AC 2009-1790: INTEGRATION OF MATHEMATICS IN PRE-COLLEGE ENGINEERING: THE SEARCH FOR EXPLICIT CONNECTIONS

Amy Prevost, University of Wisconsin - Madison

Mitchell Nathan, University of Wisconsin - Madison

Dr. Mitchell Nathan is Professor of Educational Psychology at the University of Wisconsin-Madison, where he is Chair of the Learning Sciences program. He holds appointments in Curriculum and Instruction, the Psychology Department, the Wisconsin Center for Educational Research, and the Center on Education and Work.

Dr. Nathan received his PhD in experimental (cognitive) psychology from the University of Colorado at Boulder. He holds a B.S. in electrical and computer engineering, mathematics and history from Carnegie Mellon University. As an engineer, Dr. Nathan worked in research and development in artificial intelligence and expert systems, computer vision and robotic systems mobility. This work inspired an interest in how people represent their knowledge of the physical and conceptual realms.

Dr. Nathan’s research is largely rooted in cognitive, embodied and social perspectives on learning and instruction, and employs quantitative and qualitative methods. Currently, he examines the intersection of student and teacher cognition as it plays out in classroom learning situations, primarily involving middle and high school mathematics, science and engineering. His research on students’ reasoning showed that they may invent effective strategies and representations for solving math problems, and these methods can serve as bridges for instruction. He is also exploring the embodied nature of students’ knowledge, as exhibited by gestures, and the mediating effects of action on conceptual knowledge. His studies of teachers' beliefs about the development of students' mathematical reasoning showed that content experts can show evidence of expert blind spot, which influences teachers’ expectations of what makes things difficult for their students.

He is currently co-principal investigator for the AWAKEN Project (funded by NSF-EEP), which examines the nature of high school pre-engineering, early college engineering, and professional engineering practice in order to foster a more diverse and more able pool of engineering students and practitioners.

Benjamin Stein, University of Wisconsin

Natalie Tran, California State University-Bakersfield

Allen Phelps, University of Wisconsin - Madison

Integration of Mathematics in a Pre-College Engineering Curriculum: The Search for Explicit Connections

Abstract

Educational leaders and researchers have mandated that technical education and academic subject areas be integrated so students can develop both academic and occupational competency. Mathematics, in particular, is recognized for its singular importance for modeling and generalization. In response to this clarion call, engineering and K-12 pre-engineering curricula are being developed and redesigned to invigorate the engineering pipeline and to provide an integrated program of STEM education. Explicit integration also enhances the likelihood that learners will be able to transfer new knowledge to novel situations. An important research aim, then, is to document the extent to which these objectives are being achieved. In conducting the current study, our goal was to identify all instances of *explicit integration* of mathematics concepts in three pre-engineering curricula that make up the 3-year foundations sequence for a widely adopted high school technical education program, *Project Lead the Way* (PLTW). Explicit integration is defined as any instance wherein the materials specifically point to a mathematics principle, law, or formula, and depict how it is used to carry out or understand an engineering concept, task or skill. We used the *Standards* as adopted by the National Council of Teachers of Mathematics to determine the target math concepts. For each of the three foundations courses, we analyzed the content of: (a) the *intended curricula*, including planning materials, performance objectives and classroom activities; (b) the *assessed curricula*, including student projects and presentations, and written examinations; and (c) and the teacher training materials. In addition to the structure of each course, we looked for alignment between the intended and assessed curricula and for the longitudinal progression as one advances through the 3-year program of courses.

Overall, we found that the explicit integration of math concepts with regards to engineering concepts in all three *PLTW* courses was apparent, but weakly so, and showed many areas of potential improvement. While there are many implicitly embedded opportunities for creating connections between the math concepts and the engineering activities and topics, many of these opportunities were not explicitly stated, and are likely to go unaddressed in the classroom. We found that the two later courses integrated mathematics concepts better than the entry course. We also found many areas in each of the courses where the intended and assessed curricula were misaligned, so that topics emphasized in the course were not tested, while concepts and skills on tests were not always supported by the course materials. While these findings may seem at odds with claims by the curriculum developers, we attribute the different interpretations to Expert Blind Spot, the psychological phenomenon that those highly knowledgeable in their own fields more readily see the deep conceptual underpinnings than novices do. We then use the results of these analyses to illustrate how mathematics concepts can be explicitly integrated with pre-engineering activities, and thereby enhance the likelihood that learning will be deep and foster transfer to new tasks and settings.

Introduction

In order for the US to maintain its quality of life, national security and economic vitality, the National Research Council, in *Rising Above the Gathering Storm* (2007), calls for educational leaders to optimize its knowledge-based resources and energize the United States' science, technology, engineering and mathematics (STEM) career pipeline. Furthermore, the 1990 reauthorization of the Perkins Vocational Education Act mandated that technical education and academic math and science topics must be integrated "so that students achieve both academic and occupational competencies." Mathematics, in particular, is recognized for its central role for modeling and generalization[1].

In response to this clarion call, engineering and K-12 pre-engineering curricula are being developed and redesigned to invigorate the engineering pipeline and to provide an integrated program of STEM education. An important research aim, then, is to document the extent to which these objectives are being achieved. In conducting the current study, our goal was to identify all instances of *explicit integration* of mathematics concepts in three pre-engineering courses. We define explicit integration as any instance wherein the materials specifically point to a mathematics principle, law, or formula, and depict how it is used to carry out or understand an engineering concept, task or skill. Learning skills and new concepts requires a conceptual basis for it to be impactful[2]. Furthermore, a lack of integration between one's prior knowledge and new curriculum materials is problematic given the education and cognitive science research that emphasizes the importance of explicit integration of conceptual knowledge for successful transfer of that knowledge to novel applications or new situations[3 4 5].

Project Lead the Way (*PLTW*) was chosen as the focal curriculum because of its wide dissemination in the US (it has been adopted by over 17% of US high schools, and is present in all 50 states) and its stated objective to integrate students' college preparatory and technical education programs of study: "*PLTW*'s premier high school program, Pathway To Engineering™, is a four-year course of study integrated into the students' core curriculum. The combination of traditional math and science courses with innovative Pathway To Engineering courses prepares students for college majors in engineering and E/T fields and offers them the opportunity to earn college credit while still in high school"[6]. Indeed, the NRC report, *Rising Above the Gathering Storm*[7] explicitly identifies *PLTW* as a model curriculum for providing the kind of rigorous K-12 materials needed to improve math and science learning and increase America's technological talent pool.

Prior Research on Pre-Engineering Curricula

Recent content analyses of K-12 pre-engineering curricula, including *PLTW*, suggest the presence of mathematics in particular is "noticeably thin." Typically, these are studies of the *intended* curriculum, and therefore address only the static plan that is put forth in the printed materials used for the courses under investigation.

In their analysis of the *PLTW* high school *intended curriculum*, Nathan and colleagues[8] examined the absolute and relative frequency with which *PLTW* addresses the mathematics standards (as obtained from the National Council of Teachers of Mathematics)[9] in its three core courses (*Introduction to Engineering Design, Principles of Engineering,* and *Digital Electronics*), and compared this to the mathematics curricula that high school students experience concurrently in their academic courses. The curriculum analysis study distinguished between math content standards and process standards. Math *content standards* are the topics of math, including: numbers and operations; patterns, functions, and algebra; geometry and spatial sense; and measurement. Math *process standards* address, in complimentary fashion, how math is performed and how math knowledge is created, verified, and disseminated, including: methods of data analysis; problem solving; reasoning and proof; communication; connections made across fields of mathematics and applications outside of math; and ways of representing mathematical relationships. The results of this comparative curriculum analysis show that the pre-engineering *PLTW* curriculum addresses far fewer math content standards than are addressed by the academic math courses taken by the same students. *PLTW* courses do a much better job addressing process standards, particularly problem solving and uses of representations.

Welty and colleagues[10] took a broader view, though obtained less detail about any one program. They analyzed twenty-two pre-K-12 pre-engineering curricula, including nine high school programs. The analysis explored the mission and goals of each curriculum; the presence of engineering concepts; and how each curriculum explicitly treated mathematics and science in with regards to engineering problems. The investigators offer only preliminary findings at this point. However, their remarks to date are most striking about the shallow role of mathematics across the corpus of curricula. In findings that echo the Nathan et al.[11] study of *PLTW*, Welty and colleagues[10] lament "the noticeably thin presence of mathematics" in K-12 engineering curricula (p. 10). They explained, "Most of the mathematics in engineering curricula simply involved taking measurements and gathering, organizing and presenting data. Very little attention was given to using mathematics to solve for unknowns. Furthermore, little attention was given to the power of mathematical models in engineering design" (p. 9)[10].

The Intended, Enacted, Assessed and Learned Curriculum

Curriculum analyses can be divided into the study of intended, enacted, assessed, and learned curricula[12]. The *intended curriculum* refers to the content of the course or program under investigation. For K-12 education, the intended curriculum typically includes the printed course materials and other closely connected resources (manipulatives, software, etc.) as well as national and state curriculum standards, which specify the grade-specific objectives for what each student must know and be able to do. The *enacted curriculum* refers to the specific content as it is taught by teachers and studied by students during the course of learning and instruction. In contrast to the intended curriculum, the enacted curriculum is dynamic and varies from teacher to teacher, and even changes across classrooms taught by the same instructor, as the specific interactions vary with different students. The *assessed curriculum* refers to the specific content that is tested and can differ markedly from the intended and enacted curricula as tests are drafted by the federal government (thought instruments like NAEP, for example), individual states, districts, and the teachers themselves. The *learned curriculum* captures the actual changes in knowledge by the individual students, which reflects the notion that students can and often do learn more and less than offered in the instructional context.

All four perspectives are, of course, valuable for addressing the learning and teaching experience in its entirety. In this study, our focus is specifically on the intended and assessed curricula for the three *PLTW* foundations courses, as specified by the program's course materials. In other work[13] we address the enacted curriculum as it unfolds in the classroom.

The study of the intended curriculum admittedly addresses only one aspect of the complex system within which engineering and technical education arises. Yet it is essential to document how curricula are structured, apart from their enactment, for several reasons. First, curricula institutionalize certain views of learning and development by selecting what is and is not covered, and the sequence of their organization[14]. Secondly, educators appear to internalize the views of knowledge and development as they appear in the curriculum materials, even when those are tacit, and even when they conflict with basic principles of educational reform that are adhered to by teachers[15]. These internalized views then shape the instructional and assessment practices of teachers, and so directly influences the learning opportunities and experiences of learners. Finally, curriculum analyses help to inform studies of the complexities of the classroom learning processes and instructional interactions that develop around these specific lessons and activities.

In addition to analyzing the intended curriculum, knowing the content of the assessed curriculum is important because student achievement is measured only for the content that is tested. The assessed curriculum also provides an indicator of the priorities and learning objectives held by a teacher or administrative body, at the school, district or state level. Finally, analysis of the alignment of the intended and assessed curriculum--the match between the design of the course and the structure of tests and other assessment instruments--provides an additional perspective on the future learning experiences of students.

Focus of Research

As part of our analyses, we set out to identify the occasions where mathematics and science concepts were presented during the classroom lessons and activities. We were particularly interested in identifying all of the instances of *explicit integration* between mathematics concepts (as defined by the national standards) and engineering activities through a curriculum review as a strategic means of advancing students' academic knowledge as well as fostering greater transfer of knowledge both of the mathematics and of the engineering knowledge and skills. This is in contrast to the concepts being implicitly embedded within the activities and resources, such as when core math concepts or procedures are glossed over by the instructional materials or performed automatically by CAD software.

This study differs from earlier pre-engineering curriculum studies in three important respects. First, it takes a broader view of curriculum analysis, including both the intended and assessed curricula. This broader view allows us to report on the alignment between intended and assessed curricula, as well as the structure of the curriculum more generally. Second, it includes the analysis of teacher training materials, which provides a valuable perspective on how teachers are directed, and the pedagogical emphases made by the program designers, allowing for some insight, albeit limited, into the enacted curriculum that is to be experienced by students in the classroom. This body of materials brings the analysis closer to questions that are typically only addressed by more intensive, observational studies of the enacted curriculum, such as how and why certain activities and concepts are presented in particular ways in the classroom. Finally, we examine these curricula from a developmental perspective within the program, and examine the three foundations in succession from what we believe to be introductory to intermediate to more advanced course materials. This allows us to document any changes in the emphases of the program. Specifically, as the level of student maturity increases and as students acquire greater skill and content knowledge, we can ask whether the degree to which the integration of mathematics concepts are made more or less explicit over time. These considerations lead to these organizing research questions that drive the current investigation:

1. **What is the emphasis on explicitly integrating students' conceptual understanding?** Specifically, we sought to determine the frequency with which mathematics concepts arose during the lessons, assessments, and teacher training materials we analyzed, and how often these concepts were *explicitly* integrated (as opposed to being implicitly embedded) with the engineering activities and lessons that were the focus of the curriculum materials. From a practical perspective, this may also identify new opportunities for improving the connections between engineering knowledge and skills and the underlying mathematical concepts and procedures that are so important for transfer of knowledge and for college admissions.

2. **What changes do we see over the three-year sequence of courses?** Specifically, as students and teachers move along the curriculum program, from *Introduction to Engineering Design* to *Principles of Engineering* to *Digital Electronics*, how does this change the frequency with which mathematics concepts are presented, the math standards that are addressed, and the extent to which these concepts are explicitly integrated.

3. **How well aligned are classroom and assessment materials?** By including analyses of both course materials and assessment activities (e.g., projects) and instruments (tests) we can provide analytically supported recommendations for improving the overall program design.

Methods

Materials

This study examined the three *PLTW* foundation courses, and identified instances of explicit integration between mathematics and science concepts on the one hand, and the engineering activities of the course curricula on the other. By *explicit integration,* we mean instances wherein curriculum materials specifically point to a mathematics principle, law, or formula, and depict how it is used to carry out or understand the engineering concept or activity that is the focus of the lesson. The National Council of Teachers of Mathematics note on their website that, "all students should learn an ambitious common foundation of mathematical ideas and applications. This shared mathematical understanding is as important for students who will enter the workplace as it is for those who will pursue further study in mathematics and science"[18].

The courses that we analyzed are entitled *Introduction to Engineering Design (IED), Principles of Engineering (POE)* and *Digital Electronics (DE)*; all of which are offered at the high school level and are the foundation courses for the *PLTW* program, *Pathway to Engineering*. These courses were evaluated sequentially in the order that we believe the courses are intended to be taken as indicated by the sample schedule offered on the *Project Lead the Way* website[16]; we took the IED course to be the first in the series of courses, with POE to follow and DE third.

We identified four areas of analysis for each *PLTW* foundation course as shown in Table 1. The materials for each consist of a course manual, or text. Each text is divided into units, which are in turn divided into sub-units. Each unit and sub-unit begins with a narrative or "anticipatory set," as it is called within the POE and DE curricula. For the purposes of this study, we will refer to the narrative portions of the curricula as the "anticipatory set" regardless of how they are termed in the given curriculum, as we believe this to be the more descriptive of the two terms used by *PTLW*. The anticipatory set is followed by a daily lesson plan, concepts, and performance objectives. In addition to these four items, we also evaluated the presentations that go along with each lesson. Presentations are in PowerPoint format. Taken together, these items give the student and instructor a frame of reference for the lesson as well a framework for completing it. In total, we have categorized these five items (anticipatory set, daily lesson plan, concepts, performance objectives, and the presentations) as the *plan* for the lesson.

Table 1: Materials for Analysis within each Curriculum

Intended Curriculum		Assessed Curriculum	Enacted Curriculum
Student Materials			Teacher Materials
Planning (Anticipatory Set; Concepts; Daily Lesson Plan; Performance Objectives; Presentations)	Activities (Worksheets; Hands-on work)	Assessments (Projects; Presentations given by students; and Written examinations)	Training Materials (Training documents; Activities; Projects; and Self-Assessment and Self-Reflection Items)
In the set up to the lesson or within the materials presented during the lesson, are math and science concepts explicitly connected to engineering concepts or activities?	Are students directed to actively connect math and science concepts to engineering concepts in their class work or homework?	Are the students assessed in a way that allows them to demonstrate connections of math and science concepts to engineering concepts?	Is the teacher presented with materials in training that would explicitly connect science and math concepts to engineering concepts?

Next, a list of activities is given. Students complete these activities in order to learn the material and get hands-on experiences applying the concepts. Activities are the most essential part of the curriculum – they constitute the work that is done in each course - and we thus believe that they are the most important category in our analysis. Activities include worksheets, which are included in the curriculum materials as well as hands-on work, which is merely described in the curriculum. Lastly, students are assessed to determine what they learned. Assessments are generally given at the end of each sub-unit. However, there are occasionally assessments that are given for the entire unit. Assessments can be projects, presentations or simply written examinations. The number of assessments varies widely from course to course, and accounting for these variations is described in the procedures.

In addition to student materials, we analyzed teacher training documents from the two-week intensive Summer Training Institute; attendance at the Summer Training Institute for each course is compulsory prior to teaching any *PLTW* course. The teacher training materials are presented as an aggregate. Within these teacher training materials, we reviewed activities, projects and self reflection items. Through analysis of these training materials, we intended to better understand the extent to which teachers were trained to explicitly connect mathematical and scientific concepts to the technical education topics and activities. Further, through the teacher training materials, we hoped to gain better understand what the enacted curriculum might look like.

Procedure

We performed content analyses using the framework suggested by the National Research Council (2004). These content analyses "focus almost exclusively on examining the content of curriculum materials; these analyses usually rely on expert review and judgments about such things as accuracy, depth of coverage, or on the logical sequencing of topics" (p.2). The content analyses should address the following dimensions: a) Clarity, comprehensiveness, accuracy, depth of mathematical inquiry and mathematical reasoning, organization, and balance (disciplinary perspectives); b) Engagement, timeliness and support for diversity, and assessment (learner-oriented perspectives); c) Pedagogy, resources, and professional development (teacher- and resource-oriented perspectives)[17]. In reviewing the curricula, our goal is specifically to identify instances of explicit integration between mathematics concepts and engineering concepts.

Mathematics standards recommended by the National Council Teachers of Mathematics (NCTM) for grades 9-12 were used as the frame of reference in identifying these instances. As reviewed above, the NCTM standards address the following topics: a) numbers and operations; b) patterns, functions, and Algebra; c) geometry and spatial sense; d) measurement; e) data analysis, statistics, and probability; f) problem solving; g) reasoning and proof; h) communication; i) connections; j) and representation. It is important to note that standards a-e represent content standards –the topics of the mathematics -- and standards f-j describe process knowledge or how mathematics is practiced.

The number of NCTM standards specifically connected to the curriculum in each unit was recorded at the sub-unit level. To understand the relative emphasis of the standards within each course over time, the proportion (percentage) of each standard within a given unit was calculated by dividing the frequency of a given standard by the total number of possible standards addressed in that unit. Thus, the percentage represents the emphasis of each standard relative to each other within a given unit. Since the planning materials include five categories (anticipatory set, concepts, daily lesson plan, performance objectives, and presentations), the total possible standards addressed is calculated as all of these elements combined. The number of activities is calculated by totaling the number of subunits in each curriculum. While *PLTW* presents activities as assessments, there are specified evaluation materials throughout the curricula as well. These are what we used to gauge evaluation, allowing us to distinguish between quizzes and examinations versus daily activities. In the case of assessment, each evaluation tool was considered separately, yielding its own number of possible points of explicit integration. Lastly, with regards to teacher training materials, percentages of representation were calculated in a fashion similar to that of the other materials, including instances in which teachers are asked to self-assess their abilities with regards to the curricular segments presented at the given summer institute. These materials were aggregated in order to calculate percent explicit integration for the teacher training materials.

For all elements, the frequency of occurrence of explicit integration of a given standard was calculated as a percentage based on the number of items overall. This allows us greater leverage for comparing the four areas of analysis (teacher training, planning, activities, assessments) as well as in comparing the various curricula, which each contain varying numbers of units and subunits. An example of the scoring system we used is shown below (Table 2); note that this example is to be used for understanding the methodology only and results presented here were not used for analysis.

Scoring and percent integration was accomplished first by identifying the areas of explicit integration in each curricular area (Training, Planning, Activities and Assessments). This was achieved through the comparison of the standard definition with what is presented in the curriculum. The sub-unit was our smallest unit of measurement. In the example given in Table 2 (IED, Unit 6), there are five sub-units including the introduction to the unit. Once items were scored, they were added for each standard within each type of curricular area. These became the numerator "X" in our calculation of total percent explicit integration. Thus, "X" is the number of places that were coded as explicitly integrated mathematics and engineering. The total number of items in each curricular area was also tallied. These became the denominator "N" in our calculation. Thus, the number of opportunities for explicit integration of math and engineering are given by the denominator "N" for each curriculum and within each curricular area. Percent integration then was simply calculated by dividing X by N (X/N) and multiplying by 100.

Table 2 represents one NCTM mathematics standard for one unit in a given course. In this example, we use IED, Unit 6 and assess only the content standard: Algebra. More information about this content standard can be found at the NCTM website[18].

Table 2: Sample Scoring System as Applied to a Single Unit from IED (Unit 6) for a Singe Content Standard (Algebra).

Introduction to Engineering Design © 2000 Mathematics Content Standard: Algebra Unit 6 – Modeling			
Items that Include Explicit Integration of Algebra and Engineering Concepts	No. Scored Items (X)	Opportunities for Explicit Integration (N)	Percent Integration
Training: No Items (Score as 0)	Xt = 0	Teacher Training Materials: The teacher training materials do not contain any instances of explicit integration. The teacher training materials area assessed with relationship to the entire *PLTW* course rather than at the unit level because the summer institutes are not comprehensive and do not cover the entire curriculum. In this case, there are 26 possible instances of explicit integration (taken from the IED teacher training curriculum materials), so Nt = 26.	Xt/Nt 0/26 = 0%
Planning: 6.0 Narrative (score as 1) 6.0 Daily Lesson Plan 10-11 (score as 1; although there are two Daily Lesson Plan items that meet the criteria for explicit integration, we score the item as 1 (rather than 2) because both occur within the same subunit, which is our smallest unit of analysis.) 6.4 Narrative (score as 1)	Xp = 3	Unit 6 contains 4 sub-units plus the introduction to the unit. Therefore, the total possible instance of explicit integration (N) in this unit is 25 for planning (since there are five sub-units and five contributing sub-categories to this group) Np = 25	Xp/Np 3/25 =12%
Activities: 6.4 Activity 3 (score as 1) 6.5 Activity 10 (score as 1)	Xac = 2	Unit 6 contains 4 sub-units plus the introduction to the unit. Therefore, the total possible instance of explicit integration (N) in this unit is 6 for activities so Nac = 5	Xac/Nac 2/5 = 40%
Assessment: 6.4 Assessment (score as 1)	Xas = 1	The number of assessments is not always congruent with the number of sub-units; in this example Nas =4 since there were only 4 assessments within the unit.	Xas/Nas 1/4 = 25%

In addition to scoring the curriculum, we also identified specific examples of where opportunities for integration were seized and where they could be strengthened. These examples are given throughout the results section and are included as part of the discussion. The extent of integration was judged on a) whether mathematical concepts were identified as such and b) their relevance to a given *PLTW* lesson. Lessons incorporating pure math instruction and vocabulary were judged to have explicit identification, whereas those using only technical terms and formulas were judged as not explicit. The relevance of a lesson's math standards was judged on whether they are presented as requisite or ancillary for the activities and assessments. Coders independently and blindly identified examples of explicit mathematics integration. The independent coding of these examples by two researchers was then compared with what we had scored for the training, planning, activities and assessments for each curriculum. This provided a means of creating an inter-rater reliability measure. Examples where there were opportunities for integration with mathematics concepts, but no evidence of explicit integration were identified separately. In our discussion, we review some of these examples and point out how an implicitly embedded example can be modified slightly to include explicit integration of math and engineering concepts.

Results

Overall, we found that the explicit integration of math concepts with regards to engineering concepts in all three *PLTW* courses was apparent, but weakly so, and showed many areas of potential improvement. While there are many implicitly embedded opportunities for creating connections between the math concepts and the engineering activities and topics, many of these opportunities were not explicitly stated, and are likely to go unaddressed in the classroom. In comparing each of the three *PLTW* foundation courses, we found that the two later courses, POE and DE, integrate mathematics concepts better than IED. This is expected in part due to the content of the courses, as well as the sequential nature of course offerings, wherein one course builds on another. It appears from the *PLTW* website that the courses are intended to be taken sequentially, beginning with IED, through POE and finally DE. The depth of knowledge presented in both engineering and mathematics seems to increase following the IED course. While it makes sense that the introductory course would not offer as great detail in either engineering concepts or mathematics compared to other courses, this may be especially problematic for students already struggling with low math and science achievement, since many students may only take this one pre-engineering course[19]. Our analyses do show that small changes to the curriculum could lead to much greater explicit integration of math and engineering concepts, as we discuss in the final section of this paper. First, we elaborate on these findings below. Data is presented for each course (in total) as well as for each segment of the curriculum (Teacher Training, Course Planning, Course Activities and Assessments). We thus compare each *PLTW* foundation course overall as well compare them based on the components we identified earlier.

Overall Analysis and Overall Comparison

Introduction to Engineering Design (IED)

The IED curriculum is the first *PLTW* foundation course in our analysis. The goal of the curriculum is to introduce students to the tools that engineers use in order to come up with ideas, bring them to life and subsequently assess them. Students use AutoDesk design software to learn about CAD, 2D and 3D modeling, and to complete assignments. Further, much of the project work that students are asked to do as they are introduced to engineering as a profession address “soft skills.” For example, they are asked to research engineering careers using interviews or the Internet, and they are asked to research an engineering project in their community.

As shown in Table 3, we found low rates of explicit integration of mathematics concepts with the engineering activities. This is perhaps due to the emphasis this course places on these soft projects along with the extensive use of computer software that we see gaps in the explicit integration of mathematics.

Table 3: Percent Explicit Integration in the Introduction to Engineering Design © 2000.

	Planning (X_p)		Activities (X_{ac})		Assess-ment (X_{as})		Training (X_t)	
	$N_p = 132$	Percent Integration (X_p/N_p)	$N_{ac} = 40$	Percent Integration (X_{ac}/N_{ac})	$N_{as} = 42$	Percent Integration (X_{as}/N_{as})	$N_t = 26$	Percent Integration (X_t / N_t)
Content Standards								
Number	3	2.3	1	2.5	2	4.8	5	19.2
Algebra	4	3	2	5	3	7.1	3	11.5
Geometry	18	13.6	2	5	12	28.6	8	30.8
Measure-ment	7	5.3	2	5	7	16.7	8	30.8
Data and Probability	8	6.1	4	10	4	9.5	1	3.8
Process Standards:								
Problem Solving	2	1.5	1	2.5	2	4.8	1	3.8
Reasoning	0	0	2	5	2	4.8	3	11.5
Connection	2	1.5	2	5	0	0	7	26.9
Represent-ation	23	17.4	11	27.5	9	21.4	14	53.8
Commun-ication	9	6.8	5	12.5	9	21.4	10	38.5

Yet, even taking into account that there is relatively less mathematics in this course in general, the mathematics that is present is seldom explicitly integrated, and tends to be implicitly embedded in the course tools and presentations. In the teacher training materials, instructors are trained with materials that explicitly integrate the math concepts and engineering concepts the most in the areas of geometry and measurement (from the content standards) as well as in representation, communication and connections (from the process standards). Areas that are the least explicitly integrated are of the content standards for algebra, data and probability as well as the process standards for problem solving, and reasoning. The planning portion of the curriculum somewhat parallels that trend. Geometry and representation (a process standard) are the mathematics standards that are explicitly integrated the most with the engineering materials in this section of the analysis. However, their respective percent integration is only roughly 14% and 17% respectively (Table 3). The other math standards are explicitly integrated less than 10% of the time.

Activities have very little explicit integration. The strongest area of explicit integration for Activities is in representation (since much of the work involves graphing or sketching). Assessments explicitly integrate the math and engineering concepts better than any of the other areas of analysis for this course; again the content areas of geometry and measurement show the most explicit integration at 29% and 17% as well as the process areas of representation and communication, at roughly 21% each.

There also seems to be a mismatch between the level of integration of mathematics in the activities and assessments for this course. There is little to no explicit integration of math in the activities students must complete, but there is explicit integration of math in the assessment pieces presented in each unit. This dissociation between the intended and assessed curricula is particularly true in the areas of geometry and measurement where only 5% of the activities have explicitly integrated geometry and measurement, but close to 29% of the assessments do. Unit 6, which comprises 50 days' worth of material, serves as a clear example of this disconnect – its activities use little math, and its assessments do not apply their math concepts to the design experience. Its 10 activities require product design and computer modeling, but most of the math is performed by the CAD program. Although several tasks require measurements and one of them has a volume calculation, most of them leave the computations to the software. Meanwhile the 2 of the 5 assessments in Unit 6 require calculations, but their math content is hardly relevant to the rest of the course material. Section 6.4's "Mathematical Modeling" assessment has one volume calculation, but the one other problem given has to do with the statistics of a class' test scores. The "Graphical Modeling" assessment of 6.2 is farther afield, involving the graphing of census data.

Principles of Engineering (POE)

POE is the second foundation course in the *PLTW* sequence that we analyzed. Within POE students learn about various concepts inherent to the field of engineering. Among these concepts are thumbnail and orthographic sketching, perspective drawing, free-body diagramming, the design process, X and Y components of vectors, thermodynamics, fluid and electrical systems, and mechanisms and simple machines.

The POE curriculum shows an increase with regards to explicit integration of math in all areas over IED. As Table 4 shows, the training materials, with the exception of measurement from the content standards and communication from the process standards, all show at least 20% explicit integration. However, the planning materials do not frame the lessons with integrated math. All standards are explicitly integrated at a rate of 12% or less in the planning materials. The activities do a much better job of integration, however, allowing students to work with explicitly integrated mathematics concepts over most of the standards. Some standards even had greater than 50% explicit integration of math and engineering concepts. Of note is that this curriculum had better explicit integration overall.

Table 4: Percent Explicit Integration of Mathematics in the Principles of Engineering Curriculum © 2004

	Planning (X_p)		Activities (X_{ac})		Assess- ment (X_{as})		Training (X_t)	
	$N_p = 145$	Percent Integration (X_p/N_p)	$N_{ac}= 32$	Percent Integration (X_{ac}/N_{ac})	$N_{as}=32$	Percent Integration (X_{as}/N_{as})	$N_t = 55$	Percent Integration (X_t / N_t)
Content Standards								
Number	18	12.4	15	46.9	3	9.4	17	30.9
Algebra	11	7.6	11	34.4	7	21.9	11	20.0
Geometry	15	10.3	9	28.1	1	3.1	17	30.9
Measure- ment	11	7.6	13	40.6	1	3.1	9	16.4
Data and Probability	12	8.3	10	31.3	6	18.8	11	20.0
Process Standards:								
Problem Solving	3	2.1	10	31.3	1	3.1	13	23.6
Reasoning	3	2.1	8	25	1	3.1	12	21.8
Connection	11	7.6	12	37.5	3	9.4	14	25.5
Represent- ation	16	11	14	43.8	6	18.8	21	38.2
Commun- ication	9	6.2	3	9.4	0	0	1	1.8

There were gaps, however. For instance, in Unit 2, students learn about Data Representation and Presentation. One activity in this section details different types of graphs and charts, illustrating how to use Excel to create each. Datasets are provided, but the words "function, domain, range" do not come up. Percentages are not reviewed in the pie chart context. Explicit math connections were not clear in the planning portions of the POE curriculum, as is shown by the low percentages in this area.

In contrast to the IED curriculum, the assessments in POE were weaker than the activities with regards to this. That is, the activities allowed students to practice using mathematics concepts in engineering contexts, but the students were subsequently not assessed in many of these abilities. Thus, the POE curriculum, like IED suffers from poor alignment and dissociation between the intended and assessed curricula. This dissociation is opposite that of IED, which explicitly integrated the assessments, but not the activities.

Digital Electronics (DE)

DE is the final of the three *PLTW* foundation courses. According to the *PLTW* website, this course "teaches applied logic through work with electronic circuitry, which students also construct and test for functionality"[20]. This is achieved through the introduction of resistance, circuits, Ohm's and Kirchhoff's Laws, logic gates, Boolean expression and logic theorems and proofs. Students practice Karnaugh mapping and work with combinatorial logic on several problems. For example, in one project, students design a combinatorial logic circuit that displays a birthday in as the month, day and year, in order.

With few exceptions, DE has the most frequent occurrences of explicit integration of math when compared to the other two foundation courses. For instance, each sub-unit includes engineering specific activities as well as the addition of math lessons. The math lessons are essentially worksheets that incorporate math to assist students in learning mathematics associated with the unit or sub-unit topic. In addition to asking questions about new material, the math lesson worksheets also ask students to review concepts from previous units. As an example, the math lesson in Unit 2 of DE is on number conversion. The unit topic is Number Systems, so the math lesson explicitly integrates mathematics to the engineering presented in the unit. The worksheet in Unit 2.1 asks students to convert numbers to and from the Binary Coded Decimal (BCD) system, and as a review, also asks students to calculate resistance (which was previously covered). In additional examples of math lesson problems, students are asked to use algebra to solve problems about capacitors (Unit 1), analyze logic gates (Unit 3) with a review of BCD conversion (from Unit 2), and apply Boolean algebra, DeMorgan's Theorem and Karnaugh maps to simplify logic expressions (Unit 4).

We evaluated the math lessons separately, since they were not included in any of the other courses and occasionally went beyond the scope of high school math, thus not aligning with the NCTM standards. This data is presented in Table 5.

Table 5: Percent Explicit Integration of Mathematics in the Mathematics Lessons of Digital Electronics © 2004

$N_{ml} = 39$	X_{ml}	% Integration (X_{ml} / N_{ml})
Content Standards:		
Number	20	51.3
Algebra	17	43.6
Geometry	0	0
Measurement	0	0
Data/Probability	2	5.1
Process Standards:		
Problem Solving	1	2.6
Reasoning	6	15.4
Connection	19	48.7
Representation	16	41.0
Communication	0	0

In general, the addition of the math lessons strengthened the academic rigor of the curriculum overall and created valuable explicit linkages both to math concepts covered in the DE curriculum and those previously covered through review questions. For instance, the learning of binary numbers is beyond the scope of a basic high school math curriculum. However, as the curriculum does delve into this higher level of math, it also does a better job than the other *PTLW* foundation courses of explicitly connecting high school level mathematics to the engineering concepts.

Table 6 shows the analysis of explicit integration across the entire Digital Electronics curriculum. The teacher training materials showed at least 30% explicit integration (with the exception of geometry and communication). In most cases, explicit integration happened approximately 50% of the time or more. The planning materials showed less explicit integration, in part because they did not include specific mathematical or engineering detail. Explicit integration was noted at most 26% of the time in this area. The activities had explicit integration that was on par with the teacher training materials, but the assessments showed a decrease in explicit integration, dropping to 8% in the areas of algebra and reasoning all the way down to 0% integration in the cases of measurement and data/probability.

Table 6: Percent Explicit Integration of Mathematics in the Digital Electronics Curriculum ©2004

	Planning (X_p)		Activities (X_{ac})		Assess-ment (X_{as})		Training (X_t)	
	N_p = 195	Percent Integration (X_p/N_p)	N_{ac}= 40	Percent Integration (X_{ac}/N_{ac})	N_{as}=12	Percent Integration (X_{as}/N_{as})	N_t = 102	Percent Integration (X_t / N_t)
Content Standards								
Number	35	17.9	26	65	3	25	67	65.7
Algebra	12	6.2	12	30	1	8.3	37	36.3
Geometry	0	0	0	0	0	0	0	0
Measurement	5	2.6	21	52.5	0	0	53	52.0
Data and Probability	10	5.1	9	22.5	0	0	32	31.4
Process Standards								
Problem Solving	2	1.0	19	47.5	3	25	49	48.0
Reasoning	6	3.1	22	55	1	8.3	50	49.0
Connection	70	35.9	26	65	5	41.7	85	83.3
Representation	35	17.9	21	52.5	2	16.7	66	64.7
Communication	1	0.5	2	5	2	16.7	2	2.0

One excellent example of explicit integration in DE occurs in Unit 6: Binary Addition. The unit proceeds in three steps, each with an extensive activity. Students start by learning the basics of binary addition and doing numerous problems in a purely mathematical context. From there they do experimental work that is closely related – implementing different types of adder circuits in their design software and filling in truth tables. The unit introduces both mathematical and engineering topics and reciprocally uses each one to illustrate the use of the other.

Comparative Analysis by Type of Material

Planning Materials

The planning materials (Table 7) were the most complex to analyze because of the variation in forms in which they appeared across the courses. The category, as described earlier, consists of the anticipatory set, lesson plans, concepts, performance objectives and presentations for each unit or sub-unit (when they were available). While complex and occasionally difficult to navigate, these materials were useful for examining the context for which the activities and assessments took place. They allowed us to get a feel for what the expectations were for students with regards to making explicit mathematics connections to the engineering concepts. In looking at the planning materials, we found on occasion that the math is present in the lesson's activities, but without a plan for instruction to integrate the math in an explicitly connected way. For example, in POE lesson 2.2, the trigonometric and geometric concepts necessary to complete orthographic sketching are covered only in the exercises. The planning materials for these activities do not include an explicit introduction to concepts such as trigonometry, geometry, or measurement that are necessary to successfully complete a sketch .In another example from Unit 2 of POE, one lesson shows how to use Excel to create graphs and charts. Datasets are provided, but there is no use of function language such as domain, and range; a failure to identify math concepts. Furthermore, the poor integration of this lesson with the rest of the course doesn't make this embedded material any more relevant for the students.

Table 7: Percent Explicit Integration of Mathematics in the Planning Materials

Planning Materials	Introduction to Engineering Design		Principles of Engineering		Digital Electronics	
	N_p= 132	% (X_p/N_p)	N_p=145	% (X_p/N_p)	N_p=195	% (X_p/N_p)
Content Standards:						
Number	3	2.3	18	12.4	35	17.9
Algebra	4	3	11	7.6	12	6.2
Geometry	18	13.6	15	10.3	0	0
Measurement	7	5.3	11	7.6	5	2.6
Data/Probability	8	6.1	12	8.3	10	5.1
Process Standards:						
Problem Solving	2	1.5	3	2.1	2	1.0
Reasoning	0	0	3	2.1	6	3.1
Connection	2	1.5	11	7.6	70	35.9
Representation	23	17.4	16	11	35	17.9
Communication	9	6.8	9	6.2	1	0.5

In looking at the data, we note that the planning materials do not show strong explicit integration across any of the courses or with regards to any of the math standards (Table 7). In a well-integrated curriculum, we would hope to see high percentages of explicit connections. Here, the best example is within the DE curriculum in which the “connections” standard is explicitly integrated 36% of the time in the planning materials. We think that particularly useful places do exist for creating explicit connections to math within the anticipatory sets and the presentations for each sub-unit within each curriculum; however, we did not observe that those connections were made.

Activities

The activities included in each curriculum give the students opportunities to explore concepts through working out problems. Activities include worksheets as well as hands-on work. Through the activities, students are able to develop their knowledge and make connections between the math and engineering. Thus, we view them to be the most important category with which we hope to see explicit integration between math and engineering concepts. While Table 8 shows there is heightened explicit integration in activities contained in both POE and DE, IED has very little explicit integration.

In looking for explicit connections throughout the activities associated with each of these curricula, we were able to identify some examples of where this integration exists as a benchmark, as well as where it could be improved. IED in particular presents a varied mixture of concepts that are rarely explicitly integrated with the engineering work. All of Unit 5 is devoted to pure geometry, but later units don’t make clear connections between these math concepts and the design language of the CAD software. In the next unit, course materials present the CAD terms of “Work Axis” and “Work Plane,” without mention of their pure math counterparts in Euclidean geometry, thus missing valuable opportunities to connect students’ prior knowledge to new lessons and to develop the kind of conceptual structure typically expected for transfer of learning to new settings to occur.

Unit 4 of POE is a 69-day block that covers the foundations of several Engineering disciplines. The technical nature of this material means that some of these lessons have a high level of math content. The identification of math concepts is strong in these planning materials, but the relevance of the math topics is made uneven by the mixture of activities in the Unit. In activity 4.4a, for example, the electrical systems activities combine experimentation, theoretical questions, and mathematical problem solving. In POE Lesson 4.3, the *Fluid Power* and *Hydraulics* presentations detail gas laws and the flow equations with accompanying example problems. Although these presentations have algebra, the math content is not used in the activities, nor are most of the fluid systems principles that they are used to illustrate. Math is included in the lesson, but it is neither utilized by students nor made relevant to the design course work.

Table 8: Percent Explicit Integration of Mathematics in the Activities

Activities	Introduction to Engineering Design		Principles of Engineering		Digital Electronics	
	N_{ac}= 40	% (X_{ac}/N_{ac})	N_{ac}= 32	% (X_{ac}/N_{ac})	N_{ac}= 40	% (X_{ac}/N_{ac})
Content Standards:						
Number	1	2.5	15	46.9	26	65
Algebra	2	5	11	34.4	12	30
Geometry	2	5	9	28.1	0	0
Measurement	2	5	13	40.6	21	52.5
Data/Probability	4	10	10	31.3	9	22.5
Process Standards:						
Problem Solving	1	2.5	10	31.3	19	47.5
Reasoning	2	5	8	25	22	55
Connection	2	5	12	37.5	26	65
Representation	11	27.5	14	43.8	21	52.5
Communication	5	12.5	3	9.4	2	5

Assessments

As we mentioned previously, assessment occurs throughout the *PLTW* curricula in all three foundations courses through the review of activities and through daily course work. In this section, we reviewed specific test or quiz materials. For this analysis (Table 9) we omitted the end of course examination because it encompassed all of the material from the entire course.

Table 9: Percent Explicit Integration of Mathematics in the Assessments

Assessments	Introduction to Engineering Design		Principles of Engineering		Digital Electronics	
	N_{as}= 42	% (X_{as}/N_{as})	N_{as}=32	% (X_{as}/N_{as})	N_{as}= 12	% (X_{as}/N_{as})
Content Standards:						
Number	2	4.8	3	9.4	3	25
Algebra	3	7.1	7	21.9	1	8.3
Geometry	12	28.6	1	3.1	0	0
Measurement	7	16.7	1	3.1	0	0
Data/Probability	4	9.5	6	18.8	0	0
Process Standards:						
Problem Solving	2	4.8	1	3.1	3	25
Reasoning	2	4.8	1	3.1	1	8.3
Connection	0	0	3	9.4	5	41.7
Representation	9	21.4	6	18.8	2	16.7
Communication	9	21.4	0	0	2	16.7

The assessments for all courses are particularly weak in terms of their explicit math connections. For example, the assessments in IED have some mathematics terms, but there are no instructional materials to go along with them. Focusing on the activities may provide a better picture of the opportunities for integration as it is not clear when and how the various assessments contained within each curriculum are used. In addition, much of the math presented in IED is formula-based, but not presented in ways that allow for explicit integration. For example, in Unit 6, the formula for standard deviation is presented but the meaning and utility of the formula are not discussed. The context within which math is presented could be improved and would likely allow for better assessment. For instance, to revisit Unit 6 of IED, statistics questions are given about test grades, not about an engineering topic or any of the coursework, thus deviating from the content of the IED course module.

Teacher Training Materials

We assessed the training materials presented in the Summer Institutes for each *PLTW* Foundation course. This segment of analysis is unique in that it allows us a glimpse into the enacted curriculum. The summer institutes are intensive two-week training sessions for *PLTW* teachers. *PLTW* is a National network, thus it is required that after signing a contractual agreement to become a *PLTW* school, that the school provide *PLTW* with the name(s) of teachers who will be assigned *PLTW* courses; in addition to other requirements, the teacher must complete various assessments and the two-week training sessions for each course[21]. In order to better understand how these intensive training sessions align with our analysis of the *PLTW* curriculum with regards to explicit math integration, we reviewed the teacher training manuals according to the training schedule provided along with the curriculum. With regards to the curriculum overall, the two week training session is limited in that teachers do not have time to train on the entire curriculum. Rather, sections of each are chosen for emphasis. Thus, the teacher training curricula represent only a small portion of the entire curriculum as presented to students and therefore has a different number of criteria assessed for each math standard.

As shown in Table 10, there is wide variation in the amount of explicit math integration across all three curricula as well as across all of the math standards. This is noteworthy because the teacher training program is the foundation upon which the teachers receive guidance on how to implement the curriculum in their own classrooms. If the teachers are not given the opportunity to actively make explicit connections between the math and engineering concepts, it will be predictably more difficult for them to create those connections for the students they serve, affecting both teaching and learning. In both the IED and POE curricula, mathematics content standards are integrated at a rate 31% or less. In contrast, there are some particularly strong areas of explicit integration, such as the connections standard in the DE curriculum. In fact, overall we have observed that the DE curriculum does a particularly good job of bridging the concepts of other courses both with regards to engineering as well as mathematics, showing explicit integration 83% of the time.

Table 10: Explicit Integration of Mathematics in the Teacher Training Materials

Teacher Training Materials	Introduction to Engineering Design		Principles of Engineering		Digital Electronics	
	Nt = 26	% (Xt / Nt)	Nt = 55	% (Xt / Nt)	Nt = 102	% (Xt / Nt)
Content Standards						
Number	5	19.2	17	31.1	67	65.7
Algebra	3	11.5	11	20.0	37	36.3
Geometry	8	30.8	17	31.1	0	0
Measurement	8	30.8	9	16.4	53	52.0
Data/ Probability	1	3.8	11	20.0	32	31.4
Process Standards						
Problem Solving	1	3.8	13	23.6	49	48.0
Reasoning	3	11.5	12	21.8	50	49.0
Connection	7	26.9	14	25.5	85	83.3
Representation	14	53.8	21	38.2	66	64.7
Communication	10	38.5	1	1.8	2	2.0

Discussion

Preliminary findings suggest that while the *PLTW* curriculum does present project and problem-based approaches to learning engineering, integration with academic curricula is seldom explicit, especially in the early foundation course, IED. Our initial work with the IED curriculum shows a low percentage of integration across all mathematics standards, but particularly the content standards. This is notable, since the content standards address the central conceptual framework for mathematics learning and proficiency for each grade level. One way that high school students can study mathematics that extends beyond the material expected of all students is to include specific math lessons in the curriculum material that extends the foundational material in depth or sophistication. We actually do notice that this happens in several instances in the DE course.

In addition, on analyses suggest there is commonly dissociation between the intended and assessed curricula, particularly for IED and POE. Ensuring that all four types of curricular materials (Training, Planning, Activities and Assessments) were explicitly connected would improve the courses in our opinion and remedy any curricular dissociation. In fact, we found that all courses had a substantial amount of *implicitly embedded* mathematics concepts as they pertain to engineering. Small changes in the curriculum to make these more explicit, along with the proper changes in the concomitant teacher training programs, could potentially improve the curriculum in substantially ways. This is because a lack of explicit mathematics integration inhibits learners' formation of broader generalization and abstraction of technical knowledge as well as a grounded understanding of abstract laws and notation systems. These current findings are consistent with that reported elsewhere[10] showing that most of the math involved in pre-engineering curricula takes the form of measuring, organizing and presenting data, and is not often directly applied to advance either students' understanding of engineering concepts or the mathematics. Connecting mathematics to engineering contexts is consistent with research based findings about how learning takes place [3,5,22] where the link to experience allows students to know the mathematics content and also be able to perform the mathematics and use the concepts as tools[2]. Without making these connections explicit, students may be missing learning opportunities.

These findings may seem at odds with claims by the curriculum developers for PLTW and others striving to create engineering learning experiences for grade K-12 students. For example, in the *2008 Depth of Knowledge Analysis* of IED done by *PLTW*[23] the developers contend that strategic thinking is used throughout the curriculum and especially in later units with respect to mathematics concepts, this is not enough. How do we reconcile these seemingly opposing positions? We would attribute the different interpretations to Expert Blind Spot[24,25,26]. This is the psychological phenomenon that those highly knowledgeable in their own fields more readily see the deep conceptual underpinnings than novices do. Consequently, experts are likely to literally see the conceptual structures as overt connections that we recorded as implicitly embedded in the curriculum resources and activities. Consequently, an important aspect of any curriculum redesign must address this tendency among curriculum developers and teachers.

In an attempt to illustrate how a lesson could become more explicitly integrated with regards to connecting math and engineering, we offer two examples. First, in Lesson 2.3 of POE, *Data Representation and Presentation*, algebra standards could be explicitly integrated by having students graph functions as well as data sets. Students could compare the graphs of different functions to those of the given data sets, thereby integrating additional algebra standards. Then, using Excel's "trendline" feature, they could see how their own modeling ability compares to the software algorithm for finding a line of best fit. Student could then determine the best ways to model different situations. The mathematics embedded in Excel could be explicitly identified and connected with students' prior understandings, and from there it could be reused in later lessons, such as Units 4 and 7 of the POE course – *Engineering Systems* and *Engineering for Reliability*. Second, in Lesson 4 of the POE curriculum, the "Sprockets and Gears" PowerPoint ® presentation uses ratios to explain mechanical advantage. The math lessons in this presentation are well developed but are not made clear to the novice learner. For example, one PowerPoint slide derives equations for the gear ratio, but the term's definition is elusive and the ideas of relationships between variables (direct, indirect) are missed. Torque, a central concept, is not defined. When these concepts are tested in activities, they have only equation-based questions, just as the abstract lessons have only abstract questions. The math is kept in the realm of formulas, so the connections between the mechanical principles and the math lesson are very limited. Both Algebra and Geometry standards should be integrated. 'N,' the number of teeth on a gear needs to be connected to circumference or arc length; the concepts of linear and rotational motion would provide math content while elucidating the relationship between gears of different sizes. Despite being poorly specified, these events were still coded as being explicitly integrated. However, if the mathematical relationships were expanded, the principles behind pulleys and gears would also be clarified and much greater conceptual integration would be evident.

Stone and colleagues[5] found that when mathematics concepts were used to enhance career and technical education (CTE) in explicit ways, students were better able to do math as assessed on two standardized tests of math ability. Further, these same students did no worse on assessments of their technical skills. While that study did not include any engineering curricula in their investigation, they provide firm empirical evidence for the idea that explicit integration of mathematics into technical education curricula can enhance the "rigor and relevance" (as they refer to it) of programs like the *PLTW* curriculum, as well as supporting students in both their academic course work and their successful completion of the technical curriculum. An explicitly integrated program would serve as a powerful answer to the "clarion call" of *Rising Above the Gathering Storm* and the federal Perkins Vocational Education Act to re-energize the STEM career pipeline. While employers may want workers who can begin working with software to design, implement and test products, the ability to do so depends on a solid mathematics foundation. These abilities are developed through the education "pipeline." In high school, we hope to see students use the mathematics that they learn in their college preparation course work in an explicit way within the *PLTW* courses to give them an opportunity to connect their academic knowledge to the context of engineering in concrete ways, guided by an instructor. The advantages of this, in short, are that teaching and learning of both pre-engineering and academic topics is strengthened and become more engaging for students when conceptually rich lessons are connected to prior knowledge and experience and put into a real-world context[22].

Future Directions

This study explored the intended and assessed curriculum of the three foundation courses in the *PLTW* high school engineering sequence. It lays the groundwork for further analysis of *PLTW* curricula, wherein we might further inspect the alignment of *PLTW* in mathematics as well as other relevant academic subjects; specifically, physics and other sciences. In conjunction with analyses of the enacted curriculum[13], we plan to further elucidate if and how students are using technology education to create connections to their academic coursework and build strong conceptual structures of their math, science and technology knowledge. This work also allows us to frame additional questions of interest to pre-engineering teaching and learning. For example, we can further inspect the ways in which the various elements of the curriculum identified align with one another. We can investigate whether the assessment materials ask the types of questions that ensure we are evaluating the central concepts in engineering, mathematics and science. We can also explore the alignment of skills and concepts along the proposed course sequence. As a means to further enhance the reliability of this type of analysis, it would be useful to interview *PLTW* instructors to discuss how these analyses apply to their classrooms and to explore whether assumptions made by the research team are valid in the eyes of the instructor. We also foresee the value of documenting the teacher training programs in action, in order to better understand, from an enacted perspective, how the objectives and aims of the curriculum are presented to teachers, and developed into pedagogical methods for advancing students' engineering knowledge. The intended and assessed curricula provide a rich perspective on the study of pre-engineering course curricula. These findings are further enriched when we consider them within the framework of contemporary learning theory.

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Bibliography

[1] Redish, E. F., & K.A. Smith. 2008. Looking beyond content: Skill development for engineers. *Journal of Engineering Education, 97* (3): 295–307.

[2] Streveler, R. A., Litzinger, T. A., Miller, R. L., & Steif, P. S. (2008). Learning conceptual knowledge in the engineering sciences: Overview and future research directions. *Journal of Engineering Education*, *97*, 279-294.

[3] Bransford, J. D. and Schwartz. D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. *Review of Research in Education, 24*, 61-100.

[4] Judd, C. H. (1908). The relation of special training to general intelligence. *Educational Review, 36*, 28-42

[5] Stone, J.R., Alfeld, C., & Pearson, D. (2008). Rigor *and* relevance: Enhancing high school students' math skills through career and technical education. *American Educational Research Journal, 45*(3), 767-795.

[6] From http://www.pltw.org/ Engineering/Curriculum/Curriculum-high-school.cfm accessed on 1/13/09

[7] National Research Council. (2007). *Rising Above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future*. Washington, DC: National Academies Press.

[8] Nathan, M. J. Tran, N., Phelps, L. A., & Prevost, A. (2008). The structure of high school academic and pre-engineering curricula: Mathematics. *Proceedings of the American Society of Engineering Education (ASEE) 2008* (Paper no. AC 2008-2566: pp. 1-19.) Washington, DC.: ASEE Publications.

[9] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

[10] Welty, K., L. Katehi, and G. Pearson. 2008. Analysis of K-12 Engineering Education Curricula in the United States-A Preliminary Report. In *Proceedings of the American Society for Engineering Education Annual Conference and* Exposition. Pittsburgh, PA

[11] Nathan, M. J. Tran, N., Phelps, L. A., & Prevost, A. (2008). The structure of high school academic and pre-engineering curricula: Mathematics. *Proceedings of the American Society of Engineering Education (ASEE) 2008* (Paper no. AC 2008-2566: pp. 1-19.) Washington, DC.: ASEE Publications.

[12] Porter, A. C. (2004). Curriculum assessment. In J. C. Green, G. Camill & P. B. Elmore (Eds). Complementary methods for research in education (3rd edition), Washington, DC: American Educational Research

Association.

[13] Nathan, M. J., Oliver, K., Prevost, A., Tran, N., & Phelps, L. A. (2009). Classroom learning and instruction in high school pre-engineering settings: A video-based analysis. Awaken Project working document: Authors.

[14] Nathan, M. J., Long, S. D., & Alibali, M. W. (2002). The symbol precedence view of mathematical development: A corpus analysis of the rhetorical structure of textbooks. *Discourse Processes, 33*(1), 1-21.

[15] Nathan, M. J. & Koedinger, K. R. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition and Instruction, 18*(2), 209-237.

[16] http://www.pltw.org/Engineering/Curriculum/sample-schedule.cfm, accessed 02/02/09

[17] National Research Council (2004). On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Evaluations. Committee for a Review of the Evaluation Data on the Effectiveness of NSF-Supported and Commercially Generated Mathematics Curriculum Materials, Jere Confrey and Vicki Stohl, Editors, Washington, DC.: National Academy Press.

[18] http://standards.nctm.org/document/chapter7/alg.htm; accessed 01/12/09

[19] Tran, N. & Nathan, M. J. (2008). An investigation of the relationship between pre-engineering studies and student achievement in science and mathematics. AWAKEN Project working document (manuscript under review): Authors.

[20] http://www.pltw.org/Engineering/engineering.cfm; accessed 01/19/09

[21] http://www.*PLTW*.org/faqs/schools-more-2.htm ,accessed 01/19/ 09

[22] Britton, E., Huntley, M.A., Jacobs, G. and Shulman-Weinberg, A. Connecting Mathematics and Science to Workplace Contexts: A Guide to Curriculum Materials. Thousand Oaks, CA: Corwin Press.

[23] Project Lead the Way, Inc. (2008). Introduction to Engineering Design™ Analysis of Cognitive Levels of Learning and Mathematics and Science Content. Retrieved 01/03/09 from http://www.pltw.org/Engineering/engineering.cfm. Clifton Park, NY: Project Lead the Way, Inc.

[24] Nathan, M. J., Koedinger, K. R., & Alibali, M. W. (2001). Expert blind spot: When content knowledge eclipses pedagogical content knowledge. In L. Chen et al. (Eds.), *Proceeding of the Third International Conference on Cognitive Science*. (pp. 644-648). Beijing, China: USTC Press.

[25] Nathan, M. J. & Petrosino, A. J. (2003). Expert blind spot among preservice teachers. *American Educational Research Journal. 40*(4), 905-928.

[26] Bransford, J., Vye, N., Bateman, H., Brophy, S. & Roselli, B. (2004). Vanderbilt's AMIGO^3 Project: Knowledge of How People Learn Enters Cyberspace. In Thomas M. Duffy and Jamie R. Kirkley (eds.). *Learner-Centered Theory and Practice in Distance Education*. Mahwah, NJ: Erlbaum.