AC 2011-315: MODAL ENGAGEMENTS IN PRECOLLEGE ENGINEER-ING: TRACKING MATH AND SCIENCE CONCEPTS ACROSS SYMBOLS, SKETCHES, SOFTWARE, SILICONE AND WOOD

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Modal engagements in precollege engineering: Tracking math and science concepts across symbols, sketches, software, silicone and wood

Abstract

Collaborative, project-based K-12 engineering education curricula like Project Lead the Way ® (PLTW) strive to integrate the knowledge and skills across the STEM fields to provide both college preparation with technical education. In PLTW, students engage with ideas and activities across a broad range of modalities, including: Abstract mathematical equations, graphs and diagrams; 2D design sketches, computer-aided design (CAD) and simulations; and material construction of devices in wood, metal, plastic, wire and silicone. Curricula designed around such a broad set of modal engagements are assumed to be beneficial to engineering learning, since they provide students with a varied set of contextualized encounters with ideas, representations, tools and skills that foster a rich and grounded engineering education. Studies of high school classroom learning and instruction across two multi-day units in mechanical engineering and digital electronics reveal that students struggle to see the interrelatedness across these modal engagements that are often apparent to curriculum developers and instructors. Analysis of instruction and classroom discourse drawings, gestures and physical constructions shows that the cohesion of mathematics concepts across modal engagements is something that has to be explicitly produced *in situ* and enforced locally by the participants. Modal engagement analysis reveals the consistent ways that teachers and students produce cohesion (1) by projecting to past and future modal engagements, and (2) by coordinating representations and materials that are simultaneously present during modal engagements. Together, projection and coordination create cohesion-producing opportunities to thread the mathematics through disparate representations, material forms and events. This work alerts us to the importance of explicitly addressing the need to produce and enforce cohesion across the range of material forms, representations and activities that students confront in typical learning experiences.

Motivation

The pool of engineers in the United States is neither large enough nor diverse enough to meet the current needs of a growing, high-tech, global economy¹. Yet the "talent pool" among many sectors of the population goes largely untapped². As Legand Burge, Dean of the College of Engineering, Architecture and Physical Sciences at Tuskegee University, one of the nation's premiere Black colleges, noted, "there needs to be more of a national commitment to improve the teaching of technology" at the high school level in order to promote engineering³. This means that reform of engineering education must address not only the design of post-secondary programs, but of K–12 education as well.

Along with a growing urgency for promoting student understanding of the individual facets of science, technology, engineering and mathematics has come a drive to reconceptualize instruction in terms of STEM integration that would break down traditional curriculum "silos."⁴ This emphasis is in response to several sources: Learning Sciences research aimed at fostering greater transfer of knowledge⁵; federal initiatives, such as "Race to the Top"⁶; and new policy

documents⁷ and federal laws. One influential law is the 2006 Reauthorization of the Perkins Career and Technical Education Act⁸, which mandates that technical education and academic math and science topics must be integrated "so that students achieve both academic and occupational competencies" with substantial funds allocated "to provide vocational education programs that integrate academic [math and science] and vocational education."

Many commercial K-12 engineering curricula have taken up this mandate toward STEM integration⁹. One of the most broadly adopted programs is the Project Lead the Way® (hereafter PLTW) high school program, *Pathway to Engineering*, a four-year, pre-engineering curriculum intended to be integrated into the students' academic program of study. PLTW is affiliated with over 30 nationally accredited colleges of engineering, such as Rochester Institute of Technology, Duke, San Diego State, and Purdue¹⁰. It offers seven high school courses accredited for college credit. The PLTW high school pre-engineering program has been adopted by over 15% of US high schools, and is present in all 50 states.

PLTW explicitly strives to integrate students' college preparatory and technical education programs of study¹¹. As PLTW states in their marketing materials: "The combination of traditional math and science courses with innovative Pathway To Engineering courses prepares students for college majors in engineering and E/T fields and offers them the opportunity to earn college credit while still in high school"¹². Indeed, the NRC report, *Rising Above the Gathering Storm*¹³ explicitly identifies PLTW as a model curriculum for providing the kind of rigorous K-12 materials needed to improve math and science learning and increase America's technological talent pool. Given the broad market penetration, affiliation with institutions of higher education, including provisions for college credit, and commitment to an integrated program across academic and technical education curricula, PLTW is an important exemplar for studying the degree to which integrated, and conceptually based pre-engineering programs are implemented in public high school classrooms.

The Research Focus

The focus of this research is to develop the theoretical and methodological tools for describing the STEM integration process and articulating the curricular expectation, students' struggles and teachers' remedial efforts to foster cohesion. The approach taken here is to examine instruction as it unfolds in the classroom setting. Video (often 2 cameras per session) serves as the primary data source, which is then systematically coded and analyzed to highlight the rich interactions with math, science and engineering concepts and representations used throughout the classes This study draws on video from two high school classrooms during multi-day units in mechanical and electrical engineering to help support generalizations of the findings.

The Need and Challenge of Enacting STEM Integration

Engineering education at the K-12 level in curriculum programs such as Project Lead the Way (PLTW) is often organized as a collaborative, project-based experience where students encounter ideas and activities across a broad range of modalities, including: Abstract mathematical equations, graphs and diagrams; 2D design sketches, computer-aided design (CAD) and simulations; and material construction of devices in wood, metal, plastic, wire and silicone.

These occur within a broad range of participation structures, such as lectures, individualized work and small group collaboration as they take place in a variety of physical settings, including traditional classrooms, laboratories, wood and metal shops, and out in the field. Curricula designed around such a broad set of *modal engagements* are assumed to be beneficial to learning, since they provide students with a varied set of contextualized encounters with ideas, representations, tools and skills that foster a rich and grounded engineering education. A modal engagement is defined by Hall & Nemirovsky (2010, p. 1) as "an activity someone participates in, with others, tools, and symbols"¹⁴. Examples of modal engagements include: working with notational systems, equations, and diagrams; working with digital media, such as software simulations and electronic circuits; working with raw materials such as metal and wood; and working with designed objects and measurement instruments. Different modal engagements elicit different participant structures, such as classroom lectures, computer lab work, small group work, woodshops, etc.

There are many opportunities for pre-college engineering students to connect science and math concepts across a range of representations, procedures, and material instantiations. Within this framework, a fundamental challenge in STEM education is that learners must recognize the inter-relatedness of ideas across a broad range of modal engagements and realize how concepts encountered in one form (e.g., an equation) relate to those same concepts encountered elsewhere (e.g., in a 3D device).

Yet K-12 students can struggle to see the interrelatedness across these modal engagements that are often apparent to curriculum developers and instructors. There is some evidence that students do not readily make connections across different modal engagements. For example, in precollege engineering classes many students struggle to integrate previously encountered geometry concepts in activities such as computer-aided design (CAD) or measurement activities¹⁵. Analyses of standardized tests also show that many students who take pre-college engineering courses show no demonstrable advantage in science achievement scores than their peers, while advantages in math achievement are only found among those in higher socio-economic communities¹⁶ and specialized programs of study¹⁷,¹⁸. Thus, while the opportunities for conceptual integration and transfer abound, there is little evidence that engineering courses are fostering the integration needed to benefit a great many prospective engineering students.

Recent investigations of engineering curricula, classroom instruction and student achievement point to the challenges of realizing effective STEM integration in K-12 education. Content and alignment analyses of the 2004 PLTW curricula showed that while many mathematics content standards were in evidence among the three PLTW foundations courses--with more standards and more sophisticated standards in place in later courses--those standards that were addressed were seldom made explicit to the students in the curriculum materials; rather they remained implicitly embedded in the activities, instruments and software used to carry out the tasks¹⁹.

Along similar lines, The National Academy of Engineering conducted far-reaching analyses of 22 elementary, middle and high school pre-engineering curricula, including nine high school programs²⁰. The analysis explored the mission and goals of each curriculum; the presence of engineering concepts; and how each curriculum explicitly treated mathematics and science in with regards to engineering problems. Their remarks to date are most striking about the shallow

role of mathematics often observed across the corpus of curricula. In findings that echo the studies of PLTW curricula, Welty and colleagues²¹ lament "the noticeably thin presence of mathematics" across K-12 engineering curricula (p. 10). They explained, "Most of the mathematics in engineering curricula simply involved taking measurements and gathering, organizing and presenting data. Very little attention was given to using mathematics to solve for unknowns. Furthermore, little attention was given to the power of mathematical models in engineering design" (p. 9).

Analyses of the engineering curriculum materials address the *intended curriculum*, that is, the *idealized* vision of the curriculum design put forth by designers in the printed materials used for the course. Yet, it must be noted, curricula are generally not implemented as planned, and may not even unfold the same ways under the guidance of the same teacher in different class sections. Consequently, analyses of the intended curriculum paint a foundational but incomplete picture of a course that gives so much attention to in-class group project work. To address this shortcoming, Porter and colleagues²² distinguish the intended from the enacted curriculum. The *enacted curriculum* refers to the specific content as it is actually taught by teachers and studied by students during the course of learning and instruction.

Analysis of the enacted curriculum provides an inherently richer account than the intended curriculum since its object of focus is the actual teaching and learning behaviors and student-teacher and student-student interactions. For an investigation of the enacted curriculum, it is necessary to work from primary observations in the field and videotaped records to determine the events and interactions that occur during teaching and learning. Classroom observation is especially important given the practical nature of this course and the emphasis on project-based work and peer collaboration.

Understanding the Nature of Students' Struggles to Integrate STEM Concepts

Given the high expectations and rather equivocal findings for pre-college engineering education on science and math achievement, research is needed to understand the challenges students face in establishing and maintaining cohesion across the range of science and math concepts or their changing nature as they are manifest in different modal engagements. Currently there is little systematic study of the challenges students face in building cohesion across the many modal engagements commonly encountered in engineering, or the manner in which cohesion over a range of shifting material and representational forms is established and maintained in the classroom. We focus our analyses on the cohesion of mathematical concepts as they are manifest across the range of modal engagements in multi-day engineering units. Our focus on mathematics within engineering is informed by views from engineering education scholars like Schunn²³, who argues that math is "the language of physical sciences and engineering sciences" and as such is especially "critical" to achieving synergy across STEM fields.

In accounting for the cohesion of mathematical ideas across such a broad range of STEM activities and education settings, there appear to be two distinct but interrelated objectives. First, we need to be able to locate mathematical ideas over time as they appear in various places, people and material forms. Second, we must describe what is invariant about the mathematics as it morphs across forms, activities and settings.

Locating the Mathematics in Various People, Places and Things: The Where of Mathematics

In this paper, we examine two cases drawn from our observations of multi-day lessons from high school engineering courses to illustrate how teachers and students manage the process of realizing concepts across distinct modal engagements. As a way to introduce the theoretical framework that guides our work, and the research methods we employed, we briefly present the issues in the context of the first case,, a portion of a multiday lesson on ballistics, where the students use math (trigonometry and algebra) and physics (kinematics) to calculate the distance a projectile will travel. Students then make design sketches and, ultimately, devices to launch a ping pong ball and test their predictions. The second case is drawn from a Digital Electronics class, the third course in the Project Lead the Way curriculum, in which high school students use truth tables to generate any of a number of Boolean Algebraic expressions, which are first simulated using computer software that constructs virtual digital circuits, and then hardwired and tested as a working electronic circuit.

Consider, as one of our examples, a typical engineering unit on building ballistic devices to hurl an object at a target at an unspecified distance (Figure 1). During this multiday lesson students need to understand a lecture on the physical laws governing projectile motion expressed in algebra, geometry and trigonometry; create, critique and revise 2D design sketches; use materials, measuring instruments and tools (both handheld and power tools) to construct the device; compile measurements during testing and analyze them; and so on. Each phase calls for one or more material forms, representations or tools to take center stage; each may be scheduled in a different space (wood shop, classroom) or classroom configuration (lecture, small groups); and each has a historical relationship with the events it follows and precedes²⁴. By identifying *where the mathematics is located*, we establish its existence and describe its dynamic nature and the actions needed to maintain cohesion across its many manifestations.



your distance look like.



Figure 1. (a) (Top 2 panels) The mathematics and physics of kinematics that model ballistic motion must also be connected to (b) (middle 2 panels) the 2D design sketch, and (c) (bottom 2 panels) the construction, testing, and redesign of the ballistic device. Note that the teacher attempts to connect the design sketch to the wood in the construction phase (left panel), but the student focuses on the wood, to the exclusion of any cross-modal connections (right panel).

Transitions between Modal Engagements

The process by which teachers and students manage the transitions across changing contexts while maintaining cohesion within curricular activities is both complex and precarious. The teacher and the students must continually manage and negotiate this process; and to do so they rely heavily on language in the form of speech, writing and gestures. Speech provides cohesion by using resources such as labels and explanations. As will be made clear from the cases below, however, simply referring to mathematical ideas using consistent labels across different contexts is not sufficient for most students to establish the cohesion necessary to complete their projects and to develop a clear understanding of how the mathematics permeates the various activities and representations. Teachers respond to this challenge by providing visual scaffolding, including written inscriptions such as equations, diagrams and words, created *in situ* or obtained from curriculum materials, that reify concepts, relationships and plans, in a manner that is (relatively) enduring. Teachers also use gestures to establish and maintain cohesion. They do so by linking ideas and visual elements, either with sets of pointing gestures or with gestural

catchments²⁵ that involve repeated hand shapes and movements that can signify, and thereby reinvoke, previously referenced ideas and events^{26 27}.

We have identified three ways that transitions among modal engagements are co-produced by teachers and students. One is that the participants make an *ecological shift*, which involves a reorientation of the activity context that can include different spaces, tools, media of instruction, and participant structures. At its surface, the shift can simply appear as a change in the activity, as, for example, when the digital electronics teacher called the class to stop their computer lab work and focus attention on the board. Alternatively, the shift may be more momentous, as when the engineering teacher took his students from the classroom, down the hall, to the wood shop, which altered the norms of proper (i.e., safe) conduct, the tools at their disposal, ambient sounds, and the participant structures, while also placing what had previously been plans for the future into the present task of implementing the proposed designs. Ecological shifts are not mere changes in context but transitions that can potentially alter many of the modal engagements.

A second transition process, *projection*, involves the use of language to connect events of the present to past or future modal engagements. Past projections can link across an ecological shift that has already occurred, while future projections can anticipate a coming shift in the instructional ecology (one that may even be part of the curricular design). Projections can take many forms. Some are brief utterances or simple pointing gestures, as when a teacher points to an empty white board to re-invoke the mathematical derivations from a prior lesson. Others are much more protracted, as when a teacher spends an entire introductory lesson planning the lab work for the rest of the week. Teachers and students use the verbalizations and gestures of projection, along with representations, objects, and the environment itself, both to reflect upon a history of a concept as it unfolds in their classroom, and to plan for future manifestations of the concept in different modal engagements. Ecological shifts – common as they appear to be – make it challenging for participants to preserve a sense of the cohesion and continuity of the mathematical ideas. Projections serve to construct connections over time and help to establish that sense of cohesion for students.

A third transition process is *coordination*, which involves the juxtaposition and linking of different material and representational forms. For example, students may coordinate a design created in a software environment with an actual device they are building, or they may enact coordination between symbolic and tabular representations of the logic of a digital circuit. When speakers integrate across time and material and representational forms simultaneously -- as when they make a connection from a device in their immediate context to a previously encountered but now absent equation -- we consider this both coordination and projection. We also recognize the occurrences of *intra-modal* actions as encompassing activity with a single material and representational forms.

Identifying Locally Invariant Relations: The What of Mathematics

In addition to identifying *where* the math is located and describing transitions between modal engagements, it is important to be able to say *what is the mathematics* across shifting social configurations, physical settings and material and symbolic forms. We have found that the

stability of the mathematical content across contexts and forms is something that has to be produced and maintained "locally" to the agents, time and context.

A complete answer to the question of "What is mathematics, really?" is elusive²⁸. The view that mathematics is formal, abstract, and disembodied, yet still a real and transcendent feature of the objective universe, has been characterized by Lakoff and Nunez as the "romance of mathematics". Yet this position neglects the pragmatics of math, including the manner in which terms and concepts have shifting meanings and ways that categorically related entities may share only a portion of the defining criteria. It also marginalizes the social and cultural influences on the origin of mathematics and the manner in which it is archived, taught and applied³². Another perspective, espoused by Aristotle, is that mathematical objects are not distinct from physical objects, but instead are physical objects considered in a particular, abstract way³³.

Noble and colleagues³⁴, characterize mathematical concepts by what people *do* mathematically³⁵, rather than the syntactic, or *form*-al, qualities. The criteria for membership in a mathematical category, in this view, are not pre-defined or rigid, but come instead from *family resemblances* that people register because of the "complicated network of similarities overlapping and crisscrossing"³⁶ that characterize their relationships. From this perspective, cohesion of meaning across modal engagements comes from the many inter-relations between forms and experiences that share (sometimes implicit) characteristics and differ in others, just as "the strength of the thread does not reside in the fact that some one fibre runs through its whole length, but in the overlapping of many fibres" ³⁷. Thus, cohesion of mathematical meaning and practice can occur because of the ways that people develop and employ family resemblances across the range of lived experiences and material and representational forms³⁸ (Noble et al., 2001).

However, none of these perspectives seem to account for the normative character of knowledge within scientific communities. Mathematical knowledge is neither totally over-determined as autonomous and universal, or totally underdetermined--created, so to speak, as each participant notices a unique configuration of similarities. Between these two poles lies the context of the STEM classroom, where the communication and enforcement of family resemblances is an interactional achievement that is often only partially successful. Often within STEM fields relations are made by expert practitioners because of taken-as-shared understanding of the underlying mathematical models (e.g., second order linear constant coefficient differential equations; see Appendix A), which, when properly applied across seeming disparate forms (e.g., electrical circuits versus mechanical systems), generate accurate, quantitative predictions in either domain. Thus, expert practitioners (such as mathematicians, scientists, and engineers) establish cohesion even when the conventions do *not* exhibit family resemblances. An empirically informed account of learning and instruction has to address the obstacles students face in taking up these conventions, documenting the resemblances made spontaneously by nonexperts, as well as the contributions made by instructors and those that occur in curriculum materials and classroom activities

It appears for our purposes, that essentialist, romantic and family resemblance views of mathematical concepts are insufficient for framing the study of Western science classrooms where authoritative voices from the course curriculum and the instructor assume the existence of invariant properties tied to specific concepts, and these concepts must be learned and applied

during high-stakes assessments to satisfy state and national standards. There may be powerful arguments why these invariant relations do not really exist, or do not hold *in general*. But there seems to also be a need to acknowledge that in more constrained circumstances -- specific relations in specific contexts brought forth to achieve specific curricular goals -- trained practitioners in STEM fields can identify *locally invariant* qualities of ideas even as they transition across modal engagements, and work to establish and maintain these relations. Like the temptation to declare a "flat Earth" when constructing a house, activities sufficiently localized in space and time can provide mathematical experiences that sufficiently approximate the invariant structures posited by the essentialists, but with an understanding that the dynamics of these modal engagements are central to the mathematical experiences of students and ultimately influence how they represent and enact their experience-based knowledge.

From our perspective, the cohesion of mathematical concepts cannot be assumed in STEM curricula, and the means for creating cohesion across modal engagements are neither obvious nor universal. Novices operating in STEM classrooms and workplace environments need, rather, to be socialized into perceiving the same invariants that are salient to experts. Consequently, **we set out to show that the cohesion of mathematical knowledge across contexts is something that has to be produced and enforced locally by classroom participants**. As noted by Noble and others, the meaning of a concept changes across contexts because the network of relations changes – this is part of the reason for locating concepts in different contexts. The sense of cohesion of a concept across modal engagements can be produced through the local management of talk and action. Our position, then, is a pragmatic one in that we understand this stability to be a property not of the math itself, but of something that is produced in the context of normative mathematical and scientific discourse practices.

For our current purposes, we assume that what is taken as locally invariant in each manifestation of the mathematics is some kind of central *relationship*. Following Hall and Nemirovsky³⁹, this does not mean that the mathematical concepts of concern are *amodal*, or without form or physical properties; as we will illustrate, the mathematics as it is experienced and practiced by teachers and students is highly subject to the modal engagements in which it arises. Yet, there are common relations evident in each modal engagement that can be analogically or metaphorically mapped to other modal engagements. People performing a mathematical activity can perceive, maintain, and even construct locally invariant relations by using relation- and inference-preserving cognitive mechanisms such as analogical mapping and conceptual metaphor⁴⁰.

We posit that many aspects of curriculum and instruction in STEM education exist in order to engage relation-producing mechanisms with the goal of advancing students' perceptions of locally invariant properties so that they serve as a cohesive thread throughout the STEM activities. An example of a locally invariant relation comes from the projectile motion unit for a high school engineering class we introduced earlier, where there is a need to characterize *theta*, the angle of ascent, across a range of modal engagements (Figure 1). In the classroom, we may observe the locally invariant relation in several ways; a raised arm to the base of a triangle, a Greek symbol, a numeric measure, a tangent line meeting a plane, and the relation between the trajectory of an object and the ground, as *theta* is realized, respectively, by the flight of a ball, a lecture, an equation, a sextant, or an idealized diagram in analytic geometry. By focusing on *relations* as the *what* of mathematics we direct our efforts at

understanding the nature of the relations, the entities that are in relation, and how the relations are captured or expressed in representations, in communication using speech and gesture, and in the functioning of constructed devices.

Focus of Research

The analysis that follows focuses on how mathematics is "threaded through" various modal engagements – specifically, how classroom participants establish cohesion with locally invariant mathematical relations (the *what* of mathematics) as they are projected and coordinated (transitioned) with various modal engagements (the *where*). Within this framework, we set out to address one primary research question: **How are mathematical ideas (relations) realized within and across modal engagements as they occur in STEM classrooms?** We explore this question in two classroom cases. In the final section we consider the educational implications of this work, particularly the challenges it presents to teachers, learners and curriculum developers seeking to foster STEM integration.

Theoretical Framework: Striving for Cohesion Across Modal Engagements

In addressing our research question, we focus on both the *where* and the *what* of mathematics, as discussed above. In addressing the *where* of mathematics, we consider how mathematical concepts are realized in a given modal engagement across forms, time and space, and we also consider the affordances and constraints that each modal engagement exhibits for reasoning mathematically. Within this inquiry, we examine a broad range of behaviors and contexts, such as the nature of social interactions in instruction (lectures, coaching) and student discussions (investigations, explanations and elaborations, questions, design decisions); the role of artifacts (designs, tools, devices) and symbol systems (language, symbolic, algorithmic and visual representations); changes in the learning environment; and the development of ideas and practices over time, including the history, present focus and future planning across various phases of a classroom unit.

We also address the *what* of mathematics, by exploring how mathematical relations are preserved when they are manifest in markedly different ways. The analysis of cross-modal behavior foregrounds the intended (i.e., expert) taken-as-shared, locally invariant nature of mathematics as a normative relation among entities regardless of their outward form. Together, the investigation of cross-modal and modal-specific behaviors will be used to characterize how perceived invariant mathematical relations are socialized by STEM practitioners (i.e., teachers and curriculum designers) and how the invariant relations are realized within and across modal engagements. This investigation raises important questions and insights about how mathematics both facilitates and obfuscates the integration of concepts across scientific fields.

Coordinating the many manifestations of mathematical relations across modal engagements is the hallmark of STEM integration, and is central to professional practices^{41 42}. To the student, however, the locally invariant qualities of a particular mathematics concept may not rise above the din of variation. Consequently, there may be a tendency for students to *provincialize* the mathematics, confining it to the salient, local, present forms that characterize their immediate experiences. Under this view, students are likely to develop a very narrow sense of the math,

which limits their actions, talk and ways of representing and reasoning about the concepts – what we call their *epistemological commitments*. Epistemological commitments that are highly provincial tend to highlight salient properties of the currently available settings and representations, while neglecting connections to knowledge, practice and representations that are not immediately apparent. This provincialized sense of mathematics contrasts sharply with the dominant ideology that scientific and mathematical knowledge are statements of autonomous principles of the world that humans have struggled to perceive and understand through the history of knowledge, and that this knowledge portends a world that is regular, predictable, and rational. This more global *epistemological commitment* is an overarching goal of STEM education, and it is part of the socialization process of becoming a competent member of an information-, technology-, and science-saturated society. It is also a basic part of the socialization of scientific and technical experts in our society.

The occurrence of provincialized epistemological commitments poses a serious challenge for STEM education, for, as we have seen, mathematical ideas arise in a variety of forms, settings, activities and personal interactions. It is often the case in engineering design, for example, that certain mathematical ideas – key relationships between quantities – must be under the control of a user or another device and must therefore be identified as parameters of the system. The control of the parameter then becomes a central design consideration that drives the kind of mathematical models that are used, the nature of the final product, and the means by which performance of a device is evaluated. In the case of the ballistics unit, students striving to hit a target at a specific (but as yet unknown) distance will need to enact the kinematics of projectile motion by manipulating the angle of ascent, *theta* (Figure 1), on their devices. This central parameter must therefore be realized in each representation and material form that students encounter: mathematical diagrams and equations, body-based enactments and discussions of the device behavior, design sketches, and wood and metal assemblies.

To foreshadow, our analysis identifies tensions that arise between the provincial, modal-specific epistemological commitments that naturally arise for students, on the one hand, and an integrative system that threads invariant mathematical relations across different modal engagements as is characteristic of professional practice and expected in curriculum objectives, on the other. This includes how ideas are projected forward and backward in time, and how they are coordinated across different spaces (classroom, computer laboratory, wood shop), different material and representational forms (symbols, drawings, wood and plastic), and different participation structures and social practices (e.g., those that occur in a metal shop versus in the classroom). Our analyses suggest that students are highly prone toward material- and representation-specific forms of gesture, talk, representation and tool use, which may signal highly provincial *epistemological commitments* toward the task. That is, students tend to focus on qualities that are immediately present and salient, treating each material and representational form as existing in isolation, historically and semantically, from the other forms that precede, follow and coexist with it. Teachers may actually contribute to these provincial commitments in that they, too, at times, model provincialized epistemological commitments for their students.

Teachers are also charged with providing coherent educational experiences and teaching students how to integrate these forms so they can reason, imagine, act and talk flexibly across them. Thus, we observe that teachers can keep the mathematics salient in each form while also **"threading**"

the ideas through" the modal engagements. When these instances occur, we see the teacher overtly (and sometimes covertly) coordinate math concepts across modal engagements. Teachers often highlight locally invariant relations by referring to them using labels and gestures, or by invoking them directly. This seems to be done in order to establish for students cohesion across the project activities. It also may strengthen students' understanding of the mathematics.

Method

We conducted multiday observations of teaching and learning in two different high school courses: Principles of Engineering (mechanics, 4 days) and Digital Electronics (circuit design, 4 days). We refer to our procedure for identifying these different transitions as "modal engagement analysis" and characterize it as a methodology for analyzing teachers' and students' activity in terms of modal engagements occurring within ecological contexts. For the purpose of our analysis, we define a modal engagement as an interaction between participants surrounding one specific concept, procedure, topic, or problem. For example, in the digital electronics case, instances of modal engagements are: students solving a Boolean algebra problem on a worksheet, modeling an AOI diagram on MultiSims, and debugging a digital circuit using a truth table. By analyzing the different modal engagements that occur as activity in engineering classrooms unfolds, we can see how teachers and students enact projection and coordination between material and representational forms to manage cohesion of math concepts across ecological shifts.

Videotapes of these classrooms were collected (with 2 camera views for each class), and the videotapes were transcribed and coded in Transana v2.41. We then identified segments of discourse within the lessons that manifested ecological shifts, coordination, and projection, as described above. The criteria used to code transitions are shown in Table 1.

Transition	Coding Criteria
Ecological Shift	Evidence of a major reorientation of classroom activity to involve different
	settings, participation structures, representational and material forms, tools, or
	actions (i.e., different sets of modal engagements).
Projection	Evidence that participants refer to an absent (past, planned or imagined)
	modal engagement.
Coordination	Evidence that participants link two or more co-present material or
	representational forms.
Projection +	Evidence that participants make a projection to an absent engagement while
Coordination	also linking this engagement to a currently present material or representation.

Table 1. Coding criteria for modal engagement transitions

Results From Modal Engagement Analysis: Two Case Studies

A key goal of project-based learning (PBL) in engineering classrooms is to engage complex problem-solving skills and reflective processes in teacher-guided, contextually rich, goaloriented settings⁴³. In the course of PBL, learners encounter, experiment with, and even rediscover important abstract relations that developed over the history of math and science practice. Students' uses of tools, representations, materials and forms of talk facilitate the construction of new objects that are themselves realizations of this abstract knowledge. The classrooms that we describe illustrate how teachers and students manage the process of threading concepts across these rich ecological contexts. We present descriptions of each of the lessons (with transcripts) and discuss their connections to our emerging theory.

Case #1. Theta in Symbols, Paper and Wood: A Ballistic Device Design

The ballistics project challenge

I'm actually gonna give you a distance and I'm gonna say "okay we're gonna send, we're gonna set the basket fifteen feet away," but whatever distance that is I'm gonna decide that at the time, we're gonna set the basket so many feet away and you have to try to hit it. So by doing some calculations on, what you're, um, ballistic device fires you can kinda set your angle hopefully to get, to get that distance."



Figure 3. One group's design sketch (with verbal and mathematical elaborations added).

This first case on the design of a ballistics device is drawn from a four-day unit from a high school Principles of Engineering classroom, the second of the three foundations courses in the Project Lead the Way® curriculum (www.pltw.org). On Day One of the lesson, the students in a second year pre-college engineering course learned the mathematics and physics of calculating projectile motion. The teacher highlighted for them the angle of ascent of the projectile—labeled *theta*—as the key variable that they must parameterize and represent in their sketches (one group's sketch is shown in Figure 3) and ultimately in the wood, metal, plastic and other materials that they fashioned and assembled into a catapult, trebuchet, gun or some ballistic device of their own choosing and design. If these devices properly instantiate *theta*—that is,

permit the adjustment of the angle of release while holding the other influential variables (e.g., initial velocity) constant—students will be able to predict the distance that the projectile will travel. Throughout the sequence of the lesson, knowledge of *theta* is inscribed or represented in different modal engagements with: symbols and diagrams on the white board during an initial lecture, paper and pencil during small group design meetings, and collections of materials formed into a projectile device, which are ultimately manipulated and evaluated.

First, we provide an analytic overview of the events of Day 2 of the four-day unit. Then we provide a more focused analysis of one interaction around a group's design sketch.

Table 2 provides a tabulation of the codes and descriptions applied to the modal engagements for one class day. Figure 4 shows the information spatially, with arrows showing forward and backward projection. There were 4 distinct ecological contexts (or major events with transitions). Each context has a specific setting and participation structure, such as lecture or group work. On this day, no major setting changes took place-it was all in the classroom-but other times the class may travel to the woodshop or the gymnasium and reform under different circumstances. Within each context there can be one or more modal engagements. This day saw a total of 18 modal engagements, over 4 ecological contexts. Each modal engagement is assigned one or more codes for coordination, projection, and ecological shift (a transition). Projections further can be directed forward in time, emphasizing planning or goals; or backward, emphasizing reflection of making connections to past lessons and events. There is a sense in working with the data that backward projections are more likely to support students' conceptual development and it often integrates current modalities with prior knowledge and invites reflective thinking. As the data show, this is a complex lesson with a great many modal engagements to connect. Here we see a range of forward and backward projection, and coordination of a variety of modalities, suggesting students had many opportunities to integrate conceptual and procedural knowledge.

Figure 4 shows how the entire sequence of the ballistics project was coded for modal engagements and transitions using our modal engagement analysis methodology. The analysis shows the hierarchical structure of the lessons⁴⁴, where modal engagements are nested within ecological contexts. Columns show the various ecological contexts that the activities were embedded in throughout the project Arrows in the figure illustrate the roles of projection (italicized text) and coordination (underlined text) in the management of ecological shifts. In particular, it shows how the teacher often used backward (left arrow) and forward (right arrow) projections together to bridge present modal engagements (bulleted entries within each context), such as working with design sketches, to those in the past (e.g., the physics and mathematics of projectile motion) and into the future. The figure also indicates more frequent use of forward projection as the teacher prepared students to build and test the ballistic device.

Ecological Context	Setting: Participation	Context Description	Modal Engagement	Code	Projection Type	Modality Description
	Structure	-				-
1	Classroom: Lecture	Highly teacher controlled	Prep materials for building ballistic device	Р	Forward	Speech
			Solving Projectile	C + P	Backward	Projectile motion
			Motion (PM) problem	C + P	Forward	worksheet, ping pong ball, caliper
2	Classroom: Individual/small	Students teams;	Material discussion	C + P	Forward	Rubber strap
	group work	Teacher coaches	Measuring ping pong ball diameter	C	•	Ping pong ball, caliper
			Material discussion	C + P	Forward	Behavior of springs
			Measuring ping pong ball	C	•	Ping pong ball, caliper, sticky note
3	Classroom: Lecture	Highly teacher controlled	Demonstration for measuring ping pong ball	С	•	Ping pong ball, caliper, diagrams, numeric place value
4	Classroom: Individual/small group work	Student teams; Teacher	Measuring ping pong ball	С	•	Problem worksheet, ping pong ball, caliper, sticky note
		coaches	Measuring ping pong ball	С	•	Worksheet, caliper
			Measuring ping pong ball	С	•	Caliper, place value, sticky note
			Solving PM problem	С	•	Worksheets (compared)
			Working on	C + P	Forward &	Design sketch &
			device design		Backward	gesture
			Working on design	C + P	Forward & Backward	Sketch
			Working on design	C + P	Forward	Sketch
			Working on design	C + P	Forward	Sketch
			Solving PM problem	C + P	Backward	Worksheets (compared)
			Solving PM problem	С	•	Worksheets (compared)
			Working on design	C + P	Forward	Sketch, springs

Table 2. Modal engagements (with codes) for Day 2 of the ballistics device project.

* Codes: C = Coordination; P = Projection.

		Ecological Contexts							
		Lecture in Classroom		Individual/Small Group Work in Classroom		Lecture in Classroom		Individual/Small Group Work in Classroom	Building
Modal Engagements	Previous lecture about projectile motion	 Teacher checks students' preparation of materials for building the ballistic device <u>Teacher explains</u> worksheet problems about projectile motion 		 <u>Teacher and students</u> <u>discuss materials for</u> <u>building the ballistic</u> <u>device</u> <u>Students measure a</u> ping pong ball with the teacher's guidance 		• <u>Teacher explains how</u> <u>to measure the diameter</u> <u>of a ping pong ball using</u> <u>a caliper, and how the</u> <u>tool provides place</u> <u>numeric value</u> <u>information.</u>		 <u>Students complete the</u> projectile motion problems with the teacher's guidance <u>Students work with the</u> teacher's guidance on designing the ballistic device* 	ballistic devices in woodshop

Boxes show the major ecological contexts that activity was embedded in throughout the ballistic device case.

Bullets show the main modal engagements occurring sequentially in the case.

Italics = Projection, <u>Underline</u> = <u>Coordination</u>, <u>*Italics and Underline* = <u>*Projection + Coordination*</u></u>

Arrows show the main backward/forward projections and point to projected past/future modal engagement(s).

* indicates the modal engagement discussed in the transcript of the ballistic device case.

Figure 4. Visual depiction of the modal engagements across ecological contexts for Day 2 of the Ballistics Device project.

The particular modal engagement is depicted in the transcript below took place after a group of students presented their sketch of a catapult to the teacher on Day 2 of the four-day unit (Figure 5). The discussion of the students' design is sandwiched between the more formal lecture on kinematics (including algebra, trigonometry and the idealized behavior of an object in free fall with a constant horizontal velocity) and the material construction activity. Many aspects of this discussion serve to project toward the future context where the students will use their sketch to guide construction, and many aspects project back to the past kinematics laws and mathematical relations that were presented in the prior lecture.

Over the course of the discussion it becomes clear to the teacher that these students have confused the angle of ascent of the projectile with another angle in the system -- the angle of retraction of the arm of the catapult (Figure 3). Thus, the students' design sketch not only misidentifies the relevant angle (a failure to coordinate *theta*) but it also adds another variable -the tension on the rubber band, which influences the initial velocity. The teacher attempts to point out this confusion in Lines 1 and 3 (See Transcript #1). The students in Lines 4-6 try to salvage their sketch. Based on the constrained use of their gestures and eye gaze, and restricted references used in their speech, the students seem focused exclusively on the properties of the sketch itself, exhibiting epistemological commitments that constrain their discussion (see Figure 1b). On Line 7 the teacher explains that the angle that they need to control is the angle of the release of the projectile with respect to the ground and, continuing on Line 9, that they need to design something that does not affect the initial velocity. The tacit implication is that varying initial velocity introduces new complications that were not addressed in the mathematical models presented during the previous class. On Lines 13 and 15, the teacher reaches back to the math lesson from the previous day and threads the central mathematical relation through to the sketch (Figure 5). In Lines 13-19 the teacher coordinates and projects the students' design sketch backward to the math calculations that they did the day before on the whiteboard (which are still present in the front of the room) and forward to the future behavior of the vet-to-be-realized device. This coordination is accomplished through speech and gesture. The first gesture on Line 15 (Figure 5a) is a flat-palmed hand lined up horizontally with the diagram, iconically representing *theta* as an angle relating the trajectory of the projectile with the ground, though translated into the plane of the paper sitting on the desk. This is an important reference because the hand shape and motion re-invoke a similar gesture that the teacher used during the lecture on the mathematics (algebra and trigonometry) of projectile motion-a gestural catchment. The parallel between the gesture used at the white board and the one enacted here highlights the recurrent hand shape as a sign for theta. The second gesture on Line 15 (Figure 5b) is a point indicating the calculations that are still on the whiteboard. The teacher here uses speech and gesture to coordinate the calculations on the board with the students' diagram in an effort to locate *theta* in the design sketch and reinstate its original meaning.



Figure 5. (a) Line 15, Gesture 1: teacher iconically invokes *theta*. (b) Line 15, Gesture 2: teacher points to the whiteboard in the front of the room to index *theta* as a component of the kinematics laws from the lecture from the previous day.

Case #1 Transcript 1.

(*NB.* Speech transcript is complete but only gestures relevant to this analysis are shown, where square brackets denote the start and end of the gesture and numerical indices refer to the Gesture Notes at the end of the transcript.)

- 1 T: Well I'm wondering if the further you, pull your rubber band down-
- 2 S Mmhm.
- 3 T: is gonna affect your, velocity, more than your angle.
- 4 S: Yeah it's. Well no this is the velocity but what we're sayin' is that this is how hard it pulls but then right here, where it where it, where the fulcrum is like this actually you can tilt it.
- 5 S: The rubber bands control the tension but the placement is what really controls...
- 6 S: Like. See what we're saying?
- 7 T: So it's it okay so, if I could, suggest, I think that, you might be able to adjust your angle by, by having some type, by controlling where this stops.
- 8 S: Yeah.
- 9 T: But that's probably also gonna affect your, maybe affect your velocity. What I'm saying is. Either that or else you have to tip the whole thing.
- 10 S: No we don't. That's why cuz the two sides stay put but then the top part can, tilt, right there.
- 11 T: Okay.
- 12 S: So the fulcrum can change positions basically.
- 13 T: Alright. So I think maybe what you need to do is is, take into consideration what I just said about-
- 14 S: Yeah.
- 15 T: [-being able to control the ang-] [that's why we did everything we did here-] 1 2
- 16 S: Mmhm.
- 17 T: -with the math. Because we wanna-
- 18 S: the math yeah.
- 19 T: -be able to adjust the angle of the trajectory. I would try to keep, the velocity, the same, consistent, throughout the whole every test that you do that that's consistent and so all

you're gonna change once you once you decide what that velocity has to be all you're gonna change is your angle.

- 20 S: Yeah.
- 21 T: Okay?
- 22 S: Mmhm.
- 23 T: I don't really want you to use the tension on the rubber bands, as, the only control. I want you to have an angle adjustment.

Gesture Notes

- 1. Angle gesture
- 2. point to whiteboard

By way of summary, we reflect on this case in the language of our emerging theoretical framework. The locally invariant relation (the what of mathematics) is the angle of ascent of a projectile with respect to the ground, as represented initially by the symbol *theta*. The where of mathematics is described by ecological shifts and transitions between modal engagements. Cohesiveness is a challenge for some students who show epistemological commitments that appear to confine their thoughts, actions and forms of communication to the particular representations and materials that are centrally present (the 2D design sketch). To establish and maintain cohesion, we see the teacher threading the mathematics through the various modal engagements. The teacher uses speech and gesture catchment to coordinate the angle *theta* and its meaning with regard to projectile motion to the elements of the design sketch. He also identifies an important mis-conception, where students improperly identify an element of the catapult design as an instantiation of *theta*, leading them to parameterize the initial velocity rather than the angle of ascent with respect to the ground. Projection is used to signal for the student the historical role of the design sketch. Past projections are made to the mathematical formalisms that model projectile motion. A future projection is intended to position the sketch as a guide for the construction activity awaiting the students. Here the teacher specifically uses the impending testing of the device to clarify that they will not be changing the initial velocity but trying to keep that constant while varying the angle of release (*theta*) as a way to hit targets at varying distances. In these ways, *theta* serves as a central mathematical relation threaded through a range of modal engagements as *theta* is manifest in different settings, social exchanges, and material and symbolic forms.

The modal engagement analysis of this case at the level of the entire day (Table 2 and Figure 4) and of a particular interaction (Transcript 1 and Figure 5) illustrates some of the challenges and resources that go into building coherence across these complex project-based lessons and threading mathematics concepts through the various material and representational forms and project activities.

Case #2. Logic Enacted Through Algebra, Simulation and Silicone: A Digital Voting Booth

The main activity of this digital electronics lesson was to design a voting booth privacy monitoring system. An effective monitoring circuit is indicated by two outputs: a green light-emitting diode (LED) that is activated whenever a particular voting booth is available for use, and a red LED that lights up whenever privacy is at risk and entry is denied.

The circuit design involved implementing the basic set of logical constraints and conditions into a working electronic circuit that outputs a green light when all of the conditions are met, or a red light (alarm) when any condition is violated. The process unfolds sequentially across the following modal engagements: introducing the problem ("For privacy reasons, a voting booth can only be used if the booth on either side is unoccupied."), along with a "block diagram" representing the monitoring system, and an equipment list; discussing a completed truth table with entries composed of 1's and 0's accounting for all of the possible states of the circuit (voting booth occupancy and LED output) and a related, spatial Karnaugh map (K-map); generating and manipulating a set of Boolean algebraic expressions consistent with the K-map; drawing an Automated Optical Inspection (AOI) circuit; modeling the circuit in the MultiSims software to create computer generated MultiSim diagrams; and building and debugging a working electronic circuit made of a "bread board," integrated circuits, resistors and capacitors, wires, a power source and LEDs. Similar to the projectile motion lesson, the mathematics in this lesson (here logic rather than the algebra and trigonometry of kinematics) is manifest through a sequence of modal engagements with instructional contexts, representations and a range of material forms traversed by ecological shifts. The teacher often helped students establish cohesion between the algebraic relations and the different materials and representations using coordination and projection throughout the four-day lesson.

It should be noted that in order to practically use the integrated circuits as a source for specific logic operations (e.g., AND, NOT, OR), the teacher and students had to look at something referred to as the "data sheet" (or spec sheet), which was a set of documents affixed on a poster board. This material illustrated the formal specifications of different logic gates and their layouts in each integrated circuit, which varied by manufacturer. The data sheets established the connections between idealized logic symbols for Boolean operations such as AND, NOT and OR, and the actual locations of the inputs and outputs of specific circuit components.

Modal engagement analysis of the digital electronics case study illustrates several transition processes and the coordination between various modal engagements. The following transcript is an excerpt from the last observed day of the lesson in which the teacher initiates an ecological shift (Line 1) by calling all students to gather at the lab station of one student group that had made the most progress on their voting booth monitor. The class witnessed the conversation between the teacher and a member of the group about checking the circuit for accuracy and discussing how to improve it. The teacher starts out in Line 11 addressing a practical matter that is not evident in the symbolic or simulation-based representations—the need for an orderly and "clean" wiring job ("it's just the spaghetti mess" in Line 13). The teacher in Line 13 then starts to model how to establish coordination of the simulated circuit shown in the MultiSim diagram with the physical arrangement of wires, integrated circuits ("chips"), and electronic components using speech and gesture (see Figure 6).



Figure 6. Line 13, Gesture 1: Teacher uses speech and pointing gestures to coordinate the generated MultiSim diagram with the wiring of the digital circuit.

However, in Line 14, the student chooses to work from the truth table rather than the MultiSim diagram. Practically speaking, this allows the student to turn each input switch to the circuit on and off to model the occupancy state of the voting booth (ON = Occupied, OFF = Vacant). However, by mapping the entries in the truth table directly to the circuit, the student bypasses the conceptual connection of the circuit to the Boolean expressions that are central to the MultiSim representation and to the original problem context. The narrower set of associations selected by the student exemplifies the challenges in constructing and maintaining cohesion of the concepts across the many forms in which it is manifest in complex, multimodal projects.

The dialogue from Lines 18-24 and the corresponding gestures (Figure 7) show the teacher's troubleshooting method using the systematic coordination of the entries in the truth table with the state of the digital circuit on the breadboard. The teacher models how each entry in the truth table maps directly to a physical state of the circuit, running his finger from one row to the next. The coordination between the table and the circuit provides situationally relevant feedback (the state of the green and red LEDs), while also establishing the meaning of the symbolic table entries⁴⁵.



Figure 7. (a) Lines 19-24, Gesture 3: Student reaches to the breadboard to adjust the input switches. (b) Lines 19-24, Gestures 2 and 3: Teacher (hand to far right) and student (hand below) each point to an entry in the truth table as part of a troubleshooting activity.

Initially (Line 18), the teacher calls out the circuit inputs, while the student sets the switches appropriately ("zero, zero, zero" means none of the booths are occupied and all of the switches are in the OFF position). The student echoes the teacher in Line 19, reporting on the state of the input switches, and then describes the output (e.g., "alarm is off," indicates that the red alarm LED is off and entry is permitted). By Line 21 the student has taken up the reporting of the state of the inputs and output, though the teacher is still guiding the process with his finger moving to each successive row along the right side of the truth table. In Line 24 the student notes the circuit gives the incorrect output, thus indicating a bug in implementing the logic electronically. It is not until Line 25 that the teacher withdraws his finger and the student then rapidly completes the coordination of the table and the circuit, glancing repeatedly between the two material forms, noting several more successes and one additional implementation error.

<u>Case #2 Transcript 2</u> (*NB. Speech transcript is complete but only relevant gestures are shown.*)

- 1 T: Guys everybody stop come over here 'cause we're gonna stop here and then we're gonna do an exercise, up in the front. So but I need you everybody stop what you're doing leave it come over here. (Name) come on. Okay. It says ninety percent working but I want to make some comments about the board.
- 2 S: Yeah it's messy. I get it.
- 3 T: I don't have to make comments about the board you just did it.
- 4 S: Yeah.
- 5 T: Right? What's uh the term I'm always giving you is spaghetti.
- 6 S: Spaghetti.
- 7 T: To try to solve problems and you got stuff running all over it's much harder to do but I'm glad for the most part you've got it working. So just demonstrate to me that what you've got working but you need to put your wires-
- 8 S: I just need-
- 9 T: -so they-
- 10 S: -to put the switch.
- 11 T: -they're not at angles try to get them all square so you can follow a path, laying right next to each other. Nothing goes over switches, nothing goes over the integrated circuits, get 'em straight, and if you got a long wire and you've got to make a bubble out of it shorten the wire. And I'm always saying if you have like these here are going at angles those could have been shortened straightened out. Kay.
- 12 S: Oh yeah.
- 13 T: And on your paperwork when you're doing the check, you have numbers and letters here. What hole is that in? There's a l- number and a letter. Use 'em and to check things off. Write 'em right on here. I did this one, this one's hooked up, go to the next one, look what the number on here. You know 1A. [You know is it 10B.] What are the things

plugged into? Well that's your checklist. Otherwise it's hard just look at this as a whole picture, it's just the spaghetti mess. But uh now I can follow this. If I know that you want to do something I can look. Look at the number and say oop you're in Hole 2 when it should be Hole 3. You just put it in the wrong hole and that's your troubleshooting. That's a checklist by putting it on here. Alright go through and show me what does work.

- 14 S: Oh okay, we'll uh we'll just go with this thing.
- 15 T: Alright.
- 16 S: Okay uh.
- 17 T: So we...
- 18 S: Booth alarm all of this is (at the same time) off right now? T: [Kay so you're

[Kay so you're doing zero zero.]

- 19 S: [So zero zero booth is on alarm is off.]
- 20 T: [Okay.]
- 21 S: [Zero zero one booth is on alarm is off. Uh zero zero one (at the same time)...] 3

T:

[Zero one.]

- 22 S: [zero booth alarm same thing. This too so the green one should come on here and it 3
 - does and the red one doesn't matter.]
- 23 T: [Yeah.]
- 24 S: [Uhhh (pause) yeah green one's okay, so so far it works. Oh see that one's the one that oh 3

so the green one doesn't work but the red one works for that one over there. I'll keep that a mental note okay.]

2

25 S: [Uh so these two doesn't work up the third one en this one works. The thirrr... that one 3

works. Uhhh (pause) okay this one should be off but yeah. Uh this one works. That one works. That one works. This uh that one works. That one works. And that one works], so there's three doesn't work.

Gesture Notes

- 1. Teacher points with a finger of one hand to a hole on the breadboard while holding the print out from the MultiSim software with the other hand.
- 2. Teacher points to each line of the truth values corresponding to the state of the circuit.
- 3. Student points with a finger of one hand to each line of the truth values corresponding to the state of the circuit, while reconfiguring the input switches to provide the proper inputs.

As with the ballistics unit, we have tabulated the specific occurrences of model engagements over the entire day from which Transcript 2 took place. Table 3 shows that on Day 4 there were 4 separate ecological contexts or major divisions to the day: Students worked in teams implementing their circuit designs by wiring the breadboards; the teacher called the entire class over to observe the circuit of one group; one student from the team led a demonstration and, with the help of the teacher, debugged the circuit (this is portrayed in the transcript above); and students worked individually on a new design problem for a digital alarm clock. Over these 4 contexts we identified ten modal engagements. This type of project work fosters a great deal of coordination – students are continually integrating information from MultiSims, truth tables,

circuits and output form logic probes. This is evident by the number of lone coordination (C) codes in Table 3. In addition to few projections, we can see that 3 out of 4 types of projection are forward reaching. This highlights the greater emphasis on what is to come, and how to avoid problems. It also shows that integration of prior knowledge and reflection on previous taught concepts was not common in this lesson – a finding that echoes the impressions from observing the entire four-day unit. This stands in contrast to the unit on projectile motion (Table 2), which made many backwards connections. The flow of cohesion among modal engagements is illustrated in Figure 8, where the brief transition from small group teamwork to the whole class demonstration is excluded to simplify the diagram.

Ecological Setting:		Context	Modal	Code	Projection	Modality			
Context	Participation	Description	Engagement	ngagement		Description			
	Structure	I	8 8			•			
1	Digital Lab:	Teams wire	Spaghetti mess	C + P	Forward	Breadboard			
	Student teams	breadboards	Connect power	С		Spec sheet,			
			and ground to		•	breadboard,			
			breadboard			MultiSim			
			Build circuit; fix	С		MultiSim,			
			problems		•	breadboard,			
						worksheet			
			Test/debug circuit	С	•	Breadboard, logic			
						probe			
2	Digital Lab:	Teacher	Demonstrating of	ES		Whole circuit			
	Shift from small	calls teams	working circuit		•				
	group to whole	to stop and							
	class	observe one							
		group							
3	Digital Lab:	Student-led	Aesthetics of	С	•	Breadboard, gesture,			
	Whole class	demo/debug	circuit wiring			MultiSim, truth table			
4	Classroom:	Designing a	Plan future wiring	Р	Forward	Speech			
	Individual seat	circuit for an	of breadboard						
	work	alarm clock	Introduce Alarm	C + P	Backward	Worksheet			
			Clock problem						
			Designing circuit	С	•	White board, lab			
						notebooks, Boolean			
						logic, worksheets			
			Plan future work	Р	Forward	Speech			

Table 3. Modal engagements for the digital voting booth monitoring system (Day 4)

* Codes: C = Coordination; P = Projection; ES = Ecological shift.

In the language of our emerging theory, the *what* of mathematics is the propositional logic that instantiates the privacy conditions of the voting booth, which is reified in a truth table and a simplified Boolean expression. The mathematics is located across the modal engagements afforded by the table (the students' preferred representation), Boolean algebra equations, the MultiSim diagram (the teacher's preferred representation), and the circuit configuration itself, which gives a series of outputs in the form of lighted LEDs for a given set of inputs from the switches. Cohesion is established in several ways. The discussion of proper versus messy circuit wiring is used to illustrate how cross-modal coordination can be affected by the aesthetics of the physical implementation. This also highlights the practical consideration of constructing a wellorganized breadboard to provide paths between the physical circuit and the symbolic representation of the design that are more easily traced and debugged. Coordination is also established by showing that the trajectory of building and testing a correct circuit is not monotonic; rather, verification of the circuit involves going back to an earlier encountered representational form (in this case, truth table entries). We can see how epistemological commitments to the immediately present representational and material forms were modeled by the teacher. Explicit links between the circuit and the Boolean expression that models the context of the voting booth scenario are rarely identified in his speech (or throughout the multi-day project). In a parallel fashion, the student's focus is also specifically on the truth table and circuit, rather than exhibiting the ways that these forms are particular manifestations of the logical relations that model the voting booth. The teacher regularly used forward projection with coordination to make connections between the current modal engagements and those that would be enacted at future stages of the project, establishing cohesion by communicating to the students the scope of the project.

Discussion

We have highlighted three transition processes that teachers (and students) use to establish and maintain cohesion across the range of modal engagements encountered in project-based engineering curricula: Guiding attention and behavior around ecological shifts across changing contexts, coordinating ideas across different spaces, and projecting ideas both forward and backward in time. As we have demonstrated in these cases, teachers use these approaches to support STEM integration. Yet there are, we claim, inherent challenges, which we explore in this Discussion section.

Fostering STEM integration

We have argued that, in order to forge cohesion within and across STEM disciplines, the "what" and "where" of mathematics have to be threaded through modal engagements and across ecological contexts. Tracking locally invariant relations across modal engagements is both related to and distinct from the notion of *grounding*, which refers to connecting more abstract and unfamiliar concepts and ideas to more familiar and concrete ones^{46 47 48 49}. The notion of grounding is often invoked in project-based learning and reform approaches to education, on the argument that context, materials and activity structures -- modal engagements that commonly occur in STEM classrooms -- help to establish the meaning and appropriate uses of abstract ideas in concrete and familiar ways, and help to make schooling more relevant⁵⁰. Yet we observed in these cases that grounding contexts and activities cannot be assumed to enhance understanding and learning. This is because grounding contexts also introduce new demands of their own for establishing cohesion across the familiar and new modal engagements. In order for the potential of the grounding process to be realized, learners must understand the relation of the mathematics to the grounding objects and activities, and teachers and curriculum developers need to explicitly attend to these links to promote learning.



Boxes show the major ecological contexts that activity was embedded in throughout the digital electronics case. Bullets show the main modal engagements occurring sequentially in the case.

Italics = Projection, <u>Underline</u> = <u>Coordination</u>, <u>*Italics and Underline* = <u>*Projection + Coordination*</u> Arrows show the main backward/forward projections pointing to projected past/future modal engagement(s). *indicates the modal engagement discussed in the transcript of the digital electronics case.</u>

Figure 8. Modal engagements analysis of the digital voting booth project (Day 4).

Challenges in threading mathematics through different modal engagements

Although integration is often central to the curricular objectives of these lessons, it is apparent that it is frequently challenging to achieve in the classroom. These challenges are well illustrated in the cases considered here, and have been implicated by others in analyses of STEM curricula and classroom instruction, as reviewed above. Students often struggle to "see" a mathematical idea in each of its settings and material, symbolic, and discursive instantiations. Teachers are challenged to help students to make those links.

For example, the case of the ballistic device illustrated how the saliency of other angles in the design sketch lured students in one group away from the angle *theta* that was, from the standpoint of the curriculum designers and instructors, meant to be tracked and parameterized. As a second example, the digital electronics students failed to coordinate the Boolean algebra of the computer generated MultiSim diagram and the electronic circuit. The unlit circuit board was material proof that the particulars of the coordination were not accomplished and that the Boolean algebra, though correct, was not properly implemented to produce a working electronic device. The teacher and students attempted to debug the circuit by re-coordinating between the diagram (derived from the Boolean algebra) and the circuit board. Such debugging activities, particularly those carried out in the face of situationally relevant feedback (such as the output lights), are a powerful way to drive coordination and learning.

Each representational form offers certain ways of realizing the target mathematical ideas and enables certain methods of communication about those ideas. To perform a specific task or learn a concept through multiple representations, knowledge of each of the representations and the skills to use them are required⁵¹. When interacting with representations students must have the ability to understand, select, construct, and effectively use different representational forms in order to make sense of their learning experiences and cognitive tasks⁵². Unfortunately, students are often faced with difficulty when presented with a new representation.

One of the central issues to emerge from modal engagement analysis is a greater appreciation of the challenges of STEM integration from the learners' perspective. There is a tendency to see hands-on activities and authentic contexts as powerful ways to ground new ideas and abstract representations. The analyses underscore the novel demands of working in multi-modal learning environments. As these cases illustrate, threading mathematics through different representational forms, as utilized in different modal engagements, is challenging both for teachers to facilitate and for students to recognize.

Conclusion

One of the primary goals---and one of the central challenges—of engineering education is threading mathematical concepts and ideas through the various material, discursive and representational modalities that students encounter during project-based instruction. Modal engagements analysis is used to better document this phenomenon, articulate the difficulties students experience, and describe the ways in which teachers actively produce and maintain cohesion across modal engagements. Analyses of the videos collected during two multi-day, high school engineering units, one in mechanics and one in digital electronics, reveal three processes used to foster cohesion in the classroom: Ecological shits, projection and coordination. Class participants may make an overt ecological shift, which reorients the activity to different representations, tools, media of instruction, and participant structures. Ecological shifts do not simply change the context but often alter the spaces in which instruction takes place and the modal engagements. Projection uses language and gesture to connect activities of the present to past or future modal engagements, and thereby produce cohesion over time. Coordination links different material and representational forms that are co-present in the students' workspace and helps to establish mappings between seemingly disparate modalities, as when a truth table is mapped directly to each of the states of a digital circuit. By understanding students' struggles to see cohesion across the complex elements of engineering activities and the methods for supporting cohesion production, it is possible to reframe the challenges of STEM integration and develop new methods for improving instruction.

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Appendix A

An Example of Two Different Physical Systems that Share Common Mathematical Models

Consider the series RLC circuit shown in Figure A-1, with resistance R (measured in Ohms), inductance L (Henrys), capacitance C (Farads), and voltage across the battery V (Volts). We can use the voltage equations for each circuit element and Kirchoff's voltage law to write a second order linear constant coefficient differential equation (Eqn. 1) describing the charge on the capacitor over time (q(t)). An analogous model (Eqn. 2) can be used for the mechanical system in Figure A-2, which shows displacement as a function of time (x(t)) when a force F (Newtons) is applied to an ideal mass-spring-damper circuit with mass m (kg), spring constant k (N/m) and damping coefficient R (N-s/m).



Figure A-1. A series resistor-inductor-capacitor (RLC) circuit and a real life set up.



Figure A-2. A series mass-spring-damper circuit.

(Eqn. 2) mx(t)'' + Rx(t)' + kx = F

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