Developmental Cognitive Science Goes to School

Edited by

Nancy L. Stein
Department of Psychology
University of Chicago
National Opinion Research Center

and

Stephen W. Raudenbush
Department of Sociology
University of Chicago

First published 2011
by Routledge
270 Madison Avenue, New York, NY 10016

Simultaneously published in the UK
by Routledge
2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN

Routledge is an imprint of the Taylor & Francis Group, an informa business

Typeset in Minion Pro by Prepress Projects Ltd, Perth, UK.
Printed and bound in the United States of America on acid-free paper by Walsworth Publishing Company, Marceline, MO

Library of Congress Cataloging in Publication Data
Developmental cognitive science goes to school/edited by Nancy L. Stein & Stephen W. Raudenbush.
p. cm.
Includes index.
"Based on a conference held in the Fall of 2007, in Chicago."
1. Cognitive learning—Congresses. 2. Science—Study and teaching—Congresses. 3. Mathematics—Study and teaching—Congresses. 4. Content area reading—Congresses. I. Stein, Nancy L. II. Raudenbush, Stephen W.
LB1062.D497 2010
370.15'23—dc22
2010021699

ISBN 13: 978-0-415-98883-4 (hbk)
ISBN 13: 978-0-415-98884-1 (pbk)
ISBN 13: 978-0-203-83753-5 (ebk)

Dedication

This book is dedicated to Tom Trabasso, whose lifelong interest in learning and understanding served as a guideline for addressing issues related to developmental psychology, learning, and schooling. Tom's interest in learning was apparent in his earliest work in the 1960s, with Gordon Bower and Rochel Gelman on discrimination and concept learning, in his work with Peter Bryant on transitive inferences, learning and development, in his work with Peter Ornstein on organizing, learning, and remembering, and in all of his work on models of understanding, thinking, and development. His later work on narrative and causal understanding was fueled by an attempt to account for language, memory, and thinking that went beyond simple word and sentence understanding. That is, he wanted to study complexity, and the ways in which complex systems impacted people on a daily basis.

The feature that characterized Tom the most was his passionate quest for answers and discoveries that would lead to a better understanding of how children think, reason, and remember. He was a consummate scientist and always sought evidence, no matter what the issues were, and no matter whose theories were being tested, to explain and account for data. The pursuit of answers and evidence often got him into trouble, especially with those whose theories were being tested. Tom persevered, however, until he got the answers he was seeking, often at variance with current belief systems related to how children learn.

Tom was as good at teaching as he was at doing research. His tenacity, goal directedness, and razor-sharp intellect enabled him to impart a sense of discovery and delight to students and colleagues who were engaged in studying learning and memory. Tom was faster than just about anyone in discerning confounds and problems with an approach, devising ways of explicating and testing an issue, advancing a theory that was far more robust than the one with which he started, and pointing to broad implications that theories of learning had in regard to developmental issues.

Had Tom survived, he would have been an integral part of the efforts to become more intimately involved in science and school learning. Although Tom was mathematically gifted, he was never trained in the physical sciences, and so his new venture required that he take time out to actually learn the content of physics, chemistry, and earth sciences.

Several of the chapters in the present volume do what he would have done, if he could: make developmental and cognitive science relevant and understandable to those who teach young children on a daily basis.

Richland, L. E., & McDonough, I. (2010). Learning by analogy: Discriminating between potential analogs. *Contemporary Educational Psychology*, 35(1), 28–43.
Richland, L. E., Morrison, R. G., & Holyoak, K. J. (2006). Children's development of analogical reasoning: Insights from scene analogy problems. *Journal of Experimental Child Psychology*, 94, 249–271.
Richland, L. E., Zur, O., & Holyoak, K. J. (2007). Cognitive supports for analogies in the mathematics classroom. *Science*, 316, 1128–1129.
Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology*, 99, 561–574.
Spellman, B. A., & Holyoak, K. J. (1996). Pragmatics in analogical mapping. *Cognitive Psychology*, 31, 307–346.
Stigler, J. W., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1999). *The TIMSS videotape classroom study: Methods and findings from an exploratory research project on eighth-grade mathematics instruction in Germany, Japan, and the United States* (NCES1999-074). Washington, DC: U.S. Department of Education, NCES.
Zook, K. B., & Di Vesta, F. J. (1991). Instructional analogies and conceptual misrepresentations. *Journal of Educational Psychology*, 83, 246–252.

15 Gestures in the Mathematics Classroom

What's the Point?

Martha W. Alibali, Mitchell J. Nathan, and Yuka Fujimori

Communication is an integral part of teaching. Many factors influence whether students comprehend and learn from instructional communication, including whether students have a shared understanding of the referents used by the teacher (Mortimer & Wertsch, 2003), and whether the ideas addressed in a lesson connect to students' prior knowledge (Schwartz & Bransford, 1998). Another potentially important factor that has received limited research attention is the nonverbal support for comprehension provided by teachers' gestures. Gestures are movements of the hands and body that are produced in the act of speaking and that are closely synchronized with speech (McNeill, 1992). Gestures include pointing movements that indicate objects or locations, depictive movements that illustrate the content of speakers' thoughts, and rhythmic movements that mirror the cadence of speech.

Previous studies in noneducational settings have shown that speakers' gestures facilitate listeners' comprehension of speech. However, surprisingly little is known about how teachers use gestures in instructional settings, or about whether teachers' gestures influence students' learning. As Roth (2002) stated in *Review of Educational Research*:

> It is curious . . . that there exists very little educational research concerned with the role of gesture in learning and teaching, particularly in subject areas that have been characterized as dealing with abstract matters such as science and mathematics. The few existing studies that focus on gesture in an education context . . . suggest that such research might be of tremendous importance. (p. 365)

We agree with Roth's assessment, and, in this chapter, we report on a line of research that begins to address this gap.

The chapter proceeds in three parts. First, we review existing research on gesture in instructional settings and whether it matters for students' learning. Second, we present findings from a study of how teachers gesture in mathematics lessons. Third, we argue that teachers' gestures serve to connect mathematical ideas in their instruction, and we present illustrative examples drawn from classroom mathematics lessons. Our broad aim in this chapter is to document how practicing teachers actually use gestures in mathematics instruction.

Do Teachers' Gestures Matter for Students' Learning?

Gesture Affects Comprehension of Speech

Although some investigators have downplayed the communicative importance of gestures (e.g., Krauss, Morrel-Samuels, & Colasante, 1991), there is abundant evidence that gestures affect listeners' comprehension of speech (see Kendon, 1994). When gestures convey the same information as the accompanying speech, comprehension is facilitated (e.g., Goldin-Meadow & Sandhofer, 1999). For example, when asked to "find the block that has an arrow pointing up", preschool children chose the correct block more often when the speaker used a gesture that reinforced speech (i.e., an index finger pointing up) than when she used speech alone (McNeil, Alibali, & Evans, 2000). Gestures make a greater contribution to comprehension for complex or ambiguous verbal messages than for simpler ones (Graham & Heywood, 1976; McNeil et al., 2000). Thus, it seems likely that gestures are particularly important in instructional discourse that presents complex concepts and uses unfamiliar terms. In addition, classrooms are often noisy, with multiple individuals speaking at once. Under such challenging circumstances, gestures that reinforce speech may be crucial to aid comprehension (see Rogers, 1978).

Not all gestures reinforce the content of the accompanying speech, however. Speakers sometimes express information in gestures that is not expressed in the accompanying speech (McNeill, 1992). For example, in explaining her solution to a liquid conservation task, a child might say, "This cup is taller," while indicating the *width* of the container in gesture. Such "mismatching" gestures also influence listeners' comprehension of the speech they accompany. Listeners comprehend speech less well when it is accompanied by mismatching gestures than when it is accompanied by no gesture or by matching gestures (e.g., McNeil et al., 2000).

Furthermore, both adults and children often detect information that is expressed uniquely in mismatching gestures (e.g., Kelly & Church, 1997). In one study of this issue, Alibali, Flevares, and Goldin-Meadow (1997) asked adults to view video clips of children explaining mathematical equivalence problems (e.g., 3 + 4 + 5 = 3 + __). In some clips, children conveyed information uniquely in gestures (e.g., pointed to addends they did not mention in speech). Adults often detected the information that children expressed uniquely in gestures, sometimes reiterating it in their own gestures, and sometimes translating it into speech. These findings suggest a likely explanation for the finding that mismatching gestures hinder speech comprehension: when gesture mismatches speech, people sometimes detect the message expressed in gesture, rather than the one expressed in speech.

Thus, a substantial body of evidence indicates that gestures play an important role in communication. Based on this evidence, it seems likely that gestures are important in instruction, when effective communication is crucial.

Gesture Affects Learning from Lessons

Only a handful of studies have directly examined the effects of teachers' gestures on students' learning. Most have focused on whether children learn more from lessons that include gestures than from lessons that do not. Two such studies investigated third- and fourth-grade students learning to solve mathematical equivalence problems (e.g., 3 + 4 + 5 = 3 + __). In one, the lessons were delivered by an experimenter (Perry, Berch, & Singleton, 1995), and in the other by video (Church, Ayman-Nolley, & Alibali, 2001). In both, students showed deeper learning (i.e., generalization to new problem types, retention over a one-month interval) from lessons with gestures. In fact, Church et al. (2001) found that nearly twice as many students displayed deep learning after the speech-plus-gesture lesson as after the speech-only lesson (71 percent vs. 37 percent).

Another study compared third- and fourth-grade students learning about equivalence problems from videotaped lessons with no gesture, matching gestures, and mismatching gestures (Singer & Goldin-Meadow, 2005). In the lessons with mismatching gestures, the instructor described one strategy in speech (e.g., make both sides sum to the same total), and another strategy in gesture (e.g., add the numbers on the left side and subtract the number on the right, expressed in gesture with pointing gestures to the numbers on the left side, then a flick-away gesture to the number on the right). Children learned more from the lessons with mismatching gestures than from the lessons with matching gestures or no gesture, which did not differ from one another. These data suggest that gestures that serve to link ideas (such as different strategies for solving problems) may be particularly beneficial for students' learning.

Studies of other age groups and concepts have also documented beneficial effects of instructional gesture on learning. Church, Ayman-Nolley, and Mahootian (2004) examined first-grade students learning about Piagetian conservation from videotaped lessons. For native English speakers, 91 percent learned (i.e., added new *same* judgments) from a speech-plus-gesture lesson, compared with 53 percent from a speech-only lesson. For native Spanish speakers with little English proficiency, 50 percent learned from the (English) speech-plus-gesture lesson, compared with 20 percent from the (English) speech-only lesson.

Valenzeno, Alibali, and Klatzky (2003) studied preschoolers learning about symmetry from videotaped lessons. Children viewed either a speech-only lesson or a speech-plus-gesture lesson. The lessons used the same audio track, and differed only in the teachers' use of gesture. The speech-plus-gesture lesson included pointing and tracing gestures that indicated the example shapes, delineated the center of each shape, and compared the contours of the two sides of each shape. At post-test, children judged illustrations of real-world objects as symmetrical or asymmetrical, and explained their judgments. Children in the speech-plus-gesture lesson group outperformed children in the speech-only lesson group at post-test (mean = 2.08 vs. mean = .85).

Taken together, these studies provide compelling evidence that gesture matters for students' learning. However, these studies also lack ecological validity. Most utilize videotaped lessons or lessons delivered by an experimenter, rather than lessons delivered by real teachers in realistic instructional settings. Further, these studies hinge on a comparison that is not realistic. In most experimental studies, the "control" lesson—typically a *speech-only* lesson—is not like any lesson that might actually occur in a real classroom, because real teachers *do* produce gestures when they teach. These controlled experiments have established that gesture matters for learning, but they do not provide guidance for teachers about how best to use gestures to promote student learning.

Our ultimate goal is to understand how teachers' gestural behavior relates to student learning, so that we can make empirically validated recommendations about instructionally effective gestures. However, before we can test whether variations in teachers' behavior matter, we need to understand how teachers actually use gestures. Unfortunately, little is known about how much teachers actually gesture, about what kinds of gestures they produce, and about the functions these gestures serve. To formulate hypotheses about how gesture matters for students' learning, we need more knowledge about how teachers actually gesture during instruction. Thus, we turn next to research that investigates teachers' gestures in naturalistic, classroom settings, with a specific focus on mathematics classrooms.

How Do Teachers Use Gestures in Naturalistic Mathematics Instruction?

Teachers routinely use gestures as part of their instructional communication. Many descriptions of teachers' behavior mention gestures or include gestures in transcripts of lessons (e.g., Núñez, 2005; Roth & Bowen, 1999; Yackel & Cobb, 1996); however, systematic analyses of gestures in instructional communication are scarce (for exceptions, see Roth & Lawless, 2002, on ecology lectures and Corts & Pollio, 1999, on psychology lectures). Few studies of teachers' gestures have focused on mathematics, and fewer still have focused specifically on the role of gestures in fostering students' mathematics understanding. Gestures may be particularly important in mathematics instruction, because mathematics involves spatial representations (e.g., graphs, number lines), relations between ideas (e.g., links between different representations of mathematical information, such as graphs and equations), and embodied concepts (e.g., arithmetic is motion along a path) (Lakoff & Núñez, 2001). Gestures are adept at communicating spatial, relational, and embodied concepts (Alibali, 2005; Hostetter & Alibali, 2008).

The few existing studies of gestures in naturalistic mathematics instruction document that gestures are pervasive. For example, Flevares and Perry (2001) found that first-grade teachers used five to seven "nonspoken representations" per *minute* in lessons about place value, and most of these involved gestures. Alibali and Nathan (2007) examined a middle-school early algebra lesson, and found that 74 percent of the teacher's utterances about the instructional task included gesture. Richland, Zur, and Holyoak (2007) examined American, Japanese, and Hong Kong mathematics teachers' use of gesture when they made analogies during their instruction. Teachers' use of gesture in analogies varied across cultures, with roughly 15 percent of analogies receiving gestural support in the United States, and 45 percent in Japan. Thus, gestures appear to be an integral part of teachers' instructional communication. However, further research characterizing the role of gesture in mathematics instruction is needed. To address this need, we undertook an examination of teachers' gestures in elementary mathematics lessons.

Source of Data

We analyzed videotapes of five fifth-grade geometry lessons that were collected by James Stigler and Giyoo Hatano for a cross-national study of mathematics education (Stigler, Fernandez, & Yoshida, 1996). We coded lessons from three American teachers and two Japanese teachers. Given the substantial differences in lesson planning, lesson organization, and teaching methods between the United States and Japan (e.g., Stevenson & Stigler, 1992; Stigler & Hiebert, 1999), we expected that this cross-cultural sample would represent a wide range of instructional styles, and would therefore be informative about the range of variation in teachers' gestures. All five lessons focused on finding the area of a triangle, and each lasted 40–45 minutes.

We made a full verbal transcript of each lesson, and in this transcript we identified the beginning of the main body of the lesson, defined as the moment when the teacher explicitly introduced the main topic of the lesson. For example, one teacher said, "And now, I'd like us to try to figure out how to get the area of a triangle." Teachers varied greatly in how much they spoke during the main body of the lessons. To insure an adequate behavioral sample from each teacher, we identified the first 100 utterances (complete statements or speaking turns) produced by each teacher in the main body of the lesson, excluding student utterances and off-camera utterances. All gestures accompanying these 100 utterances were transcribed and coded.

Coding Gesture Form

We classified each gesture based on its form, using a system based on that described by McNeill (1992). McNeill's system has been widely used in past research, and the primary coder (YF) received extensive training before performing the coding. Gestures that were difficult to classify were reviewed and discussed by two coders (YF and MWA). Examples are presented in Figure 15.1a–e.

1. Deictic gestures indicate their referents by pointing, typically with the index finger but sometimes with other fingers or the whole hand. Teachers used deictic gestures to indicate a variety of referents, including inscriptions, objects, and students. For example, a teacher might point to an angle to refer to that angle (Figure 15.1a).
2. Hold-up gestures display concrete objects or diagrams by holding them up. These gestures are functionally similar to deictic gestures in that they indicate a specific referent. For example, a teacher might hold up a paper triangle (Figure 15.1b).
3. Representational gestures depict semantic content through handshape or motion. For example, a teacher might depict the action of cordoning off an area (Figure 15.1c).
4. Hold-up + action gestures involve holding up and manipulating concrete objects or diagrams. For example, a teacher might hold up two identical triangles and move them together to show that two triangles form a rectangle (Figure 15.1d). These gestures are functionally similar to representational gestures, in that they depict meaning through action.
5. Beat gestures are motorically simple, rhythmic gestures that do not convey semantic content, but instead mirror the rhythm or cadence of speech (Figure 15.1e). For example, when saying "The area is base times height," a teacher might produce beat gestures on the words "base" and "height."
6. Emblems are gestures that have a conventional, culturally specified form and meaning. For example, a teacher might hold up her palm to ask students to "stop."

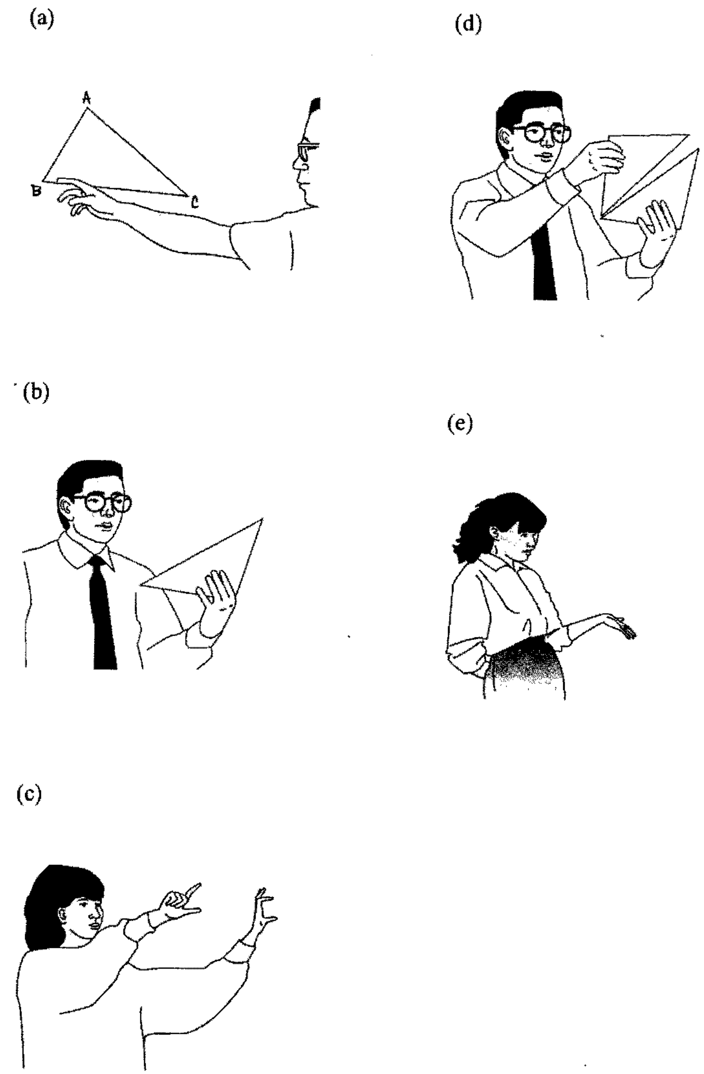

Figure 15.1 (a) Deictic gesture. (b) Hold-up gesture. (c) Representational gesture. (d) Hold-up-plus-action gesture. (e) Beat gesture.

Coding Gesture Function

We inferred the communicative function of each gesture based on the gesture form, the accompanying speech, and the instructional context. We identified four primary functions: managing interaction, expressing emphasis, conveying information, and guiding attention.

First, teachers often used gestures to *manage interaction* in the classroom. Two types of gestures were coded as managing interaction: emblems that sought to regulate students' behavior (e.g., conventional gestures meaning "shhh", "stop"), and deictic gestures used to call on students or regulate turn taking. For example, one teacher pointed at a student while asking, "What is perimeter, Jason?" Another teacher produced the "stop" gesture while saying, "Let's stop for a second."

Second, teachers used gestures to *express emphasis* or to "underscore" important parts of their speech. Beat gestures were coded as serving this function. For example, one teacher said, "Area is measured in *square* units" and (along with verbal emphasis) produced a beat gesture on the word *square*.

Third, teachers used gestures to *convey substantive information* relevant to the lessons. Two types of gestures were coded as serving this function: representational gestures and hold-up-plus-action gestures. These gestures depicted mathematical concepts visually or invoked real-world applications of mathematical ideas. For example, one teacher depicted a line in gesture while saying, "That would be a line measurement." Another teacher used a hold-up-plus action gesture to demonstrate that two identical paper triangles could make a rectangle, saying, "You put these together, like so."

Fourth, teachers used gestures to *guide students' attention* to portions of the instructional context. Two types of gestures were coded as serving this function: hold-up gestures and deictic gestures. Hold-up gestures were used to guide students' attention to objects. For example, one teacher held up a paper triangle while saying, "Here's a triangle." Deictic gestures frequently served to guide students' attention to objects or inscriptions; for example, one teacher indicated two sides of a right-angled triangle in gesture while saying, "Two sides are straight."

The Functions of Gesture in Instruction

For each of the five teachers, the predominant function of gestures was to guide students' attention to features of the instructional context. Across teachers, an average of 57 percent (SE = 7.0) of all gestures were used to guide attention. Gestures for managing interaction (mean = 8 percent of all gestures, SE = 1.4), expressing emphasis (mean = 16 percent, SE = 5.7) and conveying information (mean = 19 percent, SE = 1.8) were used much less frequently. However, all of the teachers used some gestures from each of the four functions, with the exception of one teacher who used gestures solely to express emphasis.

Japanese and American Teachers' Use of Instructional Gestures

We also compared instructional gestures in the two cultures. Of course, given the small sample size, the findings should be interpreted cautiously. Figure 15.2 presents the average number of gestures produced for each function by Japanese and American teachers. In both cultures, gestures were most often used to guide attention, and least often used to manage interaction. However, the rate of gesture production was lower among Japanese teachers for each function. American teachers produced an average of 83.3 gestures (range 71–98) over the 100 utterances, whereas Japanese teachers produced an average of 48.5 gestures (range 33–66). Thus, on average, American teachers produced 1.72 times as many gestures as Japanese teachers. American teachers

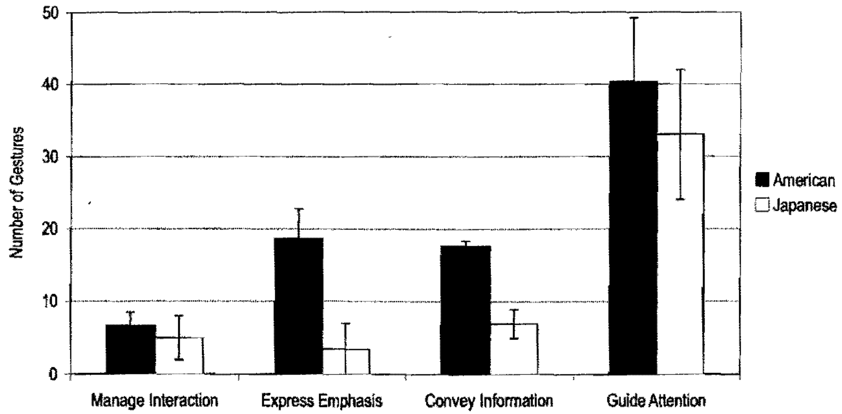

Figure 15.2 Mean Number of Gestures for Each Function Produced over 100 Utterances by American and Japanese Teachers. The error bars represent standard errors.

used many more gestures to convey substantive information [American mean = 17.7, SE = .7, vs. Japanese mean = 7.0, SE = 2.0; t(3) = 6.20, p = .008], and they also used more gestures to express emphasis [American mean = 18.7, SE = 4.1 vs. Japanese mean = 3.5, SE = 3.5; t(3) = 2.59, p = .08]. American and Japanese teachers used similar numbers of gestures to guide attention and manage interaction.

Despite these differences in gesture rates, it bears emphasizing that teachers in both cultures used gestures in largely similar ways. In both cultures, the most common function of gesture was to guide attention, and the least common function was to manage interaction.

Links between Representations

The focus of the lessons in this corpus (as in many math lessons) was on links between different representations of mathematical information. Specifically, the lessons focused on links between diagrams of geometric shapes (primarily triangles and rectangles) and formulae for calculating the areas of those shapes. Although the focus of our functional analysis was on individual gestures, we occasionally observed teachers using sets of gestures, along with speech, to link representations and to highlight correspondences among them. Such "linking episodes" often captured the mathematical goals of the lessons, and as such, they seem particularly significant.

One of the linking episodes we observed involved links between a diagram of a rectangle with length 12 and width 4, the general formula for the area of a rectangle ($A = l \times w$), and the " instantiated" formula $A = 12 \times 4$ (Figure 15.3). These links were made using speech and gesture to connect the diagram and the formula, and writing to generate the instantiated formula. In the excerpt below, brackets indicate speech that co-occurs with the gesture indicated in the lines beneath it.

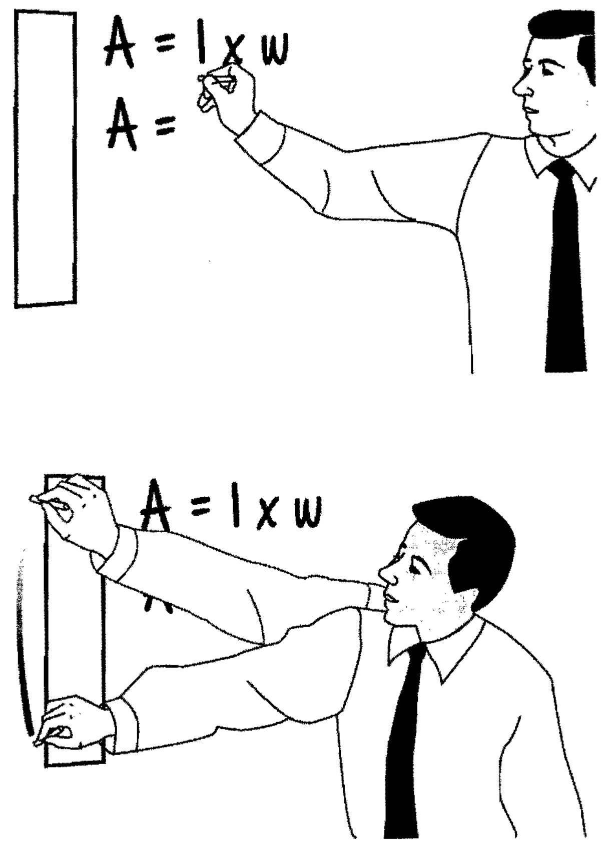

Figure 15.3 Teacher Uses Gesture to Link Area Formula and Diagram of Rectangle.

Speech:	Now we substitute,	[area equals]	[length] (pause)
Writing:		A =	
Gesture:			point to l in formula

Speech:	[12]	[(pause)]
Writing:	12	
Gesture:		trace length of long side of rectangle

Speech:	[times] the	[width, four]	[(pause)]
Writing:	×	4	
Gesture:			indicate short side of rectangle

In this example, the teacher first uses gesture to link the symbol *l* (which he indicates in the general formula), the *length* of the rectangle (which he traces on the diagram), and the number *12* (which he writes in the instantiated formula). He then uses gesture to link the *width* of the rectangle (which he indicates on the diagram) with the number *4* (which he writes in the instantiated formula). The gestures serve to guide attention sequentially to corresponding parts of the related representations. Thus, gesture is an integral part of the links the teacher establishes among the three representations.

Summary

In this corpus of elementary mathematics lessons, the primary function of teachers' gestures was to guide students' attention to features of the instructional context. Teachers also used gestures to convey information, express emphasis, and manage classroom interaction. Further, teachers sometimes used sets of gestures to highlight links between different representations of mathematical information. Such links are often at the heart of mathematics lessons, so they seem particularly important to examine and understand. Gestures are well suited to conveying relational information, so it is no surprise that gestures play an integral role in expressing links between representations.

In the following section, we consider how teachers use gestures to effectively communicate mathematical relationships, and we present illustrative examples drawn from a new study of classroom mathematics lessons.

How Teachers Use Gesture to Link Representations in Mathematics Instruction

We have argued elsewhere (Alibali & Nathan, 2007) that teachers' gestures are one means by which they scaffold student understanding of complex mathematical ideas. We based this claim on an analysis of an early algebra lesson. The teacher used gesture (a) more frequently for new material than for review material, (b) more frequently in response to students' questions than before such questions, and (c) more frequently for abstract referents than for concrete referents. Because links between representations are usually abstract, and because they often involve information that is new and potentially difficult for students, we hypothesize that teachers use gesture frequently when they communicate about such links. In an effort to better understand how teachers link representations, we have collected a corpus of 24 middle-school classroom mathematics lessons, and we are analyzing linking episodes within these lessons.

Our analysis thus far has revealed two primary ways in which teachers use gestures to establish links between different representations of mathematical information (e.g., equations, graphs, manipulatives): (1) teachers utilize sets of deictic gestures to highlight corresponding aspects of related representations and (2) teachers produced gestural catchments (i.e., repeated features in sets of representational gestures; see McNeill & Duncan, 2000) in order to show relatedness. We illustrate each of these gestural devices in turn. The examples presented here were drawn from two different lessons focusing on beginning algebra from the same sixth-grade mathematics teacher. However, it is important to note that we have observed linking episodes that utilize these techniques in all of the teachers analyzed to date. The examples we offer here are representative of those in the corpus as a whole.

Example 1: Sets of Deictic Gestures

Teachers frequently use sets of deictic gestures to highlight corresponding aspects of related representations. One representative example occurred in a lesson in which the teacher introduced a new way of using equations to model a story problem situation. The students were familiar with generating an equation that could be used to derive a solution, such as $(42-18) \div 4 = n$ (termed the *solution equation*). The lesson sought to build on this prior knowledge to help students generate a related equation that could be used to model the problem situation, namely, $4 \times n + 18 = 42$ (termed the *situation equation*).

In the lesson, the two equations were written side by side on the whiteboard at the front of the classroom. The teacher asked the students what was similar about the equations, and she revoiced the students' responses and produced deictic gestures to guide attention to the relevant, corresponding parts of the two equations. The following excerpt illustrates the teacher's use of deictic gestures to link "4 ×" and "÷ 4" and to link "+ 18" and "– 18". Gestures 6 and 7 are illustrated in Figure 15.4.

Student:	Timesing was there and dividing's there . . .			
Teacher:	Okay, so	[times],	[and] . . . so	[times four]
		1	2	3
Teacher:	and then	[divide by]	[four], cool	
		4	5	
Student:	and then plus, and then the minus over there.			
Teacher:	[Plus 18]	[and minus 18].		
	6	7		

1 Right-hand point to times sign in situation equation.
2 Right-hand point to division sign in solution equation.
3 Right-hand point toward situation equation.
4 Right-hand point to division sign in solution equation.
5 Right-hand point to 4 in solution equation.
6 Right-hand flat palm under + 18 in situation equation.
7 Right-hand flat palm under – 18 in solution equation.

In this example, the teacher used deictic gestures to establish mappings between the familiar solution equation and the less familiar situation equation, by guiding attention sequentially to corresponding aspects of the two representations. Specifically, she used gestures to delineate the correspondences between values and inverted operations across the two equations.

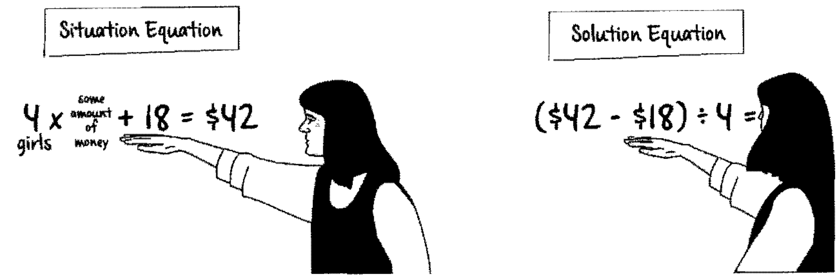

Figure 15.4 Teacher Uses Deictic Gestures to Link Corresponding Parts of Related Equations.

Example 2: Catchment of Representational Gestures

The teacher also sometimes highlighted relationships among entities with representational gestures, by repeating gesture features, such as handshape or motion, over a series of gestures. This is called a gestural *catchment*, and has been described in past research by McNeill and Duncan (2000) as a way of making connections in discourse.

The teacher produced a gestural catchment to link representations as she guided students to use equations to model a physical system, namely, a pan balance with objects on each side. The teacher gives meaning to the idea of subtracting unknown but equal quantities from both sides of an equation using a repeated gesture. She first mimes removing the same object from both sides of the pan balance, and then repeats the gesture by "removing" (i.e., pretending to pick up) the corresponding variables from both sides of the equation. These gestures are depicted in Figure 15.5.

I am gonna take away [a sphere from each side . . .]
1
[Instead of taking it off the pans]
2
[I am going to take it away from this] equation
3

1 Both hands cupped over pan balance picture, as if holding spheres on each side.
2 Both hands cupped over pan balance picture; hands move up and out to mime removing a sphere from each side.
3 Both hands cupped over equation, as if holding *s* on each side.

In this example, the teacher repeated the grasping gesture over the pan balance and then over the equation to highlight the conceptual connections between the two representations. Thus, the teacher established the correspondence between removing similar objects from both sides of the pan balance, and subtracting the same value from both sides of the equation.

It is also worth noting that, in this example, the teacher uses a *simulated action* expressed in gesture (see Hostetter & Alibali, 2008) to ground an abstract mathematical concept. Specifically, the teacher uses the familiar action of picking up two objects, represented here in gesture, to give meaning to the notion of subtracting the same value from both sides of the equation. The simulated action depicts the meaning of a potentially unfamiliar idea in terms of a familiar action.

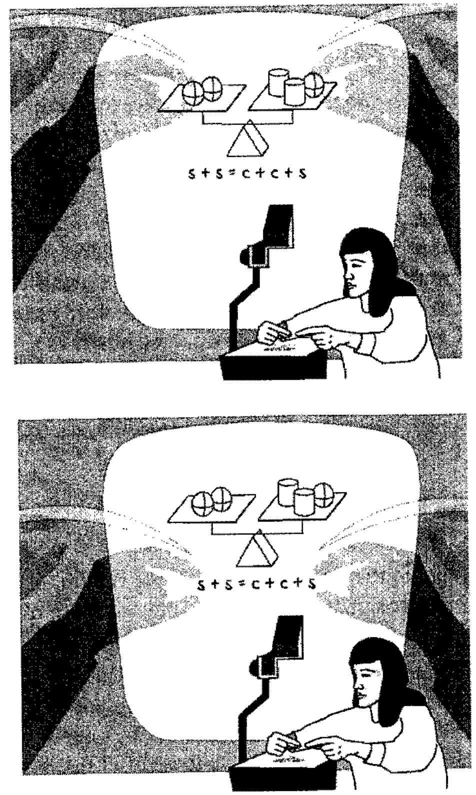

Figure 15.5 Teacher Uses a Catchment of Repeated Representational Gestures to Show that Removing the Same Object from Both Sides of the Pan Balance Corresponds to Removing the Same Value from Both Sides of the Equation.

Summary

These examples illustrate two gestural techniques that teachers use to link representations. First, teachers use sets of attention-guiding deictic gestures to delineate correspondences, as we saw both in the episode in which a teacher linked the diagram of the triangle to the formula for finding the area of the triangle and in the episode in which a teacher linked two different equations that represented the same mathematical situation. Second, teachers use repeated gesture features to highlight conceptual links, as we saw in the episode in which a teacher linked a picture of a pan balance and an equation that represented the configuration of the pan balance. Overall, gestures during instruction serve to guide attention to corresponding aspects of related representations (e.g., + 18 in one equations corresponds to – 18 in the other), and to convey substantive information about corresponding ways to manipulate related representations (e.g., removing objects from a pan balance corresponds to subtracting from an equation). These techniques are thus part of the teacher's repertoire of methods for communicating about important mathematical relations.

Our qualitative analysis of linking episodes documents teachers' use of gesture to connect new ideas and representations to more familiar ones. In one case, the teacher used gesture to connect two symbolic representations: one a familiar form and one a novel form. In the other case, the teacher used gesture to connect a familiar, concrete representation (the pan balance) to a target, abstract representation (the equation), and showed how operations on one representation are comparable to those on the other. The teacher's gestures appeared to be an integral part of her effort to bridge from students' prior knowledge to the new knowledge that was the target of her instruction.

General Discussion and Conclusions

Past research has shown that speakers' gestures play an important role in their listeners' language comprehension, and that children learn more from lessons that include gestures than from lessons that do not include gestures. Experimental work to date highlights the importance of gestures for instruction, but lacks ecological validity, and also lacks the detailed analysis of gesture types and functions that will allow us to construct a detailed model of how gesture can support learning.

Our study of the functions of gestures in classroom communication shows that teachers use gesture primarily to guide students' attention to aspects of the instructional context—often aspects of the context that are mathematically important. Teachers also use gesture to convey substantive information, express emphasis, and manage classroom interaction.

Mathematics lessons often focus on links between different representations of mathematical information, and teachers frequently use gesture to highlight such links. In many cases, this involves linking an abstract, mathematical representation (e.g., an equation) to a more concrete, grounded representation (e.g., a physical system or story). In other cases, this involves linking a less familiar mathematical representation to a more familiar one. Teachers use a variety of gestural devices to establish these links.

As yet, we do not know for certain whether variations in teachers' gestures make a difference in students' comprehension and understanding of those links. However, based on past research about the role of gesture in comprehension and learning, it seems highly probable. In ongoing research, we are testing this possibility, and attempting to ascertain what types of gesture help to convey links between mathematical ideas most effectively. By experimentally manipulating the ways in which relations between mathematical ideas are conveyed, and exploring the consequences for learning, we will gain a deeper understanding of the cognitive processes involved in acquiring mathematical understanding. Eventually, we hope to be in a position to make research-based recommendations about how teachers can use gestures most effectively. Knowledge about instructionally effective gestures is an inexpensive and valuable tool that teachers can add to their "toolkit" of methods for effective communication.

Acknowledgments

We are grateful to many people for their contributions to this line of research, including Breckie Church, Matthew Wolfgram, Eric Knuth, Autumn Hostetter, Kristen Bieda, Chelsea Johnson, Amy French, Laura Valenzeno Scheinholz, Elisabeth Sylvan, Jim Stigler, and Giyoo Hatano. We thank Beth Atkinson for preparing the illustrations. We are also grateful to the agencies and programs that have funded this work: the Institute for Education Sciences, the James S. McDonnell Foundation, the Wisconsin Alumni Research Foundation, and the Carnegie Mellon University Small Undergraduate Research Grant program. Finally, we are most grateful to the teachers and students who made this work possible.

References

Alibali, M. W. (2005). Gesture in spatial cognition: Expressing, communicating and thinking about spatial information. *Spatial Cognition & Computation*, 5, 307–331.

Alibali, M. W., Flevares, L., & Goldin-Meadow, S. (1997). Assessing knowledge conveyed in gesture: Do teachers have the upper hand? *Journal of Educational Psychology*, 89, 183–193.

Alibali, M. W., & Nathan, M. J. (2007). Teachers' gestures as a means of scaffolding students' understanding: Evidence from an early algebra lesson. In R. Goldman, R. Pea, B. Barron,& S. J. Derry (Eds.), *Video research in the learning sciences*. Mahwah, NJ: LEA. pp. 349–365.

Church, R. B., Ayman-Nolley, S., & Alibali, M. W. (2001). Multimodal representation and deep learning. Poster presented at the biennial meeting of the Cognitive Development Society, Virginia Beach, Virginia, October 26, 2001.

Church, R. B., Ayman-Nolley, S., & Mahootian, S. (2004). The role of gesture in bilingual education: Does gesture enhance learning? *International Journal of Bilingual Education and Bilingualism*, 7, 303–319.

Corts, D. P., & Pollio, H. R. (1999). Spontaneous production of figurative language and gesture in college lectures. *Metaphor and Symbol*, 14, 81–100.

Flevares, L. M., & Perry, M. (2001). How many do you see? The use of nonspoken representations in first-grade mathematics lessons. *Journal of Educational Psychology*, 93, 330–345.

Goldin-Meadow, S., & Sandhofer, C. M. (1999). Gesture conveys substantive information to ordinary listeners. *Developmental Science*, 2, 67–74.

Graham, J. A., & Heywood, S. (1976). The effects of elimination of hand gesture and of verbal codability on speech performance. *European Journal of Social Psychology*, 5, 189–195.

Hostetter, A. B., & Alibali, M. W. (2008). Visible embodiment: Gestures as simulated action. *Psychonomic Bulletin and Review*, 15, 495–514.

Kelly, S. D., & Church, R. B. (1997). Can children detect conceptual information conveyed through other children's nonverbal behaviors? *Cognition and Instruction*, 15, 107–134.

Kendon, A. (1994). Do gestures communicate? A review. *Research on Language and Social Interaction*, 27, 175–200.

Krauss, R. M., Morrel-Samuels, P., & Colasante, C. (1991). Do conversational hand gestures communicate? *Journal of Personality and Social Psychology*, 61, 743–754.

Lakoff, G., & Núñez, R. (2001). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.

McNeil, N. M., Alibali, M. W., & Evans, J. L. (2000). The role of gesture in children's comprehension of spoken language: Now they need it, now they don't. *Journal of Nonverbal Behavior*, 24, 131–150.
McNeill, D. (1992). *Hand and mind: What gestures reveal about thought.* Chicago: University of Chicago Press.
McNeill, D., & Duncan, S. (2000). Growth points in thinking-for-speaking. In D. McNeill (Ed.), *Language and gesture.* Cambridge: Cambridge University Press. pp. 141–161.
Mortimer, E. F., & Wertsch, J. V. (2003). The architecture and dynamics of intersubjectivity in science classrooms. *Mind, Culture, and Activity*, 10, 230–244.
Núñez, R. (2005). Do real numbers really move? Language, thought, and gesture: The embodied cognitive foundations of mathematics. In F. Iida, R. Pfeifer, L. Steels, & Y. Kuniyoshi (Eds.), *Embodied artificial intelligence.* Berlin: Springer-Verlag. pp. 54–73.
Perry, M., Berch, D., & Singleton, J. (1995). Constructing shared understanding: The role of nonverbal input in learning contexts. *Journal of Contemporary Legal Issues*, 6, 213–235.
Richland, L. E., Zur, O., & Holyoak, K. J. (2007). Cognitive supports for analogies in the mathematics classroom. *Science*, 316, 1128–1129.
Rogers, W. T. (1978). The contribution of kinesic illustrators toward the comprehension of verbal behavior within utterances. *Human Communications Research*, 5, 54–62.
Roth, W.-M. (2002). Gestures: Their role in teaching and learning. *Review of Educational Research*, 71, 365–392.
Roth, W.-M., & Bowen, G. M. (1999). Complexities of graphical representations during ecology lectures: An analysis rooted in semiotics and hermeneutic phenomenology. *Learning and Instruction*, 9, 235–355.
Roth, W.-M., & Lawless, D. V. (2002). When up is down and down is up: Body orientation, proximity, and gestures as resources. *Language in Society*, 31, 1–28.
Schwartz, D. L., & Bransford, J. D. (1998). A time for telling. *Cognition & Instruction*, 16, 475–522.
Singer, M. A., & Goldin-Meadow, S. (2005). Children learn when their teacher's gestures and speech differ. *Psychological Science*, 16, 85–89.
Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education.* New York: Simon & Schuster.
Stigler, J. W., Fernandez, C., & Yoshida, M. (1996). Cultures of mathematics instruction in Japanese and American elementary classrooms. In T. Rohlen & G. LeTendre (Eds.), *Teaching and learning in Japan.* Cambridge: Cambridge University Press. pp. 213–247.
Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teacher for improving education in the classroom.* New York: Simon & Schuster.
Valenzeno, L., Alibali, M. W., & Klatzky, R. L. (2003). Teachers' gestures facilitate students' learning: A lesson in symmetry. *Contemporary Educational Psychology*, 28, 187–204.
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27, 458–477.

16 Perceptual Learning and Adaptive Learning Technology

Developing New Approaches to Mathematics Learning in the Classroom

Christine M. Massey, Philip J. Kellman, Zipora Roth, and Timothy Burke

Most humans, both young and old, are capable of remarkable feats of learning in their everyday lives, and yet, all too often, the news from classrooms in the United States is about perennial difficulty and persistent failure for large numbers of students in achieving the learning goals set out in local, state, and national standards. Although the causes and potential cures are many and varied, in this chapter we consider an approach that addresses dimensions of learning that have been studied for decades in the learning sciences but have received little to no attention in K–12 classrooms. Specifically, we examine *perceptual learning* as a form of learning that contributes to the insight and fluency that characterize expertise across many settings and domains. We introduce what perceptual learning (PL) is, findings about PL that have emerged from several different lines of research, and how PL might be brought into K–12 classrooms as a significant complement to other forms of instruction. Two empirical studies of PL interventions using specially designed learning software, known as Perceptual Learning Modules (PLMs), illustrate some of its key characteristics and effects on students' learning in mathematics.

What Is Perceptual Learning?

The classic definition of perceptual learning, offered by Eleanor Gibson (1969), is "an increase in the ability to extract information from the environment, as a result of experience and practice with stimulation coming from it" (p. 3). With practice, in virtually all domains of human experience, people become significantly better and faster at extracting relevant information, ignoring irrelevant information, making fine discriminations, and perceiving higher-order structure and relationships. It is particularly noteworthy that PL emphasizes the pick-up of information that is demonstrably present in the external environment (though typically unnoticed or inefficiently processed by novices). The changes that result from such learning are changes in the ability to recognize or distinguish significant features, structures, or relationships. (For a more detailed discussion of PL in relation to other taxonomies of learning see Gibson & Gibson, 1955, and Kellman & Garrigan, 2008. See Kellman, 2002, and Goldstone, 1998, for general reviews of contemporary research on PL.)

PL involves the optimization of attention, so that the learner becomes increasingly selective in what information is attended to and what is disregarded. It is also characterized by increasing specificity of discrimination, such that experience allows the learner to make fine distinctions among features or structures that initially appeared