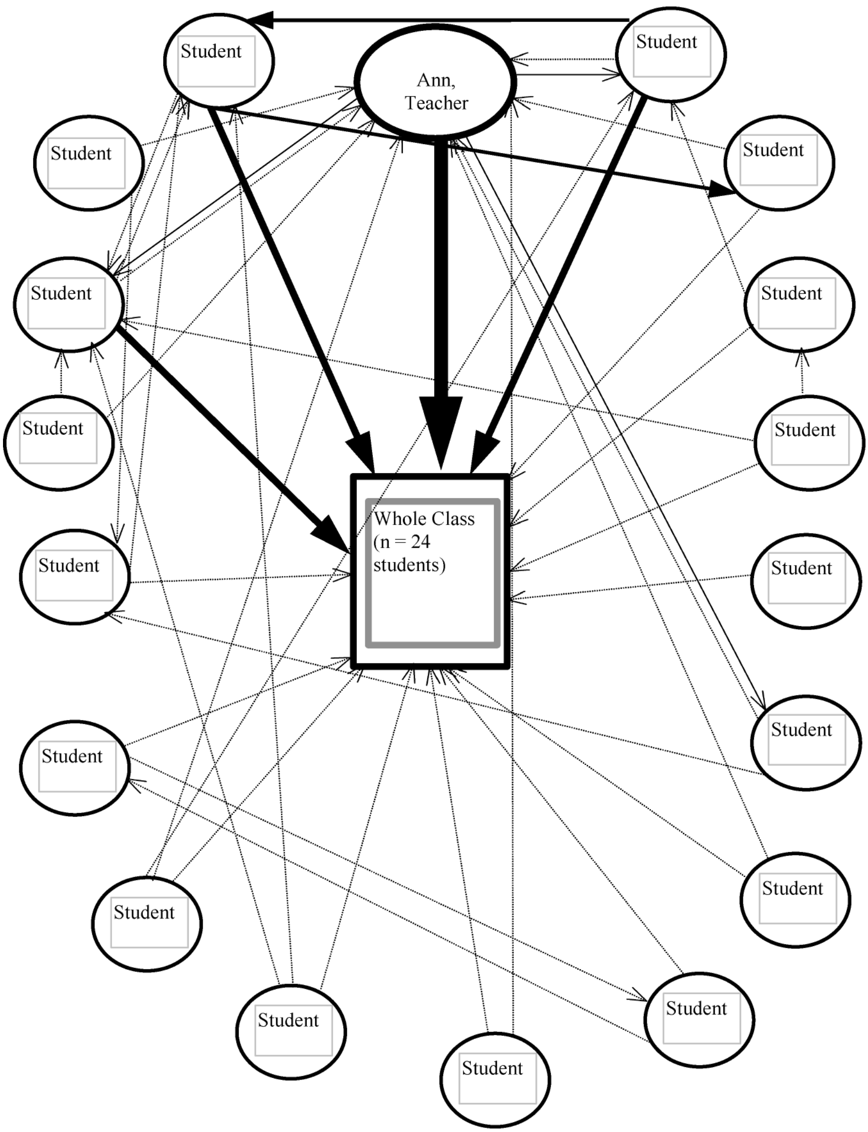

FIGURE 2 A star pattern diagram showing who speaks to whom in Ann's class during Year 2.

opinion. Further, during such discussions, Ann often did not seem to think that there might be a need to do more than simply regulate the communication of separate ideas of the students, as described by a social scaffolding role. The team analysis, however, concluded that Ann needed to encourage students to treat other's ideas as objects of discussion in and of themselves—an analytic scaffolding role—and critically evaluate the veracity of the ideas presented. When the teacher elected to move away from her analytic role, the team observed that there was nothing added to the classroom culture to fill this gap in the discourse when there were major oversights, or when conflicting views among students arose.

Teacher evaluation redux. Ann was quite self-critical after the 2nd year of the project. Based on the analyses discussed earlier, video reviews, and the team evaluation that she participated in, Ann realized how her attention to the social aspects of the class traded against her mathematical participation. This led her to question whether students were learning the content, as illustrated in the following comment at the close of Year 2.

> I liked the student-to-student interaction and wanted it to continue but…I found myself thinking "Are they getting the math?"…I wasn't always thinking about the math, and when you would ask [during the biweekly debriefing sessions] "What's the math here?", I would sometimes fumble.

Ann saw greater student participation during Year 2, and more horizontal information flow. On the surface, she realized her goals of discourse-based teaching. Yet under deeper inspection, she questioned whether students were learning the mathematical content. In the past, Ann used her own involvement in discourse to assess student knowledge during class discussions. Ironically, even though students were more vocal, Ann felt less certain of where individual students stood in their understanding of new concepts. We speculate that this is due to two things. First, Ann's large shift emphasizing social scaffolding led to a concomitant shift in her attention away from the specific mathematical talk among students. Second, Ann no longer used prompted discourse to ascertain student knowledge. By asking questions and posing challenges, Ann's expectations were fairly specific at any one time. It may have been more difficult for the teacher to assess student knowledge and level of participation when monitoring unprompted discourse from the sidelines.

CONCLUSIONS

The professional development team expressly did not try to change the teacher's beliefs or goals over the duration of the study. In Year 1, we set out to document Ann's goals and beliefs using the PA method along with classroom video reviews.

In Year 2, we helped Ann to review her actions and the effects they had on the classroom, evaluate those actions with respect to her goals and beliefs, and to occasionally consider alternative actions. Our intention in this study was to demonstrate that a strong relation can be shown between a sixth-grade teacher's beliefs and goals, and how they manifest in her instructional practices. The details of this complex relation are not apparent, however, without examining mutually constraining levels of analysis. We consider these findings in terms of the relationship of teacher goals and beliefs to instructional practice, and how reflection on one's practices can lead to instructional changes. Finally, we discuss the variability of interpretation of reform-based notions of mathematics instruction, and its relevance to teacher education programs and professional development.

The Relationship Between One Teacher's Beliefs and Her Classroom Practices

The analyses of information flow and scaffolding illustrate a pattern whereby the teacher dominated both the analytic and social realms of the classroom during Year 1 (see Figure 1 and Table 2). Although Ann is considered a highly successful teacher, the summer professional development sessions revealed that Ann's classroom was not compatible with some of her key beliefs about the role of participation-based learning through peer interactions. In Year 2, the teacher moved away from this central role as the mathematical authority in the classroom in an attempt to provide students more space to hold sustained, student-directed discussions that would support her beliefs about learning and interpretations of current educational reform. From this, Ann saw increased participation and far more peer-based scaffolding; students were talking more mathematics, and hearing more of their peers' ideas. Clearly, as shown in Figure 2, Ann had found an effective way to transform the classroom culture to be more in accordance with her beliefs about learning and instruction, and more in line with reform-based prescriptions.

An important observation from these analyses is that there was not much change in the teacher's specific goals and beliefs over the 2 years in light of her self-evaluations. However, the teacher made large changes from Year 1 to Year 2 in how she set out to enact those goals and beliefs in her practices. This finding adds to the perspectives that tend to dominate the literature on teacher change. One perspective suggests that teachers' beliefs influence their instructional practices; thus, changing teachers' practices requires influencing their beliefs (e.g., Thompson, 1992). A second perspective suggests that teachers' practices influence their beliefs; thus, changing teachers' beliefs requires engaging teachers in new practices (e.g., Guskey, 1986). A third perspective suggests that patterns of teacher change reflect both the aforementioned perspectives (e.g., Franke, Fennema, & Carpenter, 1997). Common to all three perspectives is the underlying notion that teacher change results in changes in both one's beliefs and practices. In our study, however, we find that teacher change may not necessitate changes in beliefs. Ann still held that students learn best when they participate and hear ideas from other students. Thus, we take the position that there may be a fourth perspective worth considering: Changes in teachers' practices can occur within a consistent belief system when the new practices—which may be very different from the old ones—are compatible with their old (and still current) beliefs.

Implementing Educational Reform

The professional development program served to highlight for Ann some of the key reasons why she does what she does in the classroom. This gave her the opportunity to reflect on her instructional practices, and realign them with her own beliefs and those of the mathematics education community. Ann's mathematical participation and analytic scaffolding decreased as she set out to encourage greater horizontal information flow. Student-to-student talk greatly increased in response. Although we did not see dramatic improvement in the quality of student-to-student talk about mathematics, we recognize that there is a great deal for Ann to learn about her new role. The examples show the difficulties teachers can confront when trying to define their role in guiding the mathematical content of lessons. In this final section we address the most significant challenges facing Ann and other teachers as they try to implement reform-based practices that emphasize student-centered discourse.

First, any discussion of the challenges to implementing reform must also acknowledge the complex nature of the classroom that hampers its ability to respond quickly to all influences. And the interconnections are manifold. Substantive changes in a teacher's practice requires (simultaneous) changes in a variety of areas, including teacher-student interactions, curricular activities, assessment practices and instruments, children's attitudes toward mathematics, teacher expectations for students, and so on. These changes also must occur in the larger context of accountability measures, standardized testing, and parent and community influences, to name but a few. Teachers must also develop their new practices iteratively, making adjustments to conform to the needs of students and the style and knowledge of each teacher. By the end of the study, Ann had completed 2 years of reform-based professional development, and still faced many challenges. In this light, it is no wonder that some investigators report that it takes several years for substantive change to have a demonstrable influence on student behaviors and performance (Grossman, Valencia, Evans, Thompson, Martin, & Place, 2000).

One of the more significant challenges to implementing reform-based instruction lies in its vagueness: The goals are ill defined, the effects of discourse on learning are unclear, and the means to promote it are poorly understood (e.g., Ball, 1996; Lampert & Blunk, 1999). Moreover, reform documents provide teachers with only general guidelines for shaping their own instruction and notions of reform-based pedagogy; as a consequence, there is tremendous room to interpret many of the basic tenets of current educational reform. For example, Rittenhouse (1999) described Lampert's practice (which is often viewed as an exemplar of reform-based instruction) as alternately "stepping in and out" of the circle of classroom interaction. In stepping out, Lampert and Blunk (1999) stated, "I did not answer their questions, or resolve their disagreements. I encouraged them to ask their classmates for clarification…" (p. 1). On the surface, stepping out from the conversation does not sound difficult. And indeed, the analyses presented here suggest that Ann enacted a surface level interpretation of stepping out during Year 2. Ann interpreted calls for discourse-oriented instruction to mean that she needed to provide greater support for the social aspects of the class by stepping out of her role as a primary provider of analytic scaffolding. Ann used student-to-student interactions successfully to bring students into sustainable mathematical discussions (a result of successful social scaffolding); however, this is different from sustaining discussions mathematically (a result of successful analytic scaffolding). Students showed they can fill the conversational void, but they may not and often cannot serve as the analytic authority necessary to promote correct understanding about all of the content matters.

Rittenhouse explains, however, that stepping out also means the teacher is carefully monitoring the classroom interactions, evaluating and shaping them, and, in so doing, establishing and maintaining norms for classroom discourse. "Outside," in her commentator role, Lampert strategically provided terms to label mathematical ideas that arose, and helped students put their ideas into words so students could "clearly state" what they meant (Rittenhouse, 1999, p. 175). Lampert also highlighted for students discourse practices that she wanted to encourage because they emulated many essential components of the discourse that occurs in the mathematics community. Rittenhouse also showed that in continually monitoring the conversation, Lampert is careful to "step in" as a participant at especially relevant times. Lampert contributed to the discussion, shared her mathematical knowledge, and, perhaps most importantly, modeled good mathematical discourse. In short, discourse of this nature does not come about simply because the teacher creates the space for it; there is still a need to mathematically support students' learning of content during classroom interactions. Ideally, teachers provide such support as they strike a balance between the social and analytic demands, that is, when students' own social constructions of mathematical ideas are also connected to the ideas and conventions of the mathematical community.

The challenge of developing new ways of conducting classroom interactions requires most teachers ultimately to build new pedagogy from their own interpretations of mathematics reform. For teachers attempting to enact discourse-based practices such as those exhibited by Lampert, the challenge is particularly daunting given that mathematics teacher education and professional development programs typically have not adequately prepared them to enact successfully the lofty expectations set forth in reform documents (Ross, 1998). Moreover, few teachers have seen or participated in what we call *productive discourse*; that is, forms of social exchange which provide participants with an avenue to construct and build on correct conceptions through their interactions with other class members. Although students' original contributions are a central feature of productive discourse, the teacher's role in initiating and facilitating this type of discourse involves more than eliciting and regulating the separate contributions of her students. Teachers also need to continually monitor where the discourse is going, and have some criteria (albeit flexible ones) for deciding how they know when the class has arrived (cf. Ball, 1993; Simon, 1995). In short, teachers need to learn the stepping in and out that is paramount to promoting productive discourse. One way to help facilitate this task is to identify mathematical talk as a skill unto itself that deserves instructional attention. Thus, among the challenges facing teachers and teacher educators seeking to promote discourse-based practices are questions such as (a) When does the teacher step in and out, and (b) How does the teacher participate in classroom discourse? Teacher education and professional development programs need to address such questions directly, and provide preservice and practicing teachers with both the experience participating in and the tools to facilitate productive mathematical discourse.

As researchers continue to investigate reform-based teaching, there is a need to understand and unravel the complex array of factors influencing teachers' actions as they relate to reform (e.g., Borko & Putnam, 1996; Fennema & Franke, 1992; Richardson, 1994). Accordingly, as teachers wrestle with how to reconceptualize their pedagogy in light of reform recommendations, their practices provide investigators of teaching a valuable window into the various forces that shape reform-based instructional practices. Although several valuable accounts of this reconceptualization process have been provided, they tend to describe teachers who are already experts in the field of education, and are themselves part of the reform movement (e.g., Ball, 1993, 1996; Lampert & Blunk, 1999; McClain, & Cobb, 2001; van Zee & Minstrell, 1997). As the research base extends from these exemplars to include ordinary teachers operating in more typical settings, we can expect to learn a great deal more about the relation between teachers' beliefs and their instructional practices that will lay the groundwork for further advancements in reform-based teacher education and professional development.

ACKNOWLEDGMENTS

We are grateful to Amy French for her invaluable contributions. Support for this research was provided by the James S. McDonnell Foundation. Some of these data were presented at the 1998 and 1999 American Educational Research Association annual meetings.

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