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Regulation of Teacher Elicitations in the Mathematics Classroom

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Using the perspective of instructional conversation, we investigated how one teacher regulated student participation and conceptual reasoning in the middle-school mathematics classroom. We examined the elicitations—questions and provocative statements—made by the teacher over a four-day algebra lesson. Analyses showed how the teacher systematically regulated the level of cognitive complexity of his elicitations in reaction to students' responses. When students gave inaccurate or incomplete answers, the teacher tended to reduce the level of cognitive complexity needed to respond to a subsequent elicitation, with the apparent impact being that he scaffolded participation and reasoning. When students provided responses that were mathematically accurate, the teacher usually increased the elicitation level, which subsequently engaged students in more sophisticated forms of reasoning.

REGULATION OF TEACHER ELICITATIONS AND THE IMPACT ON STUDENT PARTICIPATION AND COGNITION

Traditional forms of mathematics instruction tend to focus on getting the answer correct and recalling facts and procedures, but often leave students unengaged and unprepared for complex and novel problem solving (National Research Council, 2000). The aim of contemporary mathematics education reform is to make a ìshift from learning mathematics as accumulated facts and procedures to learning mathematics as an integrated set of intellectual tools for making sense of mathematical situationsî (National Council of Teachers of Mathematics [NCTM], 1991, p. 2).

Studies of classroom interactions have generated wide interest in how teacher-to-student and peer-to-peer talk promotes students' active participation and provides the pedagogical scaffolds that allow students to engage in sophisticated mathematical reasoning and critical reflection (Cobb, Wood, & Yackel, 1993; Hatano & Inagaki, 1991). Yet we have limited understanding of how student participation in classroom discussions is regulated by the teacher's talk, and the

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impact this has on students' conceptual reasoning. Advancing this understanding will increase our knowledge of the nature of language use in educational settings, and also contribute to our theoretical understanding of the relation between communication and collaborative learning (Baker, 2007; Sfard, 2007).

Student repair of trouble sources is no longer a sufficient marker of mathematical understanding. Students are now being directed to engage in extended, complex reasoning that often involves collaboration, and to go beyond getting an answer "correct" by also verbalizing their reasoning processes and justifications, and reflecting on their understanding of the underlying concepts (e.g., Cognition and Technology Group at Vanderbilt, 1997). Students in this context must publicly communicate their mathematical ideas, as well as listen and critically evaluate the ideas expressed by others. The teacher's role in these settings then expands to include the role of facilitator of students' mathematical participation.

Numerous studies have looked at classroom discourse to better understand how teachers stimulate students' participation and engagement, and scaffold students' activity and the construction of mathematical knowledge (e.g. Atkins, 1999; Cobb, 1995; Cobb et al., 1993; Hatano & Inagaki, 1991; Hufferd-Ackles, Fuson, & Sherin, 2004; Sfard, 2007). However, the effect of teachers' elicitations on students' learning is still not well understood, and studies of its impact show mixed results. Studies also reveal how much teachers struggle in their new roles as facilitators within discourse-based learning environments (e.g., Nathan & Knuth, 2003; Rittenhouse, 1998). Further investigation of these interactive processes can yield important insights into how the teacher elicits students' participation in academically oriented ways of talking and thinking (Cobb, 2000; Cobb & Bowers, 1999; Williams & Baxter, 1996).

This current initiative acknowledges the central role teachers play in facilitating classroom discourse by scaffolding students to verbalize their ideas and critically evaluate the ideas of others. It seeks to deepen our understanding of how teacher invitations for students to participate in classroom discussions occur, and how they may engage students and subsequently foster higher-order reasoning.

Mehan (1979) defined elicitations as those forms of communication that "engage participants in the exchange of academic information about factual matters, opinions, interpretations, or the grounds of their reasoning" (p. 64). In this investigation, we focused on the nature and fluctuations of the teacher's elicitations—the questions, pauses, and provocative statements that engage students and prompt them to respond. We examined the patterns of teacher elicitations and student responses over the course of a four-day lesson drawn from a beginning algebra unit in a middle-school classroom. We studied how one teacher's elicitations were regulated, consciously or unconsciously, in reaction to student responses to earlier prompts, the levels of reasoning subsequently exhibited by students, and the apparent impact this had on the overall structure of the classroom discourse as it unfolded over time. From this analysis we found patterns of regularity relating teacher prompts and student responses in reciprocal ways. We also examined how regulation of teacher-generated elicitations enabled students to engage in higher levels of mathematical reasoning than they exhibited on their own. In this sense, teacher elicitations contributed to the scaffolding process by providing socially mediated supports to the class as a whole (Nyikos & Hashimoto, 1997). We believe these findings contribute to emerging theories of classroom based learning, and can inform programs of teacher education and professional development.

CLASSROOM DISCOURSE

As a conversational form, classroom talk is special (Drew & Heritage, 1992), shaped, in part, by the unique relationship of authority between class members and the teacher (Schultz, Erickson, & Florio, 1982). Unlike conventional conversation, whole-class discourse typically models a participation structure predictably controlled by a single conversant, the teacher (Nystrand, Gamoran, Kachur, & Prendergast, 1997), who also has a professional obligation to foster student learning. Teachers ask most of the questions and typically maintain the right to call on students and allocate turns, "in essence organizing and orchestrating the discussions" (Greenleaf & Freedman, 1993, p. 466; also see McHoul, 1978). International comparisons show that "the kinds of questions teachers ask influence students' opportunities to think and communicate mathematically during lessons" (Kawanaka & Stigler, 1999, p. 255).

Mediation of Classroom Discourse

Interest in how classroom discourse scaffolds students' participation and reasoning follows on the heels of recent education reforms, which recognize classroom discussion as an effective way to perform instruction, conduct knowledge assessment, and facilitate learning in the content areas (e.g. NCTM, 2000; National Research Council, 2000, 2005). In addition to acquiring facts, notations and procedures, learning mathematics involves participation in the activities of the mathematics community (Lave & Wenger, 1991). This participatory aspect is evident across several levels of analysis, such as how novices move from peripheral to more central involvement in the community's discursive practices, as a way to describe the tensions that operate between minority students' academic and racial identities (Nasir & Saxe, 2003), theories for explaining behavioral shifts when people move from one regional or cultural setting to another (Gee, 2005), and as a way to characterize the course of historical development of mathematical ideas (Sfard, 1995). For example, Gee (2005) explored how young people's access to different styles of language inherent in different social settings can facilitate or obstruct their access to academic knowledge and the economic opportunities that then follow. Nathan and his colleagues (2007) showed that as students struggled to understand one another's mathematical representations, the group adopted—without any centralized directive—conventions such as labeling, color-coding, and principles of perspective drawing. This helped to foster common ground among the participants and clarify the spatial, temporal, and semantic relations that were central to the solution representations they were trying to convey. These examples illustrate how participation in classroom discourse provides opportunities for learners to be active members in a learning community and to structure their own learning experiences (Tharp & Gallimore, 1991; Wertsch, 1991, 1994).

Sociocultural theory asserts that mental processes are mediated by semiotic tools, such as objects (e.g., mathematical manipulatives like construction cubes), diagrams, formal systems of notation and, most notably, language. These semiotic tools serve a mediational role by structuring interlocutors' mental activities and transforming interpersonal (i.e., social) actions to intrapersonal (psychological) processes (Vygotsky, 1978; Wertsch, 1991, 1998; Wertsch & Toma, 1995). Activity with these tools, such as participation in certain forms of problem solving, enables

the internalization of socially constructed mental activities that emerge later as advancements in learners' cognitive development (Werstch, 1995).

In a similar vein, Gal'perin (1969) demonstrated that the materialization and verbalization that occurred with physical learning materials—actual tools and objects—mediates learners' cognitive processing and, as a consequence, facilitates their internalization of mental operations and concepts that are directly related to the physical activity. Working from within Gal'perin's framework, Talyzina (1981) found that the verbalization stage was indispensable for learners' development of higher order of thinking. Thus, through verbal externalization, learners can organize their thoughts into verbalizable units and articulate their own hypotheses. This leads them to reflect on and critically examine their underlying knowledge, and can lead learners to eventually restructure their understanding (Nathan, Eilam, & Kim, 2007; Slobin, 1996; Swain, 2000; Wells, 2000).

Classroom communication is seen as a powerful mediator of change in these complex cognitive behaviors, because communication fosters meaning making, self-monitoring and reflection, and the co-construction of new ideas. Instructional conversation (e.g., Tharp & Gallimore, 1991) plays an important role in discourse-oriented classrooms because it helps scaffold students' access to higher levels of cognitive processing. Teachers can facilitate participation in the discourse through scaffolding; that is, by inviting students into a social interaction that is also pedagogical.

Scaffolding

Scaffolding is a form of instruction that emphasizes the socially mediated nature of knowledge and teaching (Bruner, 1986; Vygotsky, 1978). With scaffolding, a more knowledgeable "other"— such as a teacher, parent, or more experienced peer—provides temporary support for learners so they can participate in more advanced reasoning and behavior than they are capable of performing on their own. The supports offered by the teacher then gradually fade away to allow for ever-increasing learner autonomy. This approach draws directly on Vygotsky's (1987) sociocultural theory of cognition, specifically, his *General Genetic Law of Cultural Development*, which states that any cognitive function that develops first appears in the social realm (through interpersonal interactions such as scaffolding and modeling) and then in the psychological realm, as internalized by the child.

Interactional Scaffolding

Teacher elicitations can directly contribute to the scaffolding process (McCormick & Donato, 2000). Cazden (1988) argued that classroom instruction should be organized around the scaffolding modes such as W-H elicitation questions (why, where, who, what, and how) for probing the next piece of information. Through scaffolding, learners internalize knowledge that they co-construct with experts (Bruner, 1984; Wertsch, 1979). Cued use of W-H questions can promote internalization of publicly displayed knowledge and forms of dialogic interaction as external classroom interactions are overtly shifted to internal mental processes.

In addition to internalization, scaffolding through elicitation questions can assist individual learners so they outperform their autonomous competence within their *zone of proximal*

development (ZPD). As defined by Vygotsky (1978), ZPD is metaphorically taken as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). More specifically, Scollon (1976) identified this instructional support as *interactional scaffolding* because teacher elicitations draw out students' self-directed solutions and their metaprocess level justifications.

Educational reformers, particularly in the mathematics and science communities, have attempted to utilize some of the principles of socially mediated learning as they emerge during classroom interaction (Ball, 1996; Cobb, 1995; Cobb, Stephan, McClain, & Gravemeijer, 2001; Cobb et al., 1993; Cognition and Technology Group at Vanderbilt, 1997; Lehrer, Strom, & Confrey, 2002; Lantolf, 2000; Wertsch, 1991, 1994, 1998). Several studies have shown the important role of teacher questions and other elicitation forms for scaffolding students' thinking. Roth (1996), for example, described a case study where the teacher's questioning was designed to get students to contribute their knowledge, and to scaffold their intellectual activity to reach independent and critical thinking. Chin (2007) examined the types of questions that teachers ask and the way teachers' queries influence the cognitive processes that are engaged as students struggle with the construction of scientific knowledge from an inquiry-based perspective. He showed how teacher questions could facilitate students' advancement up a "cognitive ladder," enabling then to exhibit higher levels of scientific reasoning and reflection.

Elicitation Techniques, Participation, and Cognitive Processing

The logic underlying teacher elicitation questions and student responses can be found in *Conversation Analysis* approaches, and is tied to the notion of a *conditionally relevant response* as a type of *adjacency pair* (Heritage, 1984a; Schegloff & Sacks, 1973) where the prompt elicits a relevant response. However, Mehan's (1979) ethnographic analyses of classrooms found deviations from the adjacency pair structure, which showed that sequences may be inserted until the elicitation is satisfied. This expanded participation structure effectively drives the teacher to communicate with students as co-participants in the interaction, and invites participants to attend to the group's intersubjectivity, structure learning opportunities, and advance the conceptual understanding needed to enable formation of the complex cognitive skills required to participate (Donato, 2000; Nathan et al., 2007; Takahashi, Austin, & Morimoto, 2000).

Teacher elicitations, when effective, can generate more information in the classroom, increase participation, and foster students' conceptual development (Cazden, 1988; Mehan, 1979; Nystrand et al., 1997). Walsh and Sattes (2005) note that this kind of instructional conversation also allows the teacher to assay students' underlying level of knowledge and to adjust the instruction to meet a student's emerging understanding (Chin, 2007; Tharp & Gallimore, 1991).

Teacher elicitations, in the form of questions and provocative responses, can be viewed as occurring at different cognitive levels that reflect the ensuing demands they place on interlocutors. Several scholars (e.g., Bloom, 1956, 1987; Kawanaka & Stiger, 1999; Nystrand et al., 1997; Wells & Arauz, 2006) have charted hierarchical systems that reflect the various levels of cognitive complexity of teachers' elicitation techniques. The most influential categorization system for teacher elicitations was offered by Mehan (1979), who identified four types. In order of increasing cognitive complexity, they are: Choice, product, process, and metaprocess elicitations. Choice

elicitation (CE) asks students to agree or disagree with what the teacher said in the previous turn, and so merely depends on students' recognition of correct information or guessing. Product elicitation (PE) invites students to provide factual knowledge, such as a name or a place, which they must generate from long-term memory. Process elicitation (PRE) asks students to provide opinions or interpretations. Metaprocess elicitation (ME) asks students to connect their responses with the intentions of a teacher's elicitation by providing, for example, the justification supporting a student's reasoning.

Contributions from Mehan (1979) and Bloom (1987) help researchers to divulge how various types of elicitation techniques, operating at different levels of cognitive complexity, reveal students' understanding and guide them toward lower- or higher-level cognitive activities. Across these systems, lower-level cognitive questions are defined as those that require only recognition or the ability to recall factual information; while higher-level activities require higher order thinking skills, such as deeper forms of comprehension, interpretation beyond given information, metacognitive monitoring, reflection, and justification.

Verplaetse (2000) claimed that when students succeed in acknowledging (CE) or recalling (PE) knowledge at the factual level, teachers would tend to move to a higher level of questioning, like a metaprocess elicitation (e.g., "How do you know that?"). This has the potential to expand the interaction with students and further explore their thinking. More generally, by regulating the cognitive demands of the elicitation down as well as up a theoretical hierarchy, teachers can use discourse participation as a way to provide scaffolds for students when they are operating at lower levels of reasoning, and build on a foundation of lower order knowledge in order to scaffold them to higher order thinking (Yip, 2004), thus extending the developmental level of performance they exhibit within the instructional interaction.

PURPOSE OF THE STUDY AND RESEARCH QUESTIONS

We assert that teachers exhibit even more sophisticated forms of regulation than those found in earlier studies (e.g., Verplaetse, 2000), and that they may adjust the cognitive complexity of their elicitations in response to students' incorrect responses as well as to their successes. That is, teachers may systematically, even if unconsciously, move the instructional conversation up and down the hierarchy of cognitive complexity in their efforts to promote engagement and reasoning.

The main purpose of this study is to examine how one teacher uses discourse over several days to engage students and promote their participation in higher levels of mathematical thinking. We show here how the teacher traverses a hierarchy of elicitation forms, with the apparent effect of fostering participation at cognitively complex levels of discourse. From this detailed case, we posit that teachers, in their efforts to promote higher order reasoning in the reform mathematics classroom, can be highly responsive to students' demonstrated needs, using classroom talk to both assay student knowledge and promote its advancement. In making this claim, we are not asserting that teachers are mindful of each of the maneuvers that they make during the discourse, but are likely guided by much more global considerations, such as establishing certain group products, reaching broad consensus about mathematical ideas, and adhering to certain norms of respectful group discussion. We propose that by examining the specific interactions and responses, we may reveal a structure to the discourse that may not be specifically planned from its outset but, rather, emerges from the discourse practices (Hutchby, 1996). Once we establish the existence

and regularity of this pattern of interactions across the corpus, and explore its apparent role in fostering student participation and reasoning, we explore specific interactions in greater detail to illustrate the nature of discourse regulation. In the final section, we discuss the implications of this work for socially mediated learning theory and for teacher practice in reform-oriented classrooms. We also take up two issues of methodological importance; that of addressing the teacher's intentionality, and of assigning causality from highly interactive data.

To guide this investigation, we first ask, How are the teacher's elicitation prompts regulated following students' responses to initial mathematical queries? Our hypothesis is that the regulation of the cognitive complexity of the teacher's elicitations is highly responsive to student statements. As our second guiding question, we ask, how do teacher elicitations provide interactional scaffolding and foster students' shifts from lower to higher order forms of reasoning? Our hypothesis is that, over time, as students exhibit mastery with the more basic knowledge, the cognitive demands of teacher elicitations will increase, providing scaffolds for students to exhibit their facility with more advanced forms of mathematical reasoning, justification and reflection.

METHOD

Participants

Participants were students in a middle-school mathematics classroom in the western United States. The participants included one male mathematics teacher and 24 middle/upper-middle-class students in a combined seventh/eighth-grade class. This was neither the lowest nor the highest tracked math section. This combined class was about 60% seventh graders who were on track, along with about 40% eighth graders deemed not yet ready for a full-on algebra course. Tracking was based on student standardized test performance from the previous year. The highest tracked class was an eighth-grade class (although it included some advanced seventh graders). The lowest tracked class was a regular seventh-grade-only class that only used the seventh-grade course materials.

The teacher had 10 years of experience teaching in elementary and middle school settings. He was nominated by both the school principal and a sixth-grade mathematics teacher as someone open to sharing his mathematics instruction with the research community and willing to participate in an experimental unit designed to enhance students' algebraic reasoning. The research team casually observed his teaching in the year prior to this study, and then were a regular presence in the classroom during the study.

Data Collection

The classroom observations reported here are part of the documentation of the fidelity of implementation of the experimental unit, an intervention that spanned nine weeks. The focus of this current analysis is a single lesson, spread out over four consecutive days, that focused on the uses of tables, graphs, and words, along with algebraic expressions and equations, to represent and predict the numerical patterns of growth exhibited by cubes of various sizes. This extended lesson occurred during the seventh week of a larger, nine-week experimental unit on beginning algebra

that was intended to promote representational fluency, algebraic modeling, and problem solving. Prior quantitative analyses of student performance data showed that using this experimental unit led to statistically reliable gains in students' algebraic reasoning, above and beyond the standard curriculum, particularly fluency among quantitative representations and reasoning about linear and nonlinear patterns of growth (for more details on findings from the student achievement data, see Nathan, Stephens, Masarik, Alibali, & Koedinger, 2002). Our interest in analyzing the nature of the teacher's elicitations is to better understand how he was able to regulate students' engagement and deepen their mathematical reasoning about patterns.

Previously in this unit, students had learned to produce algebraic representations of the patterns of 1D and 2D growth for perimeter and area of arrangements of square tiles. The topic that is reported here was the first in which they had encountered 3D patterns of growth and the use of cubic functions. We chose this particular four-day lesson because it showed sustained interaction at the level of the whole class while addressing complex mathematics, including the use of multiple algebraic representations, to describe both linear and nonlinear relationships.

The four specific algebra class sessions under the scope of this study were structured around three main curricular goals: (a) naming various parts of the $4 \times 4 \times 4$ cube, such as side length, corners, edges, faces, hidden blocks, and total volume; (b) identifying and recording the growth of linear and nonlinear patterns of the various parts as students built larger cubes using construction blocks; and (c) mathematically representing the patterns of growth using words, graphs, tables, and algebraic expressions that would allow students to form abstractions of the patterns and generalize the growth behavior for hypothetical cubes of different sizes.

Four 45-minute class sessions were analyzed. One researcher ran a video camera mounted on a moveable tripod, while a second researcher took field-notes in order to document any events the videotaping might miss. The teacher typically stood in the front of the class. For this lesson, the students sat in individual desks arranged in a half-circle along the edge of the classroom, facing the teacher and the blackboard.

The cube lesson was developed as part of the nine-week experimental intervention aimed at improving students' algebraic learning by bridging from their initial solution strategies and representations to more formal methods for representing quantitative patterns and solving problems (Nathan et al., 2002). Certain classroom norms of interaction were established to encourage the public presentation of students' mathematical ideas. Sociomathematical norms (Yackel & Cobb, 1996) of the classroom were established early in the school year. These norms included the value of students' mathematical solutions, an appreciation for considering alternative solutions and procedures, and the significance of justifying one's mathematical ideas. As part of that preparation, students practiced active and respectful listening skills. The importance of these norms and skills was revisited throughout the year in order to maintain their presence in contributing to the classroom climate.

Coding and Data Analysis

This is a descriptive, classroom-centered study that focuses on the structure of student and teacher discourse. To investigate the classroom interactions, we transcribed the verbal utterances and the accompanying nonverbal aspects of the discussions, such as changes in voice pitch and emphasis, use of hand gestures, object use, writing and drawing, and so on. Our basic unit of analysis is

teacher utterance. This decision was motivated by the unique aspects of formal classroom settings where the onus for managing classroom talk and turn taking is on the teacher. As McHoul (1978) notes: "only teachers can direct speakership in any creative way" (p. 188).

Codes were assigned to each teacher utterance. Broadly, teacher utterances were coded as elicitations and non-elicitations. The teacher's elicitations are the questions and provocative statements, as well as requests, continuers, and prolonged utterances that preceded student participation. Non-elicitations were limited to mathematical statements and utterances involving classroom management. Examples of each utterance type are presented in Appendix A.

Of specific interest are elicitations. Although it is generally expected that elicitations contain W-H interrogatives (what, which, who, when, how), reversed subject–verb order, and a rising intonation, Mehan (1979) points out that this is by no means definitive in the study of interactional events. The meaning of a given instructional act in the classroom is not wholly determined by its grammatical form. Accordingly, elicitations were defined operationally by the responses they engendered. We relied on work from the Conversation Analysis (CA) tradition (Heritage, 1984a; Sacks, Schegloff & Jefferson, 1974; Schegloff, 2007) to interpret the particular organization of the discourse in terms of the communicative actions that they evoked from participants, rather than attempting to interpret the subjective meaning of the topic the utterances referred to. In particular, we identified *elicitations* as speech acts made by a first speaker by virtue of the response as made by successive speakers (Schegloff, 2007). As Mehan (1979) defines it, elicitations are those forms of communication that engender exchanges of intellectual information among participants. In CA parlance, this is referred to as the *next-turn proof procedure* (Heritage, 1984b), where the "next turn provides evidence of the party's orientation to the prior turn" (Arminen, 2005, p. 2). In this way, we based our coding on a functional evaluation provided by the interlocutors who were participating in the actual discourse to categorize the type of talk. This provides a systematic method for assigning categories to conversational events that is in keeping with the actions and interactions exhibited by the speakers themselves.

Elicitations were further coded according to their types (for examples see Appendix A) and level of cognitive complexity. We identified several elicitation types: questions, which make an overt inquiry and exhibit rising tone; continuers and back channels, which encourage the current speaker; provocative statements, prolonged utterances, calling a student by name, and explicit requests. Inter-rater reliability for coding the types of elicitations was 95.8 % (Cohen's *kappa* = .93).

In addition to categorizing the elicitation types, we applied a coding scheme to assign a cognitive level of complexity to each teacher elicitation, based on Mehan's (1979) framework, Bloom's (1987) Taxonomy, the cognitive levels identified by Nystrand and his colleagues (2003), and the coding scheme used by Webb and her colleagues (2006) that documented a range of classroom behaviors from the simplest to the most complex. Assignment of cognitive level went beyond the surface structure of each elicitation and depended on the depth of response needed to satisfy the elicitation as well as the context of the utterance sequence (e.g., Drummond & Hopper, 2003; Nystrand, Wu, Gamoran, Zeiser, & Long, 2003). For example, when we encountered a continuer ("uh-huh," "okay") in the discourse, we looked to see what prior cognitive level was being encouraged.

We synthesized these schemes into four ever-increasing cognitive levels of complexity (see criteria and examples in Table 1): those that required a yes/no decision (Level I, Choice Elicitation, CE); those seeking factual knowledge (Level II, Product Elicitation, PE); elicitations made by

TABLE 1
The Criteria and Examples of the Four Levels of Cognitive Complexity of Teacher Elicitations

Level I: Choice Elicitation (CE)

Criteria	Asking the respondent to produce a yes/no response or select among a fixed set of alternatives.
Example	1. S: well, there's <u>three</u> different sides of top (0.3) but they both share one.
	2. →T: ok↑a::↓y. so, it's a shared side?
	3. S: yeah.

Level II: Product Elicitation (PE)

Criteria	Asking respondents to recall or describe factual mathematical knowledge or information.
Example	1 →T: what shape is it? ((Teacher holds up a 4 × 4 × 4 cube composed of wooden blocks))
	2 Ss: cube.

Level III: Process Elicitation (PRE)

Criteria	Asking students to explain or give opinions. It produces new information from the respondent.
Example	1. →T: If we wanted to look at some different cubes, how
	2. would we identify other cubes we would consider?
	3. S: Number of like the pieces of tape. If it has like . . . like Cathy has it up,
	4. if it has like three: intercept, edge, and face. Two pieces of tape: it's
	edge and face, and one piece of tape: it's a face, just a plain face. Zero,
	5. it's a hidden.

Level IV: Metaprocess Elicitation (ME)

Criteria	Asking the respondents to justify their own reasoning based on a prior statement, or make a connection to an idea from a previous turn. It encourages reflection by probing for justification of a statement made earlier by the respondent.
Example	1. →T: You said subtract two then you said cube it. Why are you cubing
	2. what you have left?
	3. S: Because they're ah, they're, it's not just one row. There's . . . if we have
	4. a side length of five, then there's um (2.7) there's three (3.0) ah, three
	5. three top ones (1.4) no, I don't know.
	6. T: You're on the right track
	7. S:[There's..
	8. →T: Anyone] what to (.) go ahead keep going.
	9. S: There's three rows of three. So it goes three, three and then three.
	10. And then there'd be another row behind that one and another row behind that one.

the teacher that probe for an explanation and interpretation that is initiated by the teacher (Level III, Process Elicitation, PRE); and those that probe for the justification of a statement that was previously made by the student, and is therefore asking the speaker to reflect upon his or her earlier thinking (Level IV, Metaprocess Elicitation, ME). Although choice elicitations may serve many roles, such as confirmation checks, the cognitive demands for making an acceptable response (regardless of whether or not it is correct) are quite low. For example, the correct answer to a CE ("A square has four equal sides, right?") can be arrived at by guessing or merely giving confirmation (e.g., Koshik, 2005), but an appropriate response to an ME ("Why are

you squaring this difference?") requires greater comprehension from the respondent and more elaborate language production.

Following our classification scheme, as the cognitive level of the teacher-directed elicitation increases, the mental processing needed to address the elicitation gets more complex. In Table 1 we provide our coding criteria and examples from our corpus for each level of cognitive complexity. Inter-rater reliability for coding the levels of cognitive complexity for 25% of the corpus was 94.4 % (Cohen's *kappa* = .94).

In the final analytic step, student responses were independently coded as correct, incorrect, partially correct (including partially complete), or neutral replies to the currently active elicitation. Mixed choral responses with both correct and incorrect replies were coded as partially correct.

FINDINGS

Analysis of our corpus revealed 627 teacher utterances over the four class sessions. Of these, 551 (87.7%) were identified as teacher-directed elicitations. Non-elicitations made by the teacher included evaluation statements, demonstrations of mathematical reasoning, and classroom management. We briefly summarize several analytically interesting patterns in the elicitation data before we delve into the specific classroom dynamics.

Evidence of Teacher Regulation

The teacher used a variety of elicitation formats across the five categories shown in Table 2, including overt requests, calling students by name, providing prolonged utterances, and so on. But asking questions (60.5%) and posing provocative statements (12.7%) clearly were the dominant elicitation modes. We also found that the teacher provided prompts across the range of the four-level hierarchy, which varied in frequency (see last row of Table 2 for totals). PE, or product elicitation, was the most common form (over 40% of all prompts), in keeping with the prevalence of calls for student recall and demonstrations of factual knowledge in the classroom, even when in the service of more conceptual reasoning and problem solving. The teacher was least likely overall (9.1% of the time) to prompt students for explanations (PRE).

In pursuit of our first research question, we examined how the cognitive level of each teacher elicitation fluctuated, from one to the next, in reaction to student responses. As Table 3 shows, over half the time there was no adjustment of the level of cognitive complexity from one elicitation to the next. Among the remaining turns, we found that the elicitations moved upward in the hierarchy about as often as they moved downward (see Sub-total row of Table 3), each 22% of the time.

However, these adjustments in complexity appear to be related to the accuracy of students' prior statements. Table 3 shows that when students made correct statements, the subsequent elicitations were twice as likely to move up the hierarchy (48 times, over the 4-day corpus) than down (24 times). When incorrect statements were made by students, the subsequent elicitation was nearly twice as likely to move down the hierarchy (32 times) than up (17 times).

A 3 × 3 Chi square of the Correct-Incorrect-Partially Correct responses crossed with Up-Down-No Change provided a test of the hypothesis that the teacher's elicitation adjustment was related

TABLE 2
The Frequency (and Percentage) of Each Form of Teacher Elicitation by Cognitive Levels

		Cognitive Level			
Teacher's turns Total	628				
Non-elicitations	77 (12.3%)	1	2	3	4
Elicitations	551 (87.7%)	CE	PE	PRE	ME
Questions					
Total	380 (60.5%)[a]	116	159	27	78
Provocative Statements	80 (12.7%)	19	46	11	4
Prolonged Utterances	18 (2.9%)	1	15	2	0
Student Names	13 (2.1%)	1	6	1	5
Requests	22 (3.5%)	0	15	6	1
Back-channels/ Continuers	38 (6.1%)	0	18	9	11
Total[b]	551	137 (21.8%)	258 (41.2%)	57 (9.1%)	99 (15.8%)

[a]Percentages are out of the total number of elicitations ($n = 551$).
[b]Cell percentages are of row totals ($n = 551$).

to accuracy of students' prior statements. The test showed that these variables are significantly related, $\chi^2(1) = 18.5$, $p < .001$. Planned pairwise comparisons were evaluated with the Dunn method controlling the family wide alpha level to 0.05. Responses that were mathematically correct were more likely to be followed by an increase in the cognitive complexity of the next elicitation then a decrease or no change. Incorrect responses were more likely to be followed by a decrease in the cognitive complexity of the next elicitation than an increase, and marginally more likely to decrease than show no change. This supports our prediction that the teacher elicitations will systematically shift up and down the cognitive hierarchy based on the accuracy of students' responses. It may be that these adjustments are conscious and intentional. However, this remains a hypothesis because the present data do not allow clear resolution of this point.

TABLE 3
The Frequency (and %) of Elicitation Adjustments Up, Down or No Change, for Correct, Incorrect, and Partially Correct Responses by the Students Over the Four Class Sessions

		Adjustment to Elicitation			
		Up	Down	No Change	Total
Accuracy of Students'	Correct	48 (45%)[b]	24 (22%)	74 (28%)	
Responses to a Teacher	Incorrect	17 (16%)	32 (30%)	47 (18%)	
Elicitation	Partially Correct	13 (12%)	19 (18%)	54 (21%)	
	Neutral or Others	29 (27%)	32 (30%)	89 (33%)	
	Sub-total	**107 (22%)**	**107 (22%)**	**264 (55%)**	**478**
Unadjusted Teacher Elicitations due to Absence of Prior Student Response[a]					74
Total Teacher Elicitations					**552**

[a]The category *Elicitations* with no prior response are initial elicitations that do not follow from a prior response.
[b]Percentages are of column totals.

TABLE 4
Frequency (and Percentage) of Each Type of Teacher Elicitation Used During Each Class Session ($n = 551$)

	Cognitive Level				
	CE	PE	PRE	ME	Total
1st day	67 (31.3%)[a]	108 (50.5%)	16 (7.5%)	23 (10.75%)	214
2nd day	25 (28.4%)	29 (32.9%)	17 (19.3%)	17 (19.3%)	88
3rd day	25 (26.9%)	41 (44.1%)	9 (9.7%)	18 (19.4%)	93
4th day	20 (12.8%)	80 (50.6%)	14 (9.6%)	42 (26.9%)	156
Total Frequency	137	258	56	100	551

[a]Percentages are of row totals.

Having established the general elicitation pattern that included upward adjustments in complexity following correct responses and downward adjustments in complexity following incorrect responses, we can examine the changes over the duration of the four-day lesson. Table 4 shows the relative presence of each elicitation level over time. Initially, PRE and ME elicitations are relatively rare, and together make up about 18% of the teacher prompts. The proportion of ME prompts grows from Day 1 to Day 2, remains steady on Day 3, and then grows again on Day 4. PRE and PE are more erratic over the four-day lesson. CE, in contrast, shows a consistent decline, slowly at first, but then drops considerably on the last day. By the final session, ME prompts rose 16% points (a 150% increase), while the lowest-level CE prompts showed a corresponding decrease of over 18 percentage points. These data suggest that there is an overall trend toward increasing complexity in the cognitive demands of the elicitations over time, although there is considerable variation within the elicitation categories across time.

Overall, there is evidence that the teacher adjusts his elicitations in a manner that appears to be responsive to students' statements. It is against this backdrop that we illustrate through excerpts from the classroom discussions how teacher elicitations are regulated following students' responses. In the first excerpt (taken from the beginning of Day 1) we show how the level of complexity of teacher elicitations changes following students' responses, illustrating the broader patterns evident in the quantitative presentation of the data. The second excerpt, drawn from the last day of the lesson, is used to illustrate how the teacher's elicitations can serve as an effective way to probe students to articulate the justifications behind their own mathematical generalizations. In this way, Excerpt 2 shows the kind of interactions that contribute to the highest order processing in our complexity hierarchy. These first two excerpts, then, serve to illustrate the patterns evident across the entire data set. In contrast, the third excerpt (drawn from the Day 3) shows a rare but important event. Here, the teacher uses an instructional strategy to challenge students to recognize a mathematically *incorrect* assertion, and use counterfactual reasoning to address the false statement and provide the correct assertion in its place.

Teacher Elicitations Adjusted from Lower Order to Higher Order

In this first excerpt we illustrate how the level of complexity of teacher elicitations changed following students' responses. Excerpt 1 comes from the beginning of the first day of the four-day

lesson. Previous research (Verplaetse, 2000) established that teachers tend to move toward higher levels of prompting when students successfully respond to CE and PE questions. Here, we illustrate the complementary phenomenon: how the level of cognitive complexity of subsequent elicitations is shifted downward when students struggle with the questions before them, when the group shows discord, or when an answer is mathematically incorrect. The teacher's follow-up move has the effect of providing students with interactional scaffolding, encouraging them to publicly present relevant information that is foundational for the topic at hand, and directing students to solve a smaller, more accessible piece of the larger problem. In a similar way, we show that when students provide information that is mathematically correct, the subsequent teacher elicitations tend to increase in complexity, often serving to draw out elaborations of students' ideas or elicit from them analytic justifications for their claims.

In Excerpt 1 (for notational conventions used in the excerpts, see Appendix B), the teacher is holding a wooden cube composed of 64 ($4 \times 4 \times 4$) smaller blocks (for an illustration see Figure 1). The assembled cube is about twice the size of his hand and easily visible from the back of the room. He orients students to the upcoming discussion on algebraic modeling of cubes of various sizes by asking the students to name the wooden object he is holding. Below, each elicitation made by the teacher is shown with the elicitation code (CE, PE, PRE, or ME) to the left.

Excerpt 1: Naming the Wooden Object That the Teacher Holds in His Hands
(Source: 5:29–6:24 of Day 1)

1. **PE-1**→ T: what is it?
2. S1: a thing of blocks=
3. **PE-2**→T:=a thing of blocks (0.4) ok, what shape is it (.)
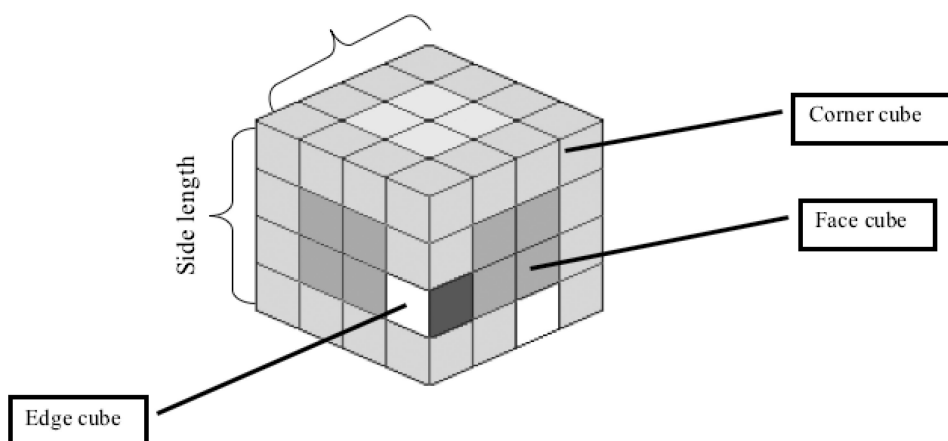4. Ss: cube

FIGURE 1   The anatomy of a cube as used by students in the class: The cube itself was made up of individual blocks that played various structural roles on the finished cube, including: Edge cubes, face cubes, corner cubes, and hidden cubes (not shown).

  5. S2: c[ube
  6. S3: a square](.)
  7. Ss: cube
  8. **PE- 3**→ T: I just heard two answers (.) what shape is this (.)
  9. Ss: CUBE
 10. **CE-4**→ T: is this a square? (.)
 11. Ss: no.
 12. Ss: yes. yes. (.)
 13. S2: well it's a [square that is three dimensional
 14. S3: the sides of it are] sq[uares.
 15. S1: it's a square cube]
 16. **CE-5**→ T: the SIDES of it are squares,
 17. S4: °square face°
 18. **PE-6**→ T: what would it. . . (0.7)
 19. S1: it's a dice.(0.7)
 20. S3: it's a [cube.
 21. T: no, i]t is not a die.(0.8) It's a cube. ()
 22. **ME-7**→T: So, why is this not a square. (0.5) This is not a square.() Why is it not.
 23. S1: ( )
 24. S5: Because a square, when you draw a square on like a paper, you just draw equal sides, but a cube it has eight or six.
 25. **PE- 8**→ T: Has six what?
 26. S5: a six flat sides, um, that are totally, that are totally equal and just a regular square has one side on each face.
 27. T: Okay.

Excerpt 1 shows how the teacher's elicitations lead students initially to engage in lower-level mathematical activities (e.g., to recall the name "cube") but provides scaffolds that allow them to eventually participate in higher order reasoning where they must justify their claims about the differences between two-dimensional squares and three-dimensional cubes. For example, the teacher started this new topic with a product elicitation (PE) question in Line 1 ("What is this") and probed students' basic prior knowledge. Failing to get an appropriate mathematical response in Line 2, the teacher narrowed down the focus with the second PE question (Line 3) by asking "what shape is it?" This question succeeded in eliciting greater participation in this activity and led students to externalize their correct and incorrect ideas (Lines 4 to 7). Although some students gave the preferred response, the public exposure of misconceptions and inaccurate vocabulary (e.g., as shown in Line 6) is one of the documented benefits of discourse-based instruction, as it supports formative assessment (Nathan & Knuth, 2003) and creates an opportunity for re-teaching. This mixed choral response was coded as partially correct for purposes of the quantitative analyses that are summarized in Table 3.

   Although S3 provided erroneous mathematical information in Line 6, the teacher did not respond with negative evaluative feedback (Line 8). Rather, he reformulated the previous question more specifically with a referential pronoun "this," re-engaged students' participation, and finally elicited the loud chorus of accurate answers from students (Line 9). Notwithstanding the correct responses, the lower level CE question in Line 10 functioned to target students' erroneous

responses from an earlier turn (Line 6), and provided one more opportunity for students to reflect on and restructure their thinking. This elicitation technique played a crucial role in generating additional participation from students, as Lines 11 to 15 show. Even though some students (e.g., S1) were still confused and generated inaccurate mathematical terminology, such as "squares," "square cube," or somewhat incomplete responses, such as "square face," the teacher continued to present elicitations (e.g., Line 16, where he emphasizes SIDES), which encouraged students to verbalize their thinking and led to at least one student (S3) to repair an earlier error (Line 20) in much the same way as interactional scaffolding (Scollon, 1976).

We see the cognitive level of elicitation shift upward into the highest ME question in Line 22. After receiving correct mathematical responses at the beginning of the excerpt, and infusing the proper distinction between squares and cubes into the public discourse, the teacher went on to probe the deeper reasoning behind the correct mathematical information. He appears to explore the cube's unique properties and contrast them with the properties of the square. When this higher order ME question elicited a partially (but not completely) correct rationale from a student (Line 24 and 25), we saw the teacher again regulating the cognitive complexity of his queries down to a lower-level PE. The PE in Line 26 served as a form of mitigated feedback (Nathan & Kim, 2006) and provided an opportunity for the student to externalize his own thinking and test his claim, as described by Swain (2000). In the subsequent turn (Lines 27–28), the respondent repaired his own error and provided the correct reasoning of how a cube is distinguished from a square. Finally, this response received a positive evaluation ("Okay") from the teacher in Line 29.

While one can see elements of the often-documented IRE sequence (Initiation by teacher, Response by students, and Evaluation or Follow-up question by the teacher; see, for example, Greenleaf & Freedman, 1993; Mehan, 1979; Sinclair & Coulthard, 1975; Wells, 1993), those analyses are typically insensitive to the regulation of cognitive levels of the initiations in response to student responses, or to the patterns that emerge over numerous initiation–response exchanges. In this case, elicitation questions played an essential role in providing further opportunities for students to verbalize their mathematical thinking, and to engage in (and also be witness to) more sophisticated reasoning.

Follow-Up Metaprocess Prompts to Elicit Higher Order Ways of Thinking

The first excerpt portrayed the general pattern of the teacher's regulation of cognitive complexity based on the accuracy of student responses (as illustrated in Table 3). The second excerpt, taken from the last day of the lesson, further illustrates this pattern, but specifically highlights the kind of classroom interactions that contribute to meta-process thinking. This example illustrates how the teacher probes for further justification of students' unspoken rationale behind mathematically accurate and inaccurate responses. This type of probe taps into the highest order of thinking in our framework, where students are called on to justify their mathematical ways of thinking. In these data, the metaprocess elicitation (ME) prompts draw out multiple explanations for a formula that students generated and then used successfully, if somewhat blindly, to compute the number of face cubes (i.e., the number of cubes that make up the outer surface area, excluding those cubes that already make up the corners and edges; see Figure 1) for a cube of any side length; that is, $(\text{side length} - 2)^2 \times 6$. The various parts of the cube as referenced by the members

of the class in this excerpt are illustrated in Figure 1. This selection portrays a more complicated topic than the one shown in Excerpt 1, which is likely to contribute to more frequent occurrences of higher order prompts. However, this seems appropriate as it comes from the final session of the four-day lesson, after students have explored more sophisticated ideas as they modeled the linear and nonlinear functions that describe the growth patterns of the various components of the cube.

Excerpt 2: Reasoning Behind why Students Square the Number (Source: 16:04–18:21 of Day 4)

1. **ME-1**→T: Why are you squaring something? Why are you squaring the difference? Why are you squaring this new number?
2. S6: It's equivalent of multiplying it by itself.
3. **ME-2**→T: WHY?
4. Ss: (5.0) (No Response)
5. T: It might help you if you see some more squares. (0.3) Some more faces.
6. ((At the white board, the teacher fills in the entries of the table showing
7. **PE-3**→ face cubes with a red color)) Face cubes. <u>Fa</u>ce cubes.
8. S6: Oh, oh, oh, oh, oh.
9. **ME-4**→T: Why are you <u>squaring</u> this difference? (.) Can't really see the red, sorry. (Student's name)
10. S6: Um, so it's the side length minus two for the first square would be one,
11. **ME-5**→T: Uh huh.
12. S6: and, and when it (indecipherable) the second square four, side length minus two would be two.
13. **ME-6**→T: Uh huh.
14. S6: Then squared, two times two is four that would give you the first face. Multiply that by six and you get all the faces. That's why you square it.
15. **ME-7**→ T: I'm not sure I'm following why. I understand multiplying it by six all the faces, but I didn't follow why you squared it. I think I understood but I'm not sure I completely followed it.
16. S6: You square it because that'll give you the two. when you have a flat Square.
17. **ME-8**→T: uh huh.
18. S6: You multiply one side by the other side to get the area, (.) it's just doing it backwards. Squaring it to get the whole area.

In Excerpt 2, the teacher addressed students' understanding of the meaning of the formula for the number of face cubes for a cube of any side length. Face cubes must be counted exclusive of edge cubes (Figure 1), so the number of edge cubes must be mathematically removed from the side length when computing the number of square faces of the cube (hence, the expression (side length – 2)). Because the number of face cubes needed to construct a cube grows with the surface area of the cube, the number of face cubes is in quadratic relation to the cube's side length (hence, the need to square (side length – 2)).

The teacher used open-ended ME prompts after students produced a valid formula for how to calculate the number of face cubes. This had the apparent consequence of acting as a probe of students' reasoning about the formula, and its meaning with respect to the spatial relations present in the cube. In Line 1, the teacher asked about the use of squaring. The student response (Line 3) was generic; it applied to any squaring operation, but said nothing about why it was warranted in this particular situation. Following this partially correct response from a student, the teacher emphatically posed students with a direct question (Line 4), "WHY?" But even with considerable wait time there was no response (Line 5).

In Lines 6–8, the teacher appeared to pursue an inductive path, asking students to examine their tables of numbers (generated during prior pattern generalization activities in class and as homework) and to look for patterns across the different size cubes. One student provided a kind of "think aloud" account (Line 12) for the first entry in the table, a $3 \times 3 \times 3$ cube, essentially applying the formula that they are all being asked to interpret. It is clear that the student understood how to apply the formula (Lines 12–19), and the meaning assigned to multiplication by six (to "get all the faces") in Line 18. The teacher provided back-channels "uh huh" (Lines 13 and 16), which seemed to indicate from a functional perspective that he was following and still expected the ME prompt to be addressed; and the student obliged by providing further elaboration. However, the student's explanation for squaring was still inadequate from a mathematical standpoint.

The teacher provided no negative feedback in response to incomplete responses until Lines 20–22. When the teacher did give negative feedback, it was in an indirect form: "I'm not sure I'm following why . . . I didn't follow why you squared it." This feedback operated as another ME question because it again invited the audience's response to the question "Why." Despite an inaccurate explanation from S6 (Lines 23–24), the teacher did not provide a negative evaluation or voice his own thoughts, but repeated a back-channel ME whereupon the student said more about his thinking (Line 25). It appears that this form of interactional scaffolding chosen by the teacher stimulated further involvement (cf. Nassaji & Wells, 2000). In fact, the student (Lines 26–27) took up the invitation and made public the important connection of squaring the side length to compute surface area.

In pattern generalization tasks, students can find themselves developing formulae that simply fit a numerical pattern—they make the numbers work procedurally—but students might not understand structurally why they work, or what the mathematics is "saying" about the patterns involved (Cai & Hwang, 2002). This is akin to Piaget's distinction between *simple abstraction*—abstraction drawn from the objects themselves—and *reflective* or *reflecting abstraction*—abstraction of invariant features drawn from the (physical and mental) actions performed (Piaget, 2001). The latter requires the construction of new and more advanced cognitive structures (e.g., Ellis, 2007). The long-term objective of algebra instruction is to develop in students the capacity for reflecting abstraction and an appreciation of the meaning, utility, and properties of the ensuing mathematical structures that model these abstract relations.

Excerpt 2 demonstrated how ME prompts can scaffold higher order mathematical reasoning, particularly that of reflecting on the meaning of mathematical structures that a student created. Each ME prompt appeared to provide a scaffold to move further out, beyond the original, unassisted thinking and explore new and more complex mathematical terrain (Verplaetse, 2000; Vygotsky, 1978; Walsh & Sattes, 2005).

Making an Erroneous Statement to Elicit Counterfactual Reasoning

Excerpts 1 and 2 were chosen to be illustrative of the overall patterns evident in the data (Table 3). This final excerpt (drawn from the first quarter of Day 3), is chosen, not to exemplify these general patterns, but to show a unique event that might otherwise go unnoticed. Excerpt 3 shows how the teacher used an instructional strategy—the "trick question"—to engage students in the classroom discourse and encourage them to think more deeply about the mathematics. Prior to the events shown below, learners provided an accurate answer and a legitimate account of their mathematical reasoning for calculating the total number of blocks needed to assemble a cube of any particular side length. In so doing, they developed the formula for the volume of any size cube. However, during an interview with the research team, the teacher related his concern that students sometimes confound the idea of volume (the total number of cubes) with surface area (the number of faces it takes to cover the outside of the cube). This seemed to be part of a broad confusion between 2D and 3D properties of the cube evident in comments made during the first day (when students were asked to name the cube), which persisted throughout the multi-day lesson. While the teacher saw it as more prevalent among students from the earlier class period, he was concerned it could be a latent misconception among some students in all his sections. For this reason, he wanted to address this confusion directly. His approach is captured in the following excerpt.

Initially, in the excerpt that follows, the teacher probed students for the meaning of their formula for volume, (side length)^3. The teacher then made an unusual, but deliberate shift: he intentionally made an *inaccurate* mathematical statement—a "trick question"—designed to lead students to recognize and challenge it using counterfactual reasoning and other higher order forms of thinking. Using physical materials as well as verbal and nonverbal responses, we observed students negotiating the mathematical concepts and finally demonstrating to the class why the teacher's proposal was, in fact, incorrect. In so doing, students provided an analytic distinction between surface area and volume that had not been previously articulated, but satisfied the teachers' aims.

This example provides a valuable perspective of how regulation of the classroom discourse can foster higher-level reasoning following a mathematically correct response. Furthermore, in this example, we were privy to some of the curricular goals of the teacher, as articulated during a debriefing session with the teacher after the lesson. Two days later, the teacher reviewed the classroom videotape with the research team. In the context of this cued retrospective report, he reflected that at the time he specifically wanted to present students with a compelling situation that would illustrate the idea of volume in a way that decoupled it from the notion of surface area (field-notes from week 8). Thus, for this exchange we are better able to view the events through the lens of the teacher's retrospectively stated intentions.

Excerpt 3. The Use of a Trick Question (Source: 9:46–10:45 of Day 3)

1. **PRE1→T:** I've had some people tell me that I can also count sixteen on the top and there's six sides so sixteen times six that's ninety six cubes plus the middle, ninety six cubes plus the middle, so the whole bunch of ones in the middle so that actually gets me up to one hundred four I think cubes. One hundred four cubes.

2. S7: that kind is cr↑azy↓

3. **PRE2**→ T: gets me up to one hundred f↑ou↓r I think cubes. one hundred f↑ou↓r cubes.

4. S8: that's all right.

5. S6: if you did th↑a::t on each s↑i::↓de then you count the t↑o↓p then one of the sides, (.) you'd be counting f↑ou↓r more over again.(0.3)

6. **ME3**→T: what do you mean counting four more over again. ((5 Lines omitted))

7. S6: see::. ((going to the front of the class and turning around the desk with the cubes)) see, (pointing to a colored cube on the desk)

8. S1: I have something else.

9. S6: se↑e:: see::↑ count thi::s to↑p, (.) ((spreading horizontally his palm on the top layer)) and then you count one of the s↑i↓des you're counting the four ↑edg↓es again. ((pointing to each face of the cube with the palm))

10. T: a::h,

11. S1: but you don't c↑ou↓nt the s↑i↓des. you count in l↑ay↓ers.

12. S6: yeah ex↑act↓ly.

13. T: [so, so

14. S6: sixt]↑ee:::n (0.3) times sixt↑ee↓n ((spreading his fingers horizontally on the top layer)) or sixteen times one times, two times, three ((indicating the second, the third, and the bottom layer with the palm)) sixteen times f↑ou↓r ((indicating the top layer with his palm several times)). there's four l↑ay↓ers, so sixteen times four.

In this excerpt, the teacher was most overt in his goals of prompting deeper student thinking by posing a provocative assertion that conveyed the *intentionally* erroneous idea (Lines 1–5) that the total number of blocks making up the $4 \times 4 \times 4$ cube was based on computing the surface area (which is 96 *square* units) rather than the volume (64 *cubic* units). In addition, the teacher made a second erroneous assertion (Lines 3 and 7–8), because he proposes further that they add the volume of the hidden blocks that was previously established to the 96 square units, to make 104 blocks.

One student, S8 (Line 9), asserted that the teacher's idea was correct, whereas another (e.g., S7 in Line 6) strongly disagreed. In subsequent turns (Lines 10 and 11), S6 argued against the teacher's reasoning based on structural considerations. S6 claimed that with the teacher's approach some blocks would be counted twice, which violated basic rules about counting. Thus, S6 demonstrated very high-level justification against the teacher's erroneous assertion, essentially using a form of proof-by-contradiction.

In response, the teacher presented an ME question (Line 12), which drew out S6's justification of the mathematical statement made earlier (Line11). This prompted S6 to explain what he meant by "counting four more over again." S6 voluntarily came to the front of the classroom to demonstrate why the teacher's reasoning contained a conceptual error (Lines 13–14, 16–18). This student used the physical materials as a representational tool to externalize his mathematical thinking and provide common ground for all of the class participants. Gestures as well as speech were employed to enact the conceptual reasoning. Specifically, the gestures pointing to the six faces (Line 18) showed the shared edges between the faces where the teacher double-counted. This information is acknowledged by the teacher in Line 19 ("a::h"), which serves as a receipt of the new information (much like the expression "Oh"), rather than as a backchannel (Heritage 1984b).

S1 initiated further repair to the teacher's erroneous statement using verbal descriptions and gestures to show that the double-counting problem would be avoided if they calculated the volume using a layer-based system of counting (Line 20), rather than quantifying all blocks on each side. S6 showed his deep understanding by accepting S1's repair (Line 21) and appropriating it as his own. With fingers and palm spreading out on the surface of the cube, S6 interrupted the teacher (the overlapping speech at Line 22 was not acknowledged and therefore not coded as an elicitation) and provided a reification of S1's layer method (Lines 23 to 27). As a result, using iconic gestures to denote the layered structure of the assembled cube along with the physical blocks themselves, S6 and S1 were able to co-construct an argument that disproved the teacher's erroneous claims. Yet, pedagogically, the teacher's use of a trick question as an elicitation technique was an effective way to engage students in the discourse and prompted them to draw collectively on the knowledge of their peers in order to exercise the deep and complex mathematical processing of which they were capable, given the proper support.

In most circumstances an erroneous statement of this sort would constitute a violation of Grice's Conversational Maxim of Quality. However, in this instructional setting, a few students took up the challenge, rather than condemning or questioning the teacher's mathematical skill or authority (although see Line 6). From a grammatical perspective, this false statement was not technically an interrogative. But it's utility, as evident in students' responses, underscores the need for a flexible definition of elicitation, as originally proposed by Mehan (1979). As a result, the teacher effectively elicited evidence from students for and against the claim, and guided them into counterfactual reasoning, which Piaget and his colleagues (e.g., Inhelder & Piaget, 1958) identified as the most advanced form of logical reasoning. The counterfactual also led students to make comparisons while applying previous knowledge to a new situation.

DISCUSSION AND CONCLUSION

In this era of educational reform that emphasizes the social and linguistic nature of knowledge construction in the classroom (e.g., Baker, 2007; Cobb et al., 1993; Sfard, 2007; Wertsch, 1979, 1994), there is a pressing need to understand how teachers use classroom talk as a mediational tool to foster higher order reasoning among students. Although there is extensive research that focuses on teachers' use of negative feedback in response to students' erroneous utterances (e.g., Lyster, 1998; Lyster & Ranta, 1997; Nathan & Kim, 2006; Panova & Lyster, 2002), the analysis presented here examines the instructor's classroom talk as it relates to correct as well as incorrect responses from learners.

Using both general patterns across the corpus and selected excerpts, we show how one teacher's elicitations mediated students' mental processing by scaffolding the mathematical discourse in a manner that helped them advance from the factual-knowledge level to higher, metaprocess levels of mathematical thinking and speaking. When students gave inaccurate or incomplete answers, the teacher seldom gave direct, negative feedback. Instead, subsequent elicitations usually made reduced demands for the level of cognitive complexity needed to respond. This helped engage students, while providing opportunities for context-specific instruction that filled apparent gaps in students' knowledge. In this way, lower order elicitations by the teacher helped to scaffold student reasoning.

The teacher appeared to regulate the level of discourse as a reaction to students' successes as well. He often increased the elicitation level when students provided responses that were mathematically accurate. In this way, the teacher could assess the conceptual foundation upon which students' responses rested, and guide students to engage in more sophisticated forms of reasoning (Nassaji & Wells, 2000). Presumably, this enabled students to "try on" the kind of mathematical thinking that was ultimately expected of them, and to see what that form of cognitive activity was like from the inside.

It is important to note that other factors may also contribute to the overall pattern observed, such as the teacher's goals to make increased demands for overt demonstrations of students' more advanced thinking over the four days. This account would be expected also to generate an overall increase in complexity over the four days (Table 4). This is a reasonable hypothesis that we regard as compatible with the regulation hypothesis. However, an account based on shifting goals by the teacher does not, by itself, explain the micro-level adjustments of complexity documented in Table 3. Indeed, if the goal of elevating the complexity were the sole impetus, we would expect to see monotonically increasing complexity at the level of teacher elicitations over the arc of the four-day lesson, regardless of students' responses. This would not lead to the pattern of behavior we observed. A more likely account posits that the teacher systematically regulated the level of cognitive complexity of his elicitations in reaction to students' responses.

Instruction of the sort reported here fosters a climate of discursive mathematical practice. At the same time, expressing a plurality of views invites students to listen and publicly evaluate multiple forms of mathematical reasoning and expression that might otherwise remain tacit. Students in this kind of learning environment need to consider alternative perspectives (Greeno & MacWhinney, 2006) and work in both intra- and inter-psychological realms to establish a shared understanding (e.g. Lerman, 2001; Matusov, 1996; Nathan et al., 2007). As an example, the teacher's trick question from Excerpt 3 engaged students in evaluative reasoning and counterfactual argumentation that cultivated higher level cognitive processes and led them to make an analytical distinction between surface area and volume that might otherwise have remained confusing.

Intentions of the Teacher

In framing our investigation and the findings of our study, we have tried to remain agnostic about questions of whether or how the teacher's intentions drive the regulation of the cognitive level of the elicitations, or if these decisions are made without conscious awareness. We believe that a compelling case for responsive discourse regulation can be made without recourse to the teacher's intentions. By way of contrast, we show that statements made by the teacher about his curricular objectives when prompted with the video of Excerpt 3 offer some insights consistent with the reform views of promoting conceptual thinking about the mathematical structures and procedures that students generate. If we had to speculate, however, our position is that the teacher is focused primarily on larger, executive goals than those that govern the regulation of individual turns. The teacher appears to be motivated to instill in students a deep understanding of what the mathematics says, and how the analytic structure must necessarily map to (in this case) the patterns and physical structures of the objects under investigation. In this way, we take teaching

to be highly strategic, in that teachers control their efforts and employ particular resources and practices to achieve their objectives (e.g., Jones, Palincsar, Ogle, & Carr, 1987; Leinhardt, 1986).

Issues of Assigning Causality from within an Interactional Perspective

Causal inference is complex when addressing highly interactive phenomena, such as classroom discourse. What, for example, is the antecedent for a feedback loop where successive exchanges contribute to as well as influence an elaborate chain of events? Although we chose to focus our analyses on teacher elicitations, it is clear that other investigations of these data could be centered on students' utterances. Indeed, the very analyses we offer show how bound up teacher elicitations are to students' statements, and vice versa. Because of this, we have tried to be less forceful than we might otherwise be to attribute causality from antecedent teacher elicitations to consequent student responses and forms of thinking. Others surveying similar kinds of data (e.g., Roth, 1996) have exhibited less restraint in imputing causality.

At the same time, we feel that some significant weight can be placed on the role of the teacher and the teacher's actions, while still acknowledging that there is no sole cause that can be isolated to explain the events. That is, given classroom learning of mathematical reasoning as it normally proceeds we can lend some support to the intuition that the teacher's role was influential of the classroom outcome. This interpretation is based, to a large extent, on our understanding of the literature on institutional talk, which emphasizes its goal-directed nature and the asymmetrical constraints that shape the contributions of participants (Drew & Heritage, 1992). In particular, there are significant inequalities in the distribution of power, responsibility, knowledge, and communication resources in instructional interactions (Drew, 1991). Yet this complex picture of a dialectic process also highlights the need to tread carefully when ascribing causality.

A useful perspective on this is offered by Mackie (1980) in his distinction between conditions and causes. Kaplan (in press) summarizes Mackie's theory of causation along the following lines. Causes take place in a *causal field*, which takes into account a host of factors that contribute to the occurrence of an event. Consider that a set of factors reliably precedes the occurrence of an event, such as observing students exhibiting a certain level of mathematical reasoning for the first time. This set may be only one of several sets of factors, each of which provides alternative ways of achieving the sought-after event. For example, one set may focus on socially constructed discourse, and includes elements such as early and repeated attention to developing students' active listening skills, carefully regulated teacher elicitations, students' thoughtful responses to the teacher and one's peers, and sufficient class time spent on the topic of interest. An entirely different pathway could outline a more didactic approach, such as a lecture on a class of worked-out examples followed by individualized seatwork with worksheets that were then graded and handed back to each student.

Note that each approach includes several constituent factors, none of which are sufficient on their own to engender the event. Each set of factors (the socioconstructivist approach, the didactic approach, etc.) is sufficient, but not necessary. And within each set, the constituent parts (e.g., teacher regulation of elicitation skills, or hand-graded feedback) are neither necessary (the outcome could be achieved by the components of an alternative set) nor sufficient. We can also assume that each constituent part makes a unique contribution, and is therefore non-redundant with the other constituents of the same set. If we could articulate all of the pathways

to achieve the same event, then, taken together, the multiple sets or pathways serve as a *condition* of that event that are both necessary and sufficient (Kaplan, in press) . Any single factor (e.g., teacher regulation) plays an important role in the overall chain of events, and serves as an "inus condition" in Mackie's (1980) terminology, because it is an "insufficient but non-redundant part of an unnecessary but sufficient condition" (p. 62). In this sense, teacher elicitations operating within this causal field are not considered causal, but are a vital condition for engendering the behaviors exhibited by students in this classroom.

Although we do not have individual student knowledge or performance data to assess the growth of each participant, we offer some speculation on how the regulation of teacher elicitations may ultimately foster learning among students. The scaffolding provided by this teacher appears to be highly responsive to the perceived state and current needs of the vocal members of the group. Even when mixed responses were given among multiple speakers, the teacher most often addressed the statements that signaled inaccuracies in student knowledge, which would otherwise hamper the teacher's ability to pose elicitations of greater complexity to the class in the future. These general patterns of elicitations and responses lead us to entertain that the teacher may be managing something akin to group-level Zone of Proximal Development, or group-ZPD. This idea of group-ZPD has gained some attention in the literature (Guk & Kellogg, 2007; Nyikos & Hashimoto, 1997), but also has its detractors and critics (e.g., Wells, 1999). Strictly speaking, ZPD is defined with the individual learner in mind (Vygotsky, 1978). Guk and Kellogg (2007) lament this situation, however, and argue that an exclusively individualist perspective on ZPD is untenable for teachers, who predominantly plan at the level of the whole class or sub-groups. Ultimately, the proper study of group-ZPD would depend on data showing teacher responsiveness for collective interactions, coupled with data demonstrating expanded group performance following these interactions.

Conclusion

Classroom discourse serves as a window through which we can observe communication between students and their teacher in their natural setting, and witness the social accumulation of knowledge as embodied in changes in discourse structure (Cazden, 1988; Nathan et al., 2007; Sfard, 2007). These data present a brief portrait of some of the forms that discourse-based styles of teaching can employ, and it reveals instructional techniques for regulating student participation and cognitive functioning. This portrayal is necessarily limited, based on one teacher's practice during several contiguous days of a single, beginning-level algebra class. Yet cases of this sort help us to develop the theory and methods necessary to study these behaviors in greater depth. It is left to future studies to further advance our understanding of the relationship between classroom talk and cognitive development more broadly, and to establish the extent to which these instructional strategies are applied skillfully in classroom settings.

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APPENDIX A

Examples of Each Type of Teacher Utterance

NON-ELICITATIONS

Mathematical Statements
T: This is four by four by four.
Classroom Management
T: Listen [directed toward the whole class].

ELICITATIONS
Questions
S2: it's 4 by 4 right? 4 by 4. And,
→T: Well I'm sorry, why do you say 4 by 4?
S3: because the four cubes.
S2: It's 4 by 6
S3: What are you talking about?
→T: What's 4 what by 4 what?
S: 4 cubes . . . one side . . . has four cubes in a row, by four cubes like that.

Continuers and Back-channels

T: Look at your verbal rules, look at your equations. How could they have possibly come up with the number ninety-six?

S: They took the side length um of nine . . . wait ten.

T: ten.

S: They subtracted two which equals eight.

→T: Uh huh

S: and then they timed . . . multiplied it by the twelve.

T: Which is:::

S: Well eight times twelve is ninety-six.

Provocative Statements

S2: I remember them, but I forgot.

→T: ok, you called them edges, you called them corners, I heard something else.

S3: Axis.

Prolonged Teacher's utterances

S: Well the ones with three, just call them intercepts. The ones with two to just count the edges, and the ones with one to just count the faces.

→T: Okay. So:::

S: So you don't count the faces on the intercepts.

Calling a student by name

S: Add two . . . if you add two.

→T: Jason↑

S: Um, okay, you take the side length and you square it and you take that answer and you multiply it by the side length again.

Requests to do something

S: like the points

→T: Here, come show us.

S: I can't remember what they're called but, it's like this. Goes around there, there, there, comes down here, there's eight of them if you count them all.

T: Ah! Very good

APPENDIX B

Jeffersonian Notation Transcription Conventions Used in the Excerpts

[	Point of overlap onset
]	Point of overlap termination
=	No interval between adjacent two turns
(2.3)	Interval between utterances (in seconds)
(.)	Very short untimed pause
word	Speaker emphasis
the:::	Lengthening of the preceding sound
?	Rising intonation, not necessarily a question

, Low-rising intonation, suggesting continuation

. Falling (final) intonation

CAPS Especially loud sounds relative to surrounding talk

° ° Utterances between degree signs are noticeably quieter than surrounding talk

↑↓ Marked shifts into higher or lower pitch in the utterance following the arrow

( ) A stretch of unclear or unintelligible speech

(( )) Nonverbal actions