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# Confronting Teachers' Beliefs About Students' Algebra Development: An Approach for Professional Development

## Objectives

Teachers' knowledge and beliefs are powerful mediators of decision-making and action (e.g., Sherin, 2002). For example, teachers generally report that their perceptions of students are the most important factors in instructional planning, and teachers consider student ability to be the characteristic that has greatest influence on their planning decisions (Ball, 1988; Borko & Shavelson, 1990; Borko et al., 1992; Carpenter et al., 1989; Clark & Peterson, 1986; Fennema et al., 1992; Romberg & Carpenter, 1986; Thompson, 1984). Yet teachers' beliefs and expectations of students' behaviors are not always accurate. This paper focuses on a method for influencing the beliefs of five urban high school algebra teachers so that they are more closely aligned with actual student performance data. We report on the initial and changing views exhibited by teachers about the nature of algebraic development and instruction, and discuss why the method has promise for affecting teachers' knowledge of students more generally.

## Theoretical Framework: Changing Teachers' Beliefs and Content Knowledge

Teachers possess knowledge of many forms, some explicit, some tacit, some bound to actual practices (e.g., Sherin, 2002). *Pedagogical content knowledge* (PCK) has emerged as a valuable construct for understanding teacher knowledge (Shulman, 1986). PCK is knowledge of content oriented specifically around pedagogical concerns, such as how to illustrate concepts. Some forms of this knowledge cross the blurry line into beliefs and expectations about how students *should* learn. Consequently, some researchers are willing to combine knowledge and beliefs into a common construct (e.g., Pajares, 1992).

Only a few studies have specifically looked at the relation between teachers' beliefs about student reasoning, and students' actual problem-solving performance (e.g., Carpenter et al., 1988; Peterson et al., 1989; Wigfield et al., 1999). Much of this past research has focused on elementary level mathematics, and has greatly enhanced our understanding of teachers' beliefs. Lacking, however, is a similar emphasis at the secondary level. One exception is the study of the expectations that high school mathematics teachers have for algebra students' problem-solving performance. Nathan and Koedinger (2000a) asked high school mathematics teachers to rank order the relative difficulty of mathematics problems that varied along two dimensions (see Table 2): arithmetic-algebra, and presentation as verbal with a context (story problem), verbal with no context (word equation), or symbolic equation. The majority (76%; N=67) inaccurate predicted that symbolic equations would be easiest for algebra students; instead equations were most difficult, even though they were carefully matched to the story and word-equation problems. Teachers justified their rankings by arguing that symbolic reasoning was a necessary precursor to solving story problems, and that symbolic representations were more "familiar," "straightforward," and "pure." This view was termed the *symbol precedence view* (SPV), and its role in algebra teachers' decision making has been independently confirmed with attitudinal survey instruments (Nathan & Koedinger, 2000b; Nathan & Petrosino, 2003). The replication of both the student performance data (N1=76; N2=171) and teacher expectations (N1=67; N2=105; N3=48) suggests this is a reliable and widespread view of mathematical development (Koedinger & Nathan, 2003; Nathan & Koedinger, 2000b).

Deep-seated beliefs do not easily change. Attempts to change teachers' views need to explicitly address teachers' existing beliefs (e.g., Fenstermacher, 1994; Richardson, 1994). This tenet guided the professional development activity described below.

# Method

## *Participants*

Five high school mathematics teachers volunteered to participate in a morning of professional development activities. All taught in the same urban school district, which serves a large portion (80%) of minority students, and qualifiers for free/reduced lunch (75%). They all agreed to stay through the duration of the day's activities, and to respond to a follow up activity 4 weeks later.

## *Procedure and materials*

Participants received a professional development packet (Table 1) and responded to a belief elicitation task where they offered predictions of students' relative problem-solving difficulties using a *difficulty ranking task* (see Table 2). Each participants' ranking was presented to the group and recorded for all to see. A brief discussion was moderated.

The professional development team then gave a 30-min presentation showing the ranking data of other teachers and mathematics educational researchers. As previously observed, the overall pattern of predictions showed the common SPV with similar rationale favoring the development and use of symbolic reasoning before verbal applications. This helped to establish for participants that they had views similar to the mathematics educational community at large; that their views were not anomalous.

Student work was then presented showing symbol equation use, common conceptual errors in symbolic representation and manipulation (along with frequency data; slip type errors were ignored). Participants also saw student uses of alternative solution strategies that led to the verbal advantage, along with frequency, error patterns, and data on likelihood of success when applied to symbolic and verbal algebra problems. Teachers were then given a summative account of the student strategies and error data. Teachers then participated in a guided activity where they applied a general rubric for evaluating students' written work (Table 3) to four example solutions. The examples were chosen because they captured the major features of the student performance data seen in previous studies (Koedinger & Nathan, 2003; Nathan & Koedinger, 2000a). These were intended to enhance the development of teachers' "algebraic eyes and ears" (Kaput & Blanton, 1999) by focusing teachers on the problem-solving processes and representations that could be inferred from students' written work.

Three weeks later, participants were sent, by postal mail, a follow up difficulty ranking task designed to assess the impact of the professional development. Teachers' rankings were accompanied by personal written justifications.

## Results and Conclusions

### *Pre-intervention ranking and justifications*

Participants provided problem difficulty rankings individually, using the six items shown in Table 2. The difficulty ranking data were analyzed two different ways. First, rankings from all five teachers were averaged to produce a single group ranking (Table 4). This group ranking was correlated with an idealized SPV ranking that predicted symbolic problem solving was easier than verbal problem solving—a pattern that repeated for both the arithmetic and algebraic problems given in the ranking task. For the example items in Table 2, the idealized SPV ranking would be `1 2 3 4 5 6`. The correlation between the idealized SPV ranking and the group average ranking was shown to be very high, Pearson's $r = 0.9$.

To corroborate this, the ranking of *each* participant was correlated with the idealized SPV rank, and each Pearson's rank correlation measure was calculated (second row of Table 4). As can be seen in the lower portion of Table 4, the correlations range from 0.49 to 0.9 with a mean of 0.7. This distribution of correlation measures yields a 95% confidence interval that ranges from 0.55 to 0.84 (SD = .17). Like the group level analysis, this analysis shows that participants provide problem difficulty rankings similar to that predicted by the SPV.

The justifications given by participants for their rankings shed further light. The constant comparative method was employed to establish a grounded coding system for teachers' justifications (Glaser and Strauss, 1967). Table 5 shows the resulting categories and reveals that teachers discussed how symbolic representations were favored because they were considered more basic and familiar to students,

### *Post-intervention ranking*

Teachers participated in a new ranking task one month later. The group level analyses (Table 4) showed a low correlation with the idealized SPV ranking, Pearson's $r = 0.15$. The individual level analyses (lower portion of Table 4) show correlations between 0.6 and -0.09 with a mean correlation of 0.13 (SD = .27). The 95% confidence interval includes 0. Both analyses lead to similar conclusions—teachers' post-intervention views show little resemblance with SPV. Additionally, teachers' justifications (Table 5) show greater awareness of the difficulties students have with formal notation and the facilitating effects of verbal representations.

## Importance of the study

Teachers' views of student development must be open to examination. This seems especially important given national (e.g., NAEP) and international data (e.g., TIMSS) showing poor student performance in secondary mathematics topics, and as schools nationally explore how to teach algebra in the middle and elementary grades. The ranking task assesses one aspect of pedagogical content knowledge for teaching algebra; namely, teachers' expectations about the relative difficulty students experience for problems presented in more or less formal representations, while controlling for the underlying quantitative structure. To enhance the validity of this study, teachers evaluated the relative difficulty of *specific* mathematics problems, rather than making statements about student algebraic reasoning in the abstract.

Our focus is on the effectiveness of a method of professional development as measured by changes in the expectations of a small sample of urban high school teachers. If teachers misperceive the relative difficulties of equations, they may introduce them prematurely, or inappropriately withhold story problems from a student. Improving teachers' expectations of problem difficulty is important because these beliefs affect instructional planning and assessment design. By demonstrating change, a promising approach has been identified that can serve as part of a larger professional development program aimed at enhancing teachers' pedagogical content knowledge.

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Table 1. Summary of the contents of the workshop packet distributed to teachers.

1. Fill out cover sheet with teacher's demographics and contact information.
2. Select among snapshots of classroom climates and curriculum materials.
3. Pre-intervention problem difficulty ranking task (Table 2)
4. Workshop presentation showing ranking data from other teachers and examples of student performance data and student written work.
5. Four-level Rubric for Students' Written Work along with instructions for evaluating student work using the rubric to four examples of student work shown in Figure M-1 (a)-(d).
6. Post-intervention problem difficulty ranking task (*Waiter problem,* $W \times 6 + 66 = 81.90$).

Table 2. Problems in the difficulty ranking task arranged in a 2 x 3 matrix.

| **Presentation type →** | **Verbal** | | **Symbolic** |
|---|---|---|---|
| **Unknown value ↓** | **Story** | **Word** | **Equation** |
| Result-unknown (Arithmetic) | P3 | P2 | P1 |
| Start-unknown (Algebra) | P6 | P5 | P4 |

Table 3. Rubric given to participants to analyze student written work.

We will use a 4-level rubric to assess students' written work. Please note that there are is no one correct answer for applying this rubric. What is important is that you feel that you can justify your reasons for assigning a particular level, and that you are consistent with your evaluation.

We will share our rubric evaluations. Based on the comments of others, you may elect to change your evaluation. However, do not feel pressured to do so.

*Rubric*

| | |
|---|---|
| Level 4. | Student's written work shows *all* of the characteristics of Level 3, plus at least one of the following:<br>• Student provides a particularly sophisticated solution strategy.<br>• Written work is presented in a very clear and well organized manner.<br>• There are aspects of the solution that indicate a deep understanding of the underlying mathematics. |
| Level 3. (Standard) | Written work presents a correct answer based on a mathematically sound method. |
| Level 2. | Written work presents an *in*correct answer that is arrived at by a method that is essentially sound, but with minor error(s) evident. For example,<br>• Computational errors<br>• Copy errors |
| Level 1. | Written work presents an *in*correct answer that is based on a conceptually flawed method. |

Table 4. Pre- and post-intervention correlation statistics (Pearson's r) with SPV (columns 1 and 2) and VPV (column 3) for the problem difficulty rankings averaged across the group (n = 5), and for each of the individual participants.

| | Curricular materials | Pre-intervention r with SPV | Post-intervention r with SPV | Post-intervention r with VPV |
|---|---|---|---|---|
| Average ranking of group (n = 5) | | .9 | .15 | .88 |
| Mean correlation from individual rankings (n = 5) | | .7[a] | .13[b] | .8[c] |
| Participants | | | | |
| A | Reform | .9 | .6 | .94 |
| B | Reform | .49 | -0.09 | .62 |
| C | Traditional | .71 | .03 | .83 |
| D | Reform | .56 | .03 | .83 |
| E | Reform | .81 | .09 | .77 |

[a] Average of the individual Pearson's r computed for all 5 participants (SD = .17). The 95% confidence interval extends from .55 to .84.

[b] Average of the individual Pearson's r computed for all 5 participants (SD = .27). The 95% confidence interval includes 0.

[c] Average of the individual Pearson's r computed for all 5 participants (SD = .12). The 95% confidence interval extends from .69 to .90.

Table 5. Participants' coded justifications for the pre- and post-intervention problem difficulty ranking task.

| | Students have greater familiarity and skill with symbols than words | Verbal problems are solved using translation to symbolic | Arithmetic word problems tell you exactly what to do | Arithmetic skill strictly precedes algebraic reasoning | Symbol manipulation is difficult (error prone) | Context helps in problem solving |
|---|---|---|---|---|---|---|
| Pre-intervention | | | | | | |
| GH | • | | • | • | | |
| LM | • | • | | | | |
| JC | | | • | • | | |
| SC | • | • | | | | |
| AH | | | • | | | |
| Post-intervention | | | | | | |
| GH | | | • | • | • | • |
| LM | | | | | • | |
| JC | | | | | | |
| SC | | | | • | | • |
| AH | | | | | | |