Embodiment in Mathematics Teaching and Learning: Evidence From Learners' and Teachers' Gestures

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Gestures are often taken as evidence that the body is involved in thinking and speaking about the ideas expressed in those gestures. In this article, we present evidence drawn from teachers' and learners' gestures to make the case that mathematical knowledge is embodied. We argue that mathematical cognition is embodied in 2 key senses: It is based in perception and action, and it is grounded in the physical environment. We present evidence for each of these claims drawn from the gestures that teachers and learners produce when they explain mathematical concepts and ideas. We argue that (a) *pointing* gestures reflect the grounding of cognition in the physical environment, (b) *representational* gestures manifest mental simulations of action and perception, and (c) some *metaphoric* gestures reflect body-based conceptual metaphors. Thus, gestures reveal that some aspects of mathematical thinking are embodied.

When teachers teach about mathematical concepts, they routinely produce gestures along with their speech (e.g., Alibali & Nathan, 2007; Flevares & Perry, 2001; Goldin-Meadow, Kim, & Singer, 1999; Richland, Zur, & Holyoak, 2007). When students talk about mathematical concepts, they also routinely produce

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gestures (e.g., Bieda & Nathan, 2009; Perry, Church, & Goldin-Meadow, 1988). In fact, when students talk about concepts they are learning, they often express new knowledge in gestures before they express it in speech (e.g., Alibali & Goldin-Meadow, 1993; Church & Goldin-Meadow, 1986; Perry et al., 1988). Thus, gestures are an integral part of communication about mathematical ideas. Yet, as Roth (2002) noted, "There exists very little educational research concerned with the role of gestures in learning and teaching, particularly in the subject areas that have been characterized as dealing with abstract matters such as science and mathematics" (p. 365).

Gestures are often taken as evidence that *the body is involved in thinking and speaking* about the ideas expressed in those gestures. That is, gestures are taken as evidence that *the knowledge itself is embodied* (Gibbs, 2006a; Hostetter & Alibali, 2008; McNeill, 2005; Núñez, 2005). But what does it mean for knowledge to be embodied? Although there is as yet no unified theory of embodiment (Barsalou, 2008), scholars of embodied cognition generally agree that mental processes are mediated by body-based systems, including body shape, movement, and scale; motor systems, including the neural systems engaged in action planning; and the systems involved in sensation and perception (Dreyfus, 1996; Glenberg, 2010). This perspective has implications for learning and instruction across the range of content areas; our focus here is on mathematics. In this article, we present evidence drawn from teachers' and learners' gestures to help make the case that mathematical knowledge is embodied. We argue specifically that mathematical cognition is embodied in two senses: It is based in perception and action, and it is grounded in the physical environment. We consider each of these claims in greater detail in this article.

Why might it be important to know whether mathematical knowledge is embodied? In our view, understanding the nature of mathematical knowledge is essential for understanding mathematics performance, instruction, assessment, and learning. An appreciation of the embodied nature of mathematical cognition will help one to understand why certain types of mathematics problems are more difficult than others, to identify suitable assessment methods that accurately gauge mathematical knowledge, to design more effective learning environments (see, e.g., Johnson-Glenberg, Birchfield, Tolentino, & Koziupa, 2011), to select appropriate methods for instructing mathematics content, and to understand why learners have greater success with some instructional methods than with others (see Núñez, Edwards, & Matos, 1999). Understanding the nature of mathematical knowledge is also crucial to understanding how learners generate and construct such knowledge and how it changes over time. These broad issues regarding learning, instruction, and assessment are central to contemporary research in the learning sciences.

It is also valuable to know how teachers and students express their knowledge of mathematics in gestures. There is extensive evidence that gestures play a role

in communication, both in facilitating speakers' language production (e.g., Kita, 2000; Kita & Davies, 2009; Krauss, 1998) and in promoting listeners' comprehension (e.g., Goldin-Meadow et al., 1999; Kelly, Özyürek, & Maris, 2010; Kendon, 1994). Moreover, mounting evidence suggests that gesture plays an integral, potentially causal role in knowledge development and change (e.g., Alibali & Kita, 2010; Goldin-Meadow, Cook, & Mitchell, 2008; Nathan & Johnson, 2010; Radford, 2009; Singer, Radinsky, & Goldman, 2008). Thus, a better scientific understanding of gesture is crucial to forging a deeper understanding of instructional communication and knowledge change.

In this article, we argue that gestures manifest embodied mathematical knowledge in three distinct ways, and we support this claim with literature and with empirical illustrations. Building on the typology of gestures presented by McNeill (1992), described in detail here, we argue that embodied knowledge is manifested in different ways by different types of gestures. We focus in particular on the gestures that teachers and learners produce in *explanations* in mathematics teaching and learning situations, including teachers' instructional explanations and learners' explanations of their thinking. We focus on explanations because they are a particularly rich source of gesture data. However, gestures are not produced solely in explanations—gestures are ubiquitous whenever speakers express ideas in spoken words (McNeill, 1992). It seems likely that the nature of gestures would be similar for mathematical activities other than explanations that involve expressing mathematical ideas in spoken words (e.g., asking questions, exploratory talk, peer collaboration, presentations).

The nature of the mathematical content we focus on is diverse and includes equation solving, word-problem solving, algebraic concepts (e.g., slope and intercept), and geometric concepts (e.g., similar polygons). The specific examples that we present are drawn from several different data sets, and they address mathematical thinking over a range of topics and developmental levels. For some of the data sets, previously published reports presented quantitative analyses of students' performance (e.g., Alibali, 1999; Alibali, Bassok, Solomon, Syc, & Goldin-Meadow, 1999) but did not discuss specific examples of speakers' behaviors in detail. None of the examples presented herein have been presented elsewhere, with the exception of one example that was presented elsewhere to make a different point.

In focusing on gestures, we do not mean to imply that other bodily actions (e.g., posture, gaze) are not relevant to mathematical cognition, and we also do not mean to minimize the importance of the integration of modalities (such as gesture, posture, and gaze) in interaction. We focus on gesture for several reasons. First, there is burgeoning interest in gesture and its role in classroom interactions and other educational settings (e.g., Crowder, 1996; McCafferty & Stam, 2008; Reynolds & Reeve, 2001; Roth, 2002; Williams, 2008). Second, scholars of embodied cognition have begun to view gestures as an indicator of embodied

mental representations (e.g., Nemirovsky & Ferrara, 2009; Núñez, 2005). Third, gesture is of particular interest because it allows for overt indexicality, and it is readily interpreted from video.

We start by briefly describing the embodied cognition perspective, and we next summarize theoretical arguments for why gestures reveal embodied knowledge. We then consider examples drawn from teachers and learners to argue that gestures reveal that some aspects of mathematical thinking are embodied.

THE EMBODIED COGNITION PERSPECTIVE

The embodied cognition perspective encompasses a diverse set of theories that are based on the idea that human cognitive and linguistic processes are rooted in perceptual and physical interactions of the human body with the world (Barsalou, 2008; Wilson, 2002). According to this perspective, cognitive and linguistic structures and processes—including basic ways of thinking, representations of knowledge, and methods of organizing and expressing information—are influenced and constrained by the particularities of human perceptual systems and human bodies. Put simply, cognition is shaped by the possibilities and limitations of the human body.

Researchers who work within the embodied cognition perspective make a variety of specific claims (see Barsalou, 2008, and Wilson, 2002, for reviews). Chief among them is the claim that cognition is based in perception and action. This holds true even for *offline* cognition—the cognitive activities that occur in the absence of relevant environmental input. Many cognitive tasks are accomplished by bringing sensory and motor resources to bear, even when the task referents themselves are far removed in space and time (Wilson, 2002). Examples include the use of mental imagery (e.g., Shepard & Metzler, 1971), the simulation of actions during language comprehension (e.g., Glenberg & Kaschak, 2002), and the construction of mental models during reasoning (e.g., Johnson-Laird, 1983) and reading comprehension (e.g., Glenberg, 1999; van Dijk & Kintsch, 1983). Consider, for example, how spatially scanning a mental image of guests seated around the tables of an elaborate dinner party can facilitate planning, even though the people involved are spread across the globe and the event itself is still months away.

A related set of claims is that cognition occurs in real-world environments, is used for practical ends, and exploits the possibility of interacting with and manipulating external props (Anderson, 2003; Nathan, 2008). These connections to the physical environment can serve to *ground* novel or abstract ideas or information in the physical world. *Grounding* describes a mapping between an abstraction and a more concrete, familiar referent, such as an object or event, that facilitates meaning making (Koedinger, Alibali, & Nathan, 2008; Nathan, 2008).

Grounding of this sort may support the transfer of knowledge to new situations. For example, Goldstone, Landy, and Son (2008, 2010; Landy & Goldstone, 2007) have argued that learners' perceptions become "tuned" to encode deep structure in objects and events, and these perceptions are then carried forward in subsequent encounters with other instances of the same sort. Thus, rather than casting abstractions as amodal formalisms that strip away perceptual qualities, they argue that perceptual information is central to forming abstractions. From this perspective, grounding in the physical, perceptual world fosters knowledge that is robust and "transportable."

Although the literature is replete with examples demonstrating the roles of action, perception, and grounding in cognitive tasks, our focus in this article is on *mathematical* cognition, for which thinking and communicating are often about abstract and imaginary entities. We argue that even mathematical cognition is embodied, and it is embodied in two distinct senses—it is based in perception and action, and it is grounded in the physical environment. Evidence for each of these claims can be drawn from the gestures that teachers and learners produce when they work with or communicate about mathematical concepts and ideas. Before proceeding to specific examples to illustrate our claims, we first address the issue of how gestures might reflect embodied knowledge.

HOW GESTURES REVEAL EMBODIED KNOWLEDGE

How do gestures reveal embodied knowledge? In this section, we present general arguments about ways in which various types of gestures reveal embodied knowledge. We draw on the typology of gestures presented in the seminal work of McNeill (1992). McNeill's (1992) typology is widely used in gesture studies, and it is flexible, in the sense that it can be applied to gestures in any type of discourse or any content area.

McNeill's (1992) typology delineates four major categories of gestures: (a) *pointing* (deictic) gestures, which are gestures that serve to indicate objects or locations, often with an extended index finger but sometimes with other fingers or the entire hand (e.g., pointing to a cube in order to refer to that cube); (b) *iconic* gestures, which are gestures that depict semantic content directly via the shape or motion trajectory of the hand(s) (e.g., tracing a triangle in the air to mean *triangle*); (c) *metaphoric* gestures, which depict semantic content via metaphor (e.g., cupping hands as if to "hold" an idea, which reflects the metaphor IDEAS ARE OBJECTS, discussed in detail later); and (d) *beat* gestures, which are motorically simple, rhythmic gestures that do not express semantic content but that instead align with the prosody of speech. It should be noted that many investigators who utilize McNeill's (1992) typology view the categories of iconic and metaphoric gestures as together making up a broader category of *representational* gestures,

defined as gestures that depict aspects of their meaning, either literally (in the case of *iconics*) or metaphorically (in the case of *metaphorics*; e.g., Alibali, Heath, & Myers, 2001; Kita, 2000).

In recent work, McNeill (2005) has argued that these gesture "categories" may actually be better thought of as dimensions, because individual gestures often incorporate elements of multiple categories. For example, a depictive gesture can be performed *over* an object or location; such a gesture is both iconic and deictic at the same time.

In this article, we make three claims about how different types of gestures manifest embodied cognition: (a) *Pointing* gestures reflect the grounding of cognition in the physical environment, (b) *representational* (i.e., iconic and metaphoric) *gestures* manifest mental simulations of action and perception, and (c) some *metaphoric* gestures reflect body-based conceptual metaphors. (Note that we do not address beat gestures in this article.) In this section, we consider each of these claims in turn. In the following sections, we provide illustrative examples drawn from studies of teachers and learners communicating about mathematics.

Pointing Gestures Reflect the Grounding of Cognition

According to the *Indexical Hypothesis* (Glenberg & Robertson, 1999, 2000), people comprehend language in part by *indexing* words and phrases to actual objects or to "perceptual symbols," which are perceptual memories that involve reactivating aspects of the perceptual states that occur when one is interacting with objects (Barsalou, 1999). For example, when a listener comprehends the statement "The baby is napping," that listener may index the noun phrase "the baby" to an actual baby who is physically present or to a mental representation of a baby that includes perceptual information about the baby, such as information about how the baby looks, sounds, or smells.

Pointing gestures are often used along with speech, and they manifest speakers' indexing of speech content to objects, locations, or inscriptions in the physical environment. By "physical environment" we mean the setting for the interaction (e.g., a classroom, tutoring session, or experimental session) including the interlocutors (e.g., students, teacher, experimenter); the focal tasks; and the representations, notational systems, tools, and technological resources that are used. The environment also has social dimensions that may be relevant to the interaction, such as norms for talking and interacting in the community as a whole (e.g., Are questions encouraged? Do students typically come to the front of the class to present their ideas?).

Speakers use pointing gestures both to index physically present objects or inscriptions and to evoke nonpresent objects or inscriptions, and such gestures utilize the physical environment in a variety of ways (Butcher, Mylander, & Goldin-Meadow, 1991; Morford & Goldin-Meadow, 1997). Speakers sometimes

point to perceptually similar objects to index nonpresent objects. For example, Butcher and colleagues (1991) described a deaf child who pointed to a toy soldier (one holding a xylophone) in order to request a different toy soldier (one holding a bass drum). Speakers may also point to locations to index nonpresent objects or people that are associated with those locations; for example, a child may point to the cupboard where the cookies are usually stored (even if there are no cook-ies inside) to request a desired cookie, or a teacher may point to the location on the board where the homework assignment is customarily written in order to refer to yesterday's assignment. Speakers also sometimes set up locations within their gesture space to serve as "placeholders" for objects or people and then point to these locations to index those objects or people. In one extended example of this type of indexing, a speaker talking about a movie plot set up different spaces for the "bad guys" and the "good guys" and pointed to those spaces to index those characters over the course of his narrative (McNeill, 1992, p. 155).

Pointing gestures physically link speech and associated mental processes to the physical environment. Without the environmental ground that gives it meaning, pointing would often be uninterpretable. As such, pointing gestures are "environ-mentally coupled gestures" in the sense described by Goodwin (2007). Pointing gestures "anchor" the information expressed in the verbal channel in the material world (Williams, 2008), and in so doing they manifest the grounding of speech in the physical environment. Thus, they provide support for claims that cognition is situated in the real-world environment and that the environment is part of the cognitive system (Wilson, 2002).

Representational Gestures Manifest Simulations of Action and Perception

A number of contemporary theorists have argued that *simulated action* is the basis of many cognitive processes, including language and mental imagery (e.g., Barsalou, 1999; Gibbs, 2006b; Glenberg, 1997; Havas, Glenberg, & Rinck, 2007; Solomon & Barsalou, 2004; Wu & Barsalou, 2009). A *simulation* can be defined as the neural experience of performing or witnessing a particular action, such that sensory, premotor, and motor areas of the brain are activated in action-appropriate ways. Both behavioral and neuropsychological evidence support the claim that simulations are involved in language comprehension and in the formation and manipulation of mental images.

Simulating actions and perceptions involves activating neural areas that are involved in planning actions (Jeannerod, 2001) and in perceiving and using objects (Gerlach, Law, & Paulson, 2002; Grafton, Fadiga, Arbib, & Rizzolatti, 1997; Kosslyn, 2005). According to the *Gesture as Simulated Action* framework (Hostetter & Alibali, 2008), in some cases this premotor activation is realized in motor output, specifically in gestures. The particular type of gestures thought to

arise in this way is *representational gestures* (i.e., gestures that depict semantic content, either literally or metaphorically, via handshape or motion trajectory).

This view is compatible with Streeck's (2002) argument that certain representational gestures are "abstracted from instrumental actions" (p. 19). Streeck analyzed the gestures produced by an auto mechanic in talking about his work and showed that many of them reflected actions that the mechanic commonly performed (turning a key, pushing a car, listening for a cranking sound). One interpretation is that the mechanic simulated these actions when speaking about them, even when the relevant objects were not immediately present, and as a consequence he produced gestures that reflected those actions.

Hostetter and Alibali (2008, 2010) argued that whether a simulation is actually produced as a gesture depends in part on the strength of the action component of the simulation. Specifically, speakers produce gestures when the action component of a given simulation exceeds a threshold that is determined by a number of cognitive and social factors. For the present purposes, the details regarding which factors affect the threshold are not important—the crucial claim is that representational gestures manifest the motoric and perceptual simulations that underlie language and imagery.

Note that the claim is that representational gestures can derive not only from simulated *actions* but also from simulated *perceptions*. This claim is based on the assumption that there are bidirectional, reciprocal relations between perception and action (e.g., Dewey, 1896; Gibson, 1979). From this perspective, action and perception are intimately linked: The purpose of perception is to guide action (see, e.g., Craighero, Fadiga, Umiltà, & Rizzolatti, 1996), and actions (e.g., movements of the eyes, heads, and hands) are necessary in order to perceive (e.g., Campos et al., 2000; O'Regan & Noë, 2001). When humans perceive objects, they automatically activate actions appropriate for manipulating or interacting with those objects (Ellis & Tucker, 2000; Tucker & Ellis, 1998). Thus, imagining an object can evoke simulations of perception (i.e., of the actions associated with perceiving the object) or of potential actions involved in interacting with the object. From this perspective, it does not really matter whether a gesture is simulating action or perception—they are two sides of the same coin.

In sum, according to Hostetter and Alibali (2008), representational gestures derive from the same simulations of action and perception that also underlie language and mental imagery. Put simply, representational gestures occur *because* thinking is based in perception and action. Thus, representational gestures provide support for the claim that cognition is based in the body.

Some Metaphoric Gestures Reflect Body-Based Conceptual Metaphors

Lakoff and Johnson (1980) argued that a set of broad-based metaphors underlies the conceptual system. These metaphors structure understanding and perceptions

of the world, and they are manifested in a range of expressions in everyday language. Some examples are LIFE IS A JOURNEY (e.g., he *needs some direction*), GOOD IS UP (e.g., things are *looking up*), and so forth. These conceptual metaphors are based on fundamental aspects of human experience, such as common actions, spatial relations, and bodily experiences. However, it is worth noting that they are not cross-culturally universal (e.g., Núñez & Sweetser, 2006), suggesting that although there are ties to bodily experiences, their realization is also influenced by social and cultural factors.

McNeill (1992) presented evidence that conceptual metaphors are frequently manifested in representational gestures that depict abstract concepts in terms of images (see Cienki & Müller, 2008, for recent work on this topic). McNeill (1992) termed such gestures *metaphoric* gestures. One example metaphor that McNeill (1992) discussed extensively is the *conduit* metaphor (see Lakoff & Johnson, 1980; Reddy, 1979), which holds that IDEAS, CONCEPTS, MEANINGS (and so forth) ARE OBJECTS; WORDS, SENTENCES, AND OTHER LINGUISTIC EXPRESSIONS ARE CONTAINERS; and COMMUNICATION IS SENDING AND RECEIVING. This conduit metaphor is frequently manifested in gestures that depict holding or transferring objects. For example, a speaker might extend her hand as if holding something while saying, "I have a great idea" or "I just thought of something."

Many of the conceptual metaphors discussed by Lakoff and Johnson (1980) are based on image schemas about space, movement, forces, and other aspects of human experience that are inherently spatial and therefore readily expressed in gestures. For example, HEALTH IS UP, SICKNESS IS DOWN (she's in *top* shape; his health is *declining*); HAPPINESS IS UP, SADNESS IS DOWN (that *boosted* my spirits; I was feeling *low*). Many of the conceptual metaphors that underlie mathematics are also inherently spatial; for example, NUMBERS ARE LOCATIONS IN SPACE (e.g., *approaching* zero) and ARITHMETIC IS COLLECTING OBJECTS (e.g., *put* two and two *together*; Lakoff & Núñez, 2001). Metaphors that involve space and action are readily expressed in metaphoric gestures that reflect the spatial structure of the underlying images.

Summary

In brief, we argue that speakers' gestures provide evidence for several of the key claims of the embodied cognition perspective. *Pointing gestures* manifest the grounding of cognition in the physical environment. Such gestures therefore support the claims that cognition is situated in the environment and that the environment is an integral part of the cognitive system. *Representational gestures* manifest the motoric and perceptual simulations that underlie language and imagery, and some *metaphoric gestures* (a subclass of representational gestures) reflect conceptual metaphors that are grounded in the body. These types of gestures provide support for the claim that cognition is based in the body. They also

demonstrate that bodily resources enable offline cognition about objects, events, and relations that are not immediately present.

All of these types of gestures are prevalent in talk about mathematical concepts and procedures, thus providing support for the claims that mathematical ideas are grounded in the environment and that they are based on bodily experiences. In the following sections, we present evidence on these points from mathematics teachers' and learners' pointing gestures, representational gestures, and metaphoric gestures.

In the combined speech/gesture transcripts presented in this article, square brackets indicate the words that accompany each gesture. Gestures are numbered below the bracketed corresponding speech and described in detail below the speech transcript.

POINTING GESTURES MANIFEST GROUNDING
IN THE PHYSICAL ENVIRONMENT

We have argued that pointing gestures reveal speakers' indexing of speech to the environment. In this section, we provide examples of mathematics teachers and learners utilizing pointing gestures to index talk about mathematical ideas to the physical world.

Teachers regularly use pointing gestures in mathematics lessons. Indeed, in elementary mathematics lessons, pointing gestures are the most commonly used type of gesture (Alibali, Nathan, & Fujimori, 2011). The referents of mathematics teachers' pointing gestures include common classroom objects, math manipulatives that were designed for instructional purposes (such as balance scales or algebra tiles), and inscriptions that symbolize mathematical concepts or relationships (such as equations, graphs, and diagrams).

When teachers point to objects or inscriptions as they speak, those pointing gestures link their verbal stream to its referents. The same holds true for students' pointing gestures. Figure 1 depicts a series of pointing gestures produced by an elementary school student as he explained how he obtained his (incorrect) solution (17) to the problem $6 + 3 + 4 = \_\_ + 4$.[1] In this excerpt, the student solved the problem, and the experimenter then asked him to explain how he arrived at his solution. He said, "6 plus 3 is 9, plus 4 is 13, plus . . . 13 plus 4 is 17," and he pointed to each of the numbers in the problem in turn. It is worth noting that he used his left hand to point to the 6, 3, and 4 on the left-hand side of the equation and his right hand to point to the 4 on the right-hand side and to his solution (17), suggesting that he was aware of the fact that the equation had two sides, even

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[1]This example is drawn from the data set reported in Alibali (1999), an experimental study of how children's explanations of equations change in response to different types of instruction.
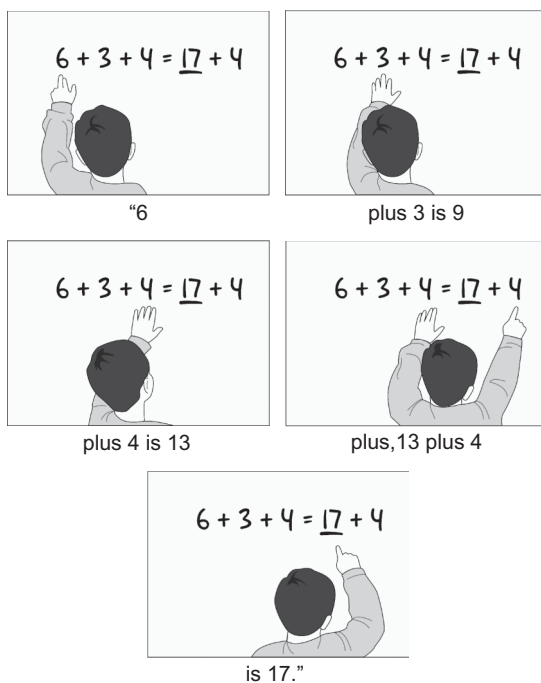
FIGURE 1   Pointing in a student's explanation of a mathematical equation. Accompanying speech is indicated under each frame.

though he did not utilize this fact in his solution procedure. Such gestures can reveal the "leading edge" of a learner's knowledge (Alibali & Goldin-Meadow, 1993). These pointing gestures link the student's verbal utterance to its referents on the blackboard.

In this example, as seen in Figure 1, the student was standing close to the referents of his speech, so his gestures could index their referents quite unambiguously. In other cases, students speak from their seats and refer in both speech and gesture to referents at a distance, such as inscriptions on the blackboard. In such cases, teachers sometimes index students' speech with their own pointing gestures *as* students speak (a phenomenon we call *addressee gesture*; Nathan, 2008). At other times, teachers revoice the content of a student's speech immediately following the student's turn (O'Connor & Michaels, 1993) and use pointing gestures along with that revoicing. Both types of teacher pointing appear to index speech not only for the benefit of the student who is speaking (and who may be using ambiguous gestures in an effort to index speech to far-away referents) but also for the benefit of other students in the class.

Pointing gestures that index speech to objects and inscriptions in the environment are also adept at conveying relationships between mathematical ideas (see, e.g., Alibali & Nathan, 2007; Richland et al., 2007; Williams, 2008). Teachers frequently use sets of pointing gestures to highlight corresponding aspects of related representations (Alibali et al., 2011). Most often, teachers utilize sequential points to corresponding elements of related representations, first indicating an element of one representation and then indicating the corresponding element in another representation (e.g., the *y* in an equation and the y-axis on a graph, or the base of triangle and the *b* in the formula $A = \frac{1}{2} bh$). Less often, teachers utilize simultaneous points to express the link, using both hands to point to corresponding elements in two representations at the same time.

Figure 2 illustrates both a teacher's use of gestures to index a student's utterance as she revoices that utterance and the teacher's use of a series of pointing gestures to highlight relationships among representations.[2] This particular excerpt is part of an episode in which the teacher highlights the relationships between two similar rectangles. As seen in Figure 2, the teacher is projecting several figures of rectangles via an overhead projector. She asks a student to justify why two of the

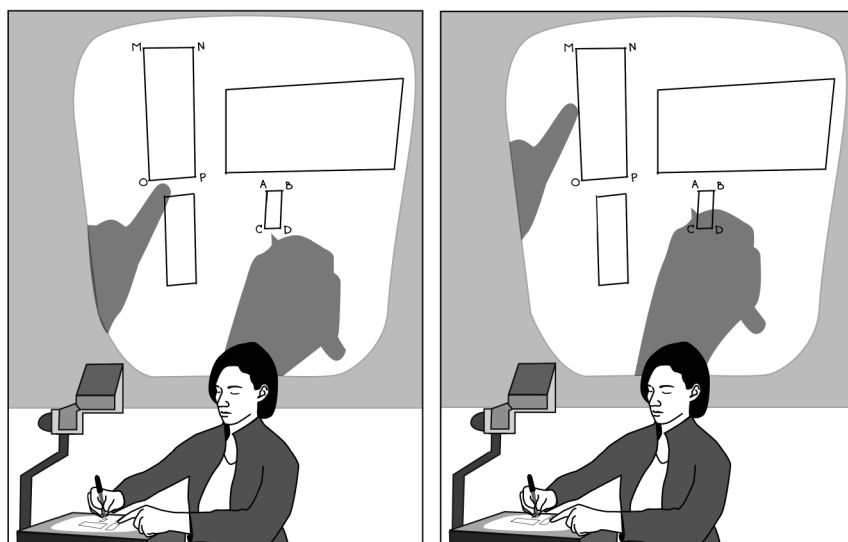FIGURE 2   Pointing gestures that highlight corresponding aspects of related representations: Corresponding sides in similar rectangles.

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[2]This example is drawn from an unpublished corpus of 18 middle school mathematics lessons collected in order to document teachers' use of gesture in naturalistic instructional settings (Alibali, Nathan, Wolfgram, Church, Knuth, Johnson, Jacobs, & Kim, 2010).

figures are similar. Recall that square brackets indicate the words that accompany each gesture. Gestures are numbered below the bracketed corresponding speech, and gesture descriptions are presented below the speech transcript.

Teacher: Why do you think they're similar?

Student (*off camera*): Because if you add, I mean if you times ABCD times three, then it equals the twelve six-, by six.

T: Oh, [so you're looking at the fact that this side [is three times longer.]]
 1   2

S (*at the same time*): Is three times.

T: [And this side [is three times longer?]] Okay.
   3   4

1. Right hand points with pen to the short side of the small rectangle; holds during the next gesture.
2. Left hand points with index finger to the short side of the large rectangle.
3. Right hand points with pen to the long side of the small rectangle; holds during the next gesture.
4. Left hand points with index finger to the long side of the large rectangle.

Note that in this case, the teacher's revoicing of the student's speech highlights the similar relationship between the rectangles and breaks it down into two components: the similarity of the short sides and the similarity of the long sides. Thus, her revoicing fleshes out the "times three" relationship that the student mentions and also uses more mathematically precise language ("this side is three times longer" vs. "times ABCD times three"). Note also that for each pair of corresponding sides, the teacher uses gestures that are sequential in their onsets, but in each case she holds the first point to the side on the smaller rectangle while she points to the corresponding side on the larger rectangle. Thus, she indexes both of the related items simultaneously.

There is evidence that instructional points like the ones in this example affect learners' uptake of lesson content. For example, in one study of kindergarten students learning about symmetry, children succeeded on more than twice as many posttest items after a lesson that included pointing gestures than after a comparable lesson that did not include pointing gestures (Valenzeno, Alibali, & Klatzky, 2003). Along similar lines, previous studies in laboratory settings have demonstrated that pointing gestures affect listeners' comprehension of the speech that accompanies them, particularly when that speech is ambiguous (Thompson & Massaro, 1986) or complex relative to the listeners' skills (McNeil, Alibali, & Evans, 2000).

Pointing gestures may serve as an aid to comprehension and learning because they reduce "cognitive load" for learners. Cognitive load theory (Sweller, van Merrienboer, & Paas, 1998) presents a general framework intended to explain and predict how instructional methods that make varying demands on learners'

working memories influence comprehension and learning. One of the central findings from this program of research is the *split-attention effect*—the phenomenon that learners' comprehension is better when they do not have to integrate disparate information in a serial manner, because this divides their limited attention resources. Learning is greater when lessons aid learners in integrating information, such as when related verbal and numeric information is physically integrated in a diagram (Mayer & Moreno, 2002). We suggest that when teachers use pointing gestures to perform this integration, learners may also require fewer resources to shift attention between two related sources of information. This should reduce working memory demands, freeing up resources for encoding and processing information as well as accessing prior knowledge. Simultaneous pointing (such as that shown in Figure 2) may be particularly effective for reducing split attention because information is integrated both spatially and temporally; we are testing this hypothesis in ongoing work.

The examples we have presented thus far illustrate learners' and teachers' use of pointing gestures to index the referents of their speech in talk about mathematical ideas. Thus, these examples illustrate speakers' grounding of mathematical ideas in the physical environment, which includes mathematical inscriptions that are perceptually available to both speakers and listeners.

REPRESENTATIONAL GESTURES MANIFEST MENTAL SIMULATIONS OF ACTION AND PERCEPTION

We have argued that representational gestures reveal speakers' mental simulations of perceptions and actions. In this section, we provide examples of mathematics teachers and learners expressing simulated actions and perceptions in representational gestures as they speak about mathematical concepts and procedures.

Although less frequent than pointing gestures, representational gestures are nevertheless quite common in mathematics instruction (Alibali et al., 2011). Such gestures sometimes reveal simulations of *actions on mathematical objects*. A representative example, shown in Figure 3, is drawn from a tutoring session in which a teacher (seated on the right) sought to illustrate for a student (seated on the left) how a line on a graph looked different when the slope was altered from 2 to 4. The example is drawn from an unpublished corpus of tutorial interactions between teachers and middle school students that were designed in order to investigate gesture in teacher–student interactions on a common set of tasks. The tutorials focused on slope and intercept and how they are represented in equations and graphs (Alibali, Nathan, Wolfram, Church, Knuth, Johnson, Jacobs, & Hostetter, 2010). The graph the teacher and student are discussing is shown in Figure 4.

Teacher: Right. Right. [So this one–] [this one at *2x*]
                                      1   2
T: [is kind of like starting like right here, right?
        3
T: But that]—[then so the *4x* is kinda going up].
                         4
1. Point to line on graph that depicts $y = 2x$
2. Represent line in neutral space with forearm (elbow on table) at an angle with a
   slope of roughly 3 (between 2 and 4)
3. Lower forearm to represent slope of 2
4. Raise forearm to represent slope of 4

Figure 3 depicts Gestures 3 and 4. In this example, the teacher's gestures simulate the action of altering the slope of the line. It is worth noting that, in this example, the teacher also establishes a link between the slopes of the lines on the graph and the symbolic expressions $2x$ and $4x$. Thus, teachers utilize not only pointing gestures but also representational gestures to link related ideas in mathematics instruction.

Representational gesture can also reveal simulated perceptions—most often visual perceptions. In talk about mathematics, such gestures often reveal perceptual characteristics of inscriptions, which are visual images. In the following example, shown in Figure 5, the teacher (the same one shown in Figure 2, in a different lesson from the same unit on similar polygons) seeks to highlight the correspondence between the "bottom sides" of two similar triangles. To do so, she simulates a visual image of the bottom side of a triangle as well as of the two triangles as they were positioned on a page in students' textbooks.

FIGURE 3   Representational gestures that simulate action on a mathematical object: Altering the slope of a line.
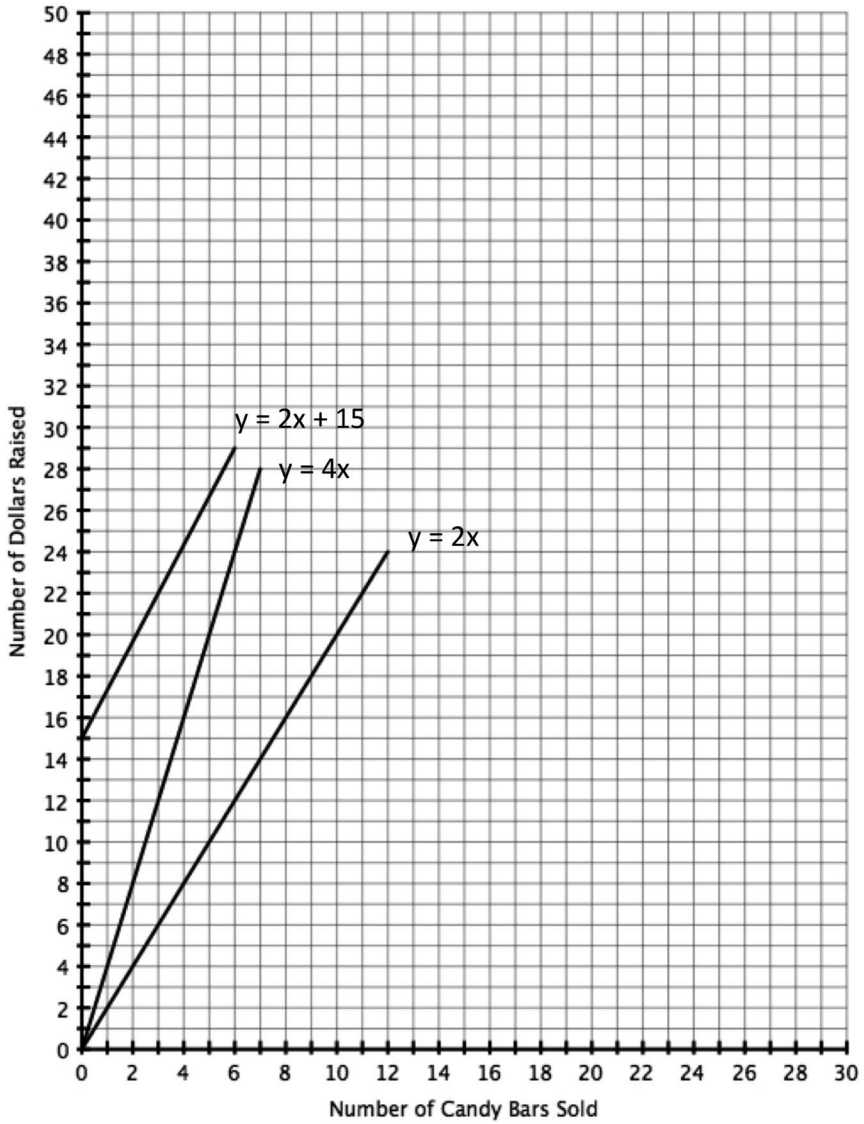
FIGURE 4   Graph that is the focus of interaction in Figures 3, 9, and 10.
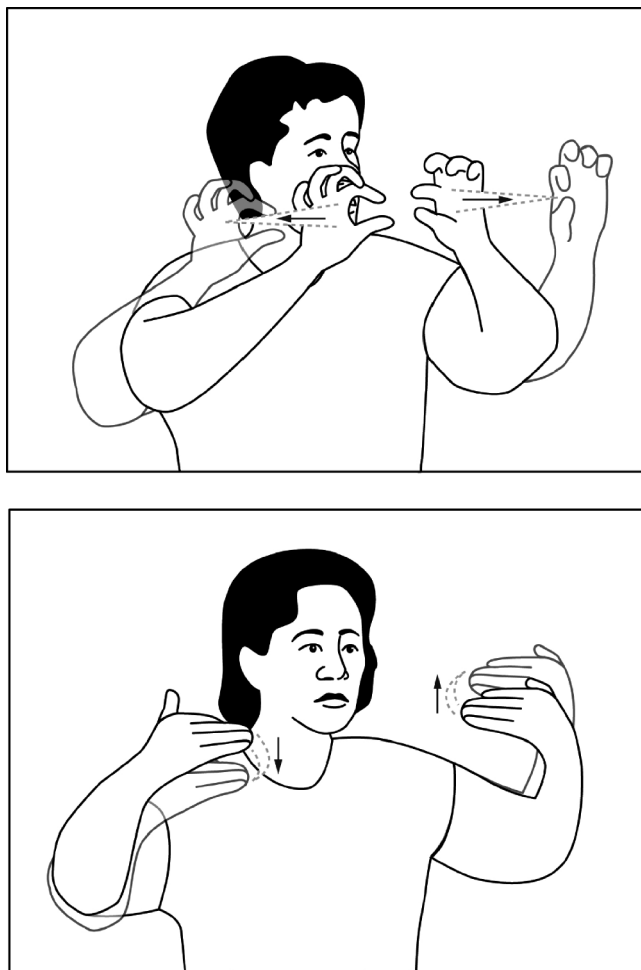
FIGURE 5   Representational gestures that depict inscriptions: Bottom side of a triangle (top panel) and two triangles (bottom panel).

Teacher: Well, they're saying that [that bottom side]
                                    1
T: on [both of those triangles]
       2
T: [correspond to each other]
       3
1. Both hands start from the center and move apart, drawing the bottom side of a triangle
2. Slightly cupped hands are held up to depict two triangles
3. Hands move alternately up and down

In this case, the teacher's gestures simulate an inscription drawn from the students' textbooks. The teacher uses gesture to highlight a particular aspect of that inscription, namely the (corresponding) bottom sides of the two triangles, which is relevant to her larger point about relations between mathematically similar objects. Representational gestures that simulate inscriptions have also been described by Edwards (2009) in work on teachers' talk about fractions.

In other cases, teachers' representational gestures simulate real-world objects that ground or give meaning to mathematical ideas. In the following example, presented in Figure 6, the same teacher grounds the concept of a right angle with a familiar object, the corner of a piece of paper:

Teacher: . . . [doing a right angle is really easy cause it's like the [corner] of your
             1 (held)   2

   [paper]].
     3
  1. Palms of hand pressed together to represent right-angle corner of paper; held
     for the remainder of the utterance
  2. Moves hands slightly up and down
  3. Moves hands slightly up and down

In this example, it is interesting to note that the very same gesture is used both for "right angle" and for "corner of paper"—the gesture is produced initially with
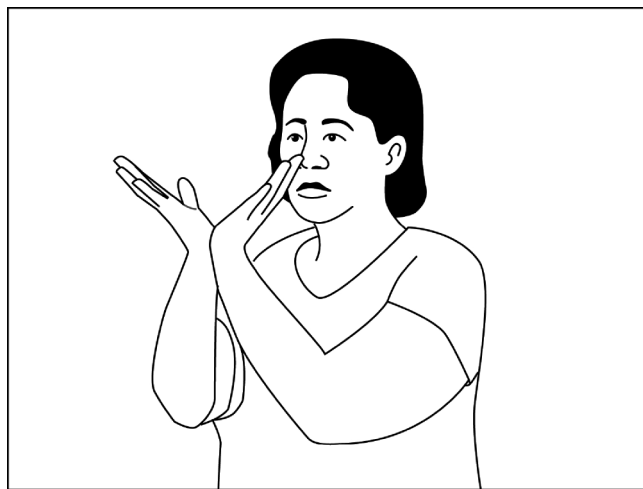

FIGURE 6   Representational gesture depicting a real-world object that grounds a mathematical concept: Corner of a piece of paper/right angle.

the phrase "right angle" and it is then reinvoked (with superimposed beat ges-
tures, which are up-and-down movements superimposed upon representational
gestures, typically for emphasis; see McNeill, 1992) on the expression "corner
of your paper." Thus, the link between the mathematical object and the real-
world object is not simply *depicted* in the gesture but is actually *enacted* in the
gesture.

In some cases, speakers also utilize representational gestures to simulate the
real-world objects and situations that mathematical problems are about. In the
following example, a college student was asked to describe the gist of a word
problem to another participant and then to describe how he would go about solving
the problem.[3] The problem in question was the following:

> For a lecture, 10 rows of chairs have been arranged in a lecture hall. The chairs have
> been set up such that the number of chairs in each row increases by a constant from
> the number of chairs in the previous row. If there are 25 chairs in the first row and
> 115 chairs in the 10th row, how many chairs total are there in the lecture hall?

In describing the problem, the student shown in Figure 7 began by saying the
following:

FIGURE 7   Representational gesture depicting a real-world situation addressed in a math
problem: Rows of chairs in a lecture hall.

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[3]This example is drawn from the data set reported by Alibali et al. (1999), which addressed
relations between adults' problem representations and their strategy use.

Student: There are, [er, [a constant rate of increase of chairs in each row]
                          1   2
    [subsequent to the first row]]
          3

1. Left hand, palm sideways and facing self, on table about 18" from the body, held in this position for the remainder of the utterance (*represents first row*)
2. Right hand, palm sideways and facing self, starts next to the left palm and makes a series of four hops toward the self (*represents subsequent rows*)
3. Repeat gesture 2

In this gesture, this student simulated the perceptual experience of viewing rows of chairs in the lecture hall. Thus, this gesture suggests that the student constructed a "situation model" of the problem, or a mental model of what the problem is about. This situation model presumably draws on the student's past experience with chairs and lecture halls, which enables him to simulate the perceptual experience of viewing the lecture hall and to express that simulation in gestures. Note that in this example the simulation does not capture a specifically *mathematical* idea; instead, the simulation captures a real-world situation that is the focus of a mathematical problem.

One might ask how this framework might account for simulations that speakers have never experienced. Simulations are mental activities, and as such they can be manipulated and controlled by the simulator, just as a mental image can be manipulated and controlled by the imager. Consider a speaker who has never skydived before. The speaker can use motoric knowledge about what it feels like to jump from a diving board into a swimming pool and visuospatial knowledge about how things look from the window of an airplane to construct a simulation of the experience of skydiving. The simulation might be "faulty" (i.e., jumping from a plane could differ from jumping from a diving board in very important ways), but this faulty simulation would presumably support gestures anyway.

Can simulations be used to conceptualize things that are completely imaginary or impossible for humans to experience? Indeed, by drawing on real-world experience, and by using analogy and metaphor, people regularly simulate such things. For example, McNeill (1992) described the gestures that two mathematicians used in conversation about various abstract mathematical ideas. The mathematicians expressed the idea of a mathematical limit in gestures that incorporated "straight-line trajectories followed by 'end-marking' (a tensed stop)" (McNeill, 1992, p. 166)—suggesting that they conceptualized mathematical limits by analogy to physical limits, which they simulated via movements that were physically limited.

Why do speakers express simulated actions and perceptions in gestures? As described earlier, according to Hostetter and Alibali (2008), the motor activation involved in simulating actions or perceptual experiences, in conjunction with the

motor activation involved in speaking, sometimes exceeds a speaker's threshold for producing overt action, so the speaker produces a gesture. This framework implies that speakers produce gestures as a *direct consequence* of the motor activation involved in planning and producing speech (of course, influenced by aspects of the setting, addressee, and topic, as well). It is also possible that speakers may intentionally lower their activation threshold for producing overt action when they wish to have the benefits of gesture for their cognition—that is, they may choose to move when they think doing so may be useful. Many experiments have suggested that representational gestures facilitate thinking or speaking, for example by focusing speakers' attention on perceptual information (Alibali & Kita, 2010), by helping speakers to package ideas into units suitable for speaking (Kita, 2000; Kita & Davies, 2009), by activating mental images (de Ruiter, 1998; Wesp, Hess, Keutmann, & Wheaton, 2001), or by priming lexical items (Krauss, Chen, & Gottesman, 2000). Speakers may realize—perhaps implicitly— that gesture facilitates thinking and speaking in these ways, and they may alter their gesture thresholds to take advantage of these benefits.

There is also extensive evidence that representational gestures contribute to listeners' comprehension of the accompanying speech (e.g., Cook & Tanenhaus, 2009; Kelly & Church, 1997; Kendon, 1994). Gestures may be beneficial for communication in part because they help listeners to simulate the actions and perceptions that are expressed in speakers' gestures (see Alibali & Hostetter, 2010, for a discussion of this issue).

In brief, evidence from representational gestures suggests that explaining mathematical thinking involves simulations of actions on mathematical objects, simulations of visual images of mathematical ideas (often mental images of inscriptions), and simulations of the real-world situations that mathematical problems address. Speakers produce such gestures when they think and speak about mathematical ideas, and indeed they may *intentionally* produce such gestures in order to facilitate thinking about such ideas or to promote effective communication about such ideas.

METAPHORIC GESTURES REVEAL BODY-BASED CONCEPTUAL METAPHORS

We have argued that some metaphoric gestures reveal speakers' body-based conceptual metaphors. According to Lakoff and Johnson (1980), conceptual metaphors derive from image schemas regarding space, moving, forces, and other aspects of human experience. Metaphors that involve action and space are readily expressed in gestures (see Cienki & Müller, 2008).

Lakoff and Núñez (2001) presented a theoretical analysis showing that conceptual metaphors that involve action and space may underlie many mathematical

concepts. These metaphors are manifested in the language used to speak about mathematical concepts. Two illustrative metaphors are NUMBERS ARE LOCA- TIONS IN SPACE (e.g., *approaching* zero) and ARITHMETIC IS COLLECTING OBJECTS (e.g., *put* two and two *together*, *take* three apples *from* six and there will be three *left*").

Many of the conceptual metaphors that underlie mathematical ideas rely on a cognitive mechanism called *fictive motion*, which allows one to conceptualize static entities in dynamic terms (Lakoff & Núñez, 2001; Talmy, 1996). Fictive motion is commonly expressed in everyday language (e.g., "the highway *runs beside* the river for a stretch," "the hedge *goes along* the border between the two properties"), and it underlies many examples of the NUMBERS ARE LOCATIONS IN SPACE metaphor, such as "$f(x)$ never *goes beyond* 1".

The conceptual metaphors that underlie mathematical ideas are often expressed in the gestures that speakers produce when speaking about those ideas. Thus, metaphoric gestures provide evidence about the "psychological reality" of the conceptual metaphors that underlie mathematical concepts (Núñez, 2005, 2008). In this section, we discuss examples of metaphoric gestures produced by mathematics teachers and learners that reveal conceptual metaphors in mathematics.

Núñez (2005, 2008) presented several examples of conceptual metaphors expressed in gesture drawn from mathematics professors teaching at the university level. In one example, a professor describes a sequence that "oscillates" between two values, and he depicts this oscillation in gesture with his right arm moving back and forth. In another example, a professor describes an unbounded monotonic sequence that "goes in one direction," and he depicts this sequence in gesture using a circular motion of his hand, which he produces while walking forward at the front of the classroom. These examples illustrate the NUMBERS ARE LOCATIONS IN SPACE metaphor, and both involve fictive motion.

An example from our own data, which we have presented elsewhere to make a different point (Alibali et al., 2011), illustrates the ARITHMETIC IS COLLECTING OBJECTS metaphor. In this example, a middle school teacher is giving a lesson about using equations to model the configuration of a pan balance with objects on each side. With an overhead projector, she projects a figure of a (balanced) pan balance with two spheres on one side and two cylinders and a sphere on the other side and, below it, the associated equation, $s + s = c + c + s$.

The teacher first talks about removing identical objects from both sides of the pan balance, saying, "I am going to take away a sphere from each side." With this utterance, she makes a grasping handshape over the spheres on each side of the pan balance figure. She then says, "Instead of taking it off the pans, I am going to take it away from this equation." With this utterance, she first mimes remov- ing a sphere from each side of the pan balance figure and then makes the same

grasping handshapes over the *s* symbols on the two sides of the equation. With this last gesture, she expresses the metaphor of taking objects away—reflecting the ARITHMETIC IS COLLECTING OBJECTS metaphor described by Lakoff and Núñez (2001)—to give meaning to the principle of subtracting equal quantities from both sides of an equation.

These examples illustrate teachers' use of metaphoric gestures to represent specifically mathematical content. In other cases, speakers utilize metaphoric gestures to express other nonmathematical conceptual metaphors that are involved in some mathematical problems. The following example is taken from a college student who was asked to describe the gist of a mathematics word problem to another participant (drawn from the study described in Alibali et al., 1999). The problem was as follows:

> After a seven-day harvest, a potato farmer notices that his rate of gathering potatoes increased steadily from 35 bushels/day to 77 bushels/day. How many bushels of potatoes total did the farmer collect during the seven-day harvest?

In describing this problem, the student utilizes a gesture that reveals the metaphor TIME PASSING IS MOVEMENT IN SPACE (Boroditsky, 2000; see also Núñez & Sweetser, 2006), shown in Figure 8.

FIGURE 8   Metaphoric gesture for "seven-day harvest" based on the metaphor TIME PASSING IS MOTION THROUGH SPACE.

Student: How many would be the [total] if the [seven-day period] was

                            1   2

  [increasing at a constant rate]
    3

1. Both palms sideways on the table facing center, approximately 12" apart
2. Right-hand palm sideways on table in front of self, makes two hops to the right
   (*represents seven-day period*)
3. Right-hand palm sideways on table in left neutral space, sweeps to the right

The target gesture for the present purposes is the second gesture in the sequence, which depicts the days of the harvest, moving through time from left to right. Her gesture is a physical realization of the time period of a 7-day harvest moving through space.

As these examples illustrate, evidence from metaphoric gestures can provide evidence for the "psychological reality" (Núñez, 2008) of conceptual metaphors. Evidence from teachers' and learners' talk about mathematics provides evidence both for conceptual metaphors that underlie mathematical concepts and for conceptual metaphors in other domains.

GESTURE IN TEACHING–LEARNING INTERACTIONS

Thus far, we have provided illustrative examples of three types of gestures that, we argue, manifest the embodiment of mathematical knowledge. The examples are drawn from teachers' instructional explanations (Figures 2, 3, 5, and 6) and students' elicited explanations of their thinking about various mathematical problems (Figures 1, 7, and 8). To conclude, we present an example of an extended sequence of interaction between a teacher and a student to illustrate that all three of the types of gestures we have discussed occur routinely in teacher–student talk about mathematical ideas. The full (speech and gesture) transcript of the excerpt is provided in the Appendix.

The excerpt is drawn from a corpus of tutorial interactions that we staged to investigate gesture in teacher–student interactions (the same corpus from which the example in Figure 3 was drawn). The teacher is working individually with a student on the concepts of slope and y-intercept as they apply to graphs and equations. The lesson centered on a story problem involving the school band earning money by selling candy bars. The band earns $2 for each candy bar, and at the outset of the tutorial, the scenario is represented with the equation $y = 2x$. The teacher was asked to work with each student to represent this equation on a graph. The teacher was also asked to alter the story problem in two ways, once by changing the amount earned for each candy bar from $2 to $4 (altering the slope) and once by including a $15 donation from a parent when the band began selling candy bars

(altering the intercept). The teacher was asked to work with the student to alter both the equation and the graph for each of these changes to the story. Finally, the teacher was asked to discuss with the student how slope and y-intercept are represented in the equation and the graph.

This excerpt begins with a "trouble source" (Seedhouse, 2004) in the teacher–student discourse—that is, a place where the student reveals a lack of understanding of the material. At this point in the tutorial, the teacher and student had graphed all three lines ($y = 2x$, $y = 4x$, and $y = 2x + 15$), as in Figure 4 . The teacher asks the student to state the slope of the line $y = 2x + 15$, and the student provides an incorrect response, stating the y-intercept (15) rather than the slope (2; Line 2 in the Appendix). In past research, we showed that teachers use more gestures in the utterances that follow trouble sources than the utterances that precede them (Alibali & Nathan, 2007), suggesting that gesture is one tool that teachers use as part of their effort to communicate effectively.

Throughout this excerpt, both teacher and student use pointing gestures to ground their speech to the inscriptions of the equations and graphs with which they are working. When the student offers the incorrect answer (15), the teacher asks how she knows that, seeking to elicit more information about the student's thinking. The student responds by pointing to the equation that represents the line and emphasizes the 15 in that equation, both in her speech and in pointing gestures that trace under the 15 (Line 4, depicted in Figure 9.

The teacher responds to this trouble source with an extended explanation about slope and y-intercept that includes many *pointing gestures*. Noteworthy among
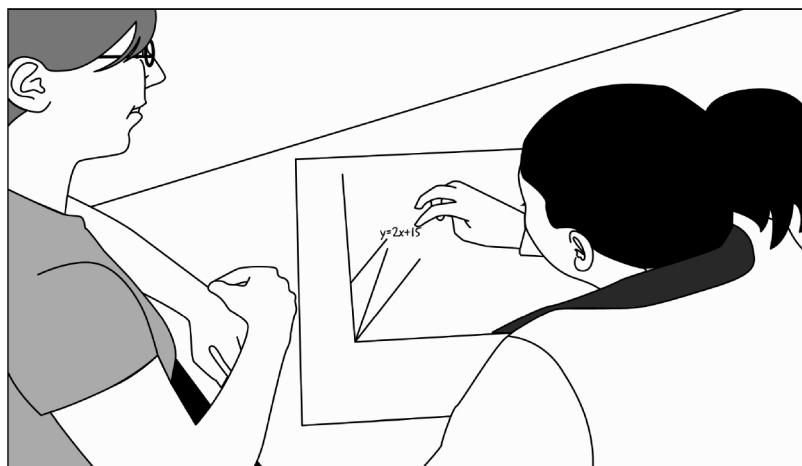

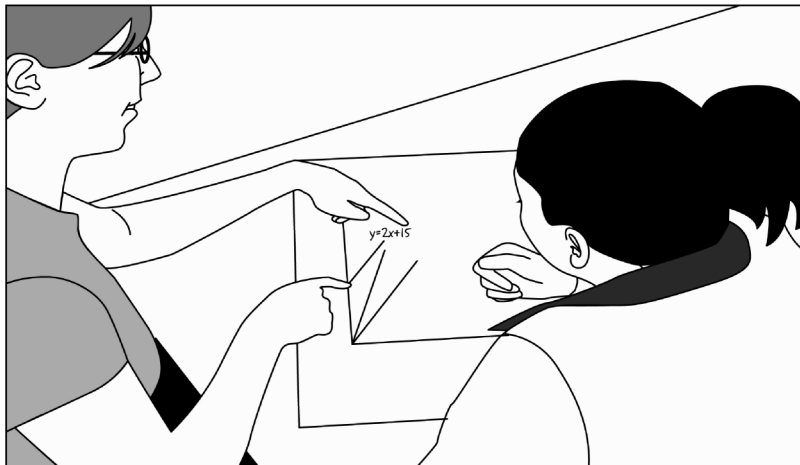FIGURE 9   Pointing gesture: The student indicates 15 in the equation.

FIGURE 10   Simultaneous pointing gestures to corresponding aspects of related representations: The teacher indicates the y-intercept (15) in the equation and the y-intercept on the graph (0, 15).

these are the simultaneous pointing gestures the teacher uses to highlight the relationship between the 15 in the equation (which had been the student's focus of attention) and the y-intercept (0, 15) (Line 23, shown in Figure 10). These pointing gestures both indicate the referents of the teacher's speech and, by virtue of their simultaneity, highlight the link between the 15 in the equation and the y-intercept of the graph.

The teacher also uses *representational gestures* at several points during this excerpt. At one point, she seeks to differentiate the slope and intercept, saying, "It (y-intercept) is not the same thing as slope, because the slope tells us how tilted our line looks" (Line 25). With this utterance, the teacher produces a representational gesture that depicts a line as if holding it between her palms and simulates altering the slope of the line from more steep to less steep and back by shifting the angle of her wrists in unison. This gesture is depicted in Figure 11. During the excerpt the teacher also uses several other representational gestures, many of which are performed over the graph, and therefore also include a pointing component (e.g., Gesture 21, which involves tracing gestures over the two lines with the same slope, $y = 2x$ and $y = 2x + 15$).

At several points during this excerpt, the teacher also uses *metaphoric gestures* that reflect the metaphor QUANTITY IS (HORIZONTAL) DISTANCE. When talking about how much money was donated by a parent (which gives the y-intercept for the line), the teacher uses a gesture that depicts a bounded horizontal space (Line 26, seen in Figure 12). She also makes a similar gesture at two other points earlier

FIGURE 11 Representational gesture that simulates action on a mathematical object: Moving line to different slopes.

FIGURE 12 Metaphoric gesture for "how much money was donated" based on the metaphor QUANTITY IS HORIZONTAL DISTANCE.

in the excerpt when talking about how much money was made for each candy bar (Gestures 14 and 20 in the Appendix).

Following her explanation of y-intercept and how it differs from slope, the teacher asks the student to tell her the y-intercept of another line ($y = 4x$), and the student makes the same error as before (!) but offers her incorrect answer somewhat uncertainly (Line 30). The teacher states that the student's answer reflects the slope and then asks again about intercept, using much more scaffolding—both

verbal ("How much money did we start with though? Where does this red line cross this y-axis?") and gestural (she traces the target line and the y-axis, culminating at the y-intercept, which is the correct answer!; Line 35). Now the student offers the correct answer, but in a questioning tone (Line 36). The teacher then asks the student to state the y-intercept of another line ($y = 2x$). This time, the student answers correctly, with a more certain tone in her voice (Line 39), indicating that she has successfully repaired the initial trouble source.

This excerpt is representative of many episodes of teacher–student interaction that we have observed in that the teacher uses gesture extensively in communicating instructional material, especially in response to trouble sources in the discourse (Alibali & Nathan, 2007). As in other cases we have studied (Alibali et al., 2011), pointing gestures predominate, but representational gestures and metaphoric gestures are also common. Teachers and students use gestures to ground the co-occurring speech by establishing reference, to simulate actions and perceptions, and to express body-based metaphors that are relevant to the instructional material.

DISCUSSION

In this article, we have provided theoretical arguments and illustrative examples to show that (a) pointing gestures manifest grounding in the physical or imagined environment, (b) representational gestures manifest mental simulations of action and perception, and (c) metaphoric gestures reveal conceptual metaphors that are grounded in the body and human experience. All of these types of gestures routinely occur in discourse about mathematical ideas, for example in instruction and explanation.

Gestures of these types in mathematical discourse provide evidence that mathematical thinking is embodied in several important senses. Evidence from pointing gestures produced by teachers and students suggests that mathematical thinking is grounded in the physical environment, which includes mathematical inscriptions that are perceptually available to both speakers and listeners. Evidence from representational gestures suggests that mathematical thinking involves simulations of actions on mathematical objects, simulations of visual images of mathematical ideas or inscriptions, and simulations of the real-world situations that mathematical problems address. Finally, evidence from metaphorical gestures supports the "psychological reality" of conceptual metaphors that underlie mathematical concepts, as well as conceptual metaphors in other domains (e.g., time). Taken together, these three lines of evidence support the claim that embodied knowledge is an integral component of mathematical thinking and learning. Gestures thus provide a unique and informative source of evidence regarding the nature of mathematical thinking.

We have argued that gestures *reflect* speakers' embodied thinking about mathematical concepts and procedures, and we further suggest that they may also play a crucial role in *communicating* embodied knowledge to learners. Thus, studying the gestures produced in mathematics instruction and teacher–student interaction may shed light not only on the nature of mathematical thinking but also on the mechanisms involved in learning from instruction. Students may construct understanding by generating simulations that are informed by the simulated actions that their teachers express in gestures (see Alibali & Hostetter, 2010).

Gestures may also play a role in mathematical thinking by externalizing aspects of speakers' mathematical knowledge. In so doing, gesture may help learners to manage the working memory demands of mathematical thinking and explanation (e.g., Alibali & DiRusso, 1999; Goldin-Meadow & Wagner, 2005; Wagner, Nusbaum, & Goldin-Meadow, 2004). Although we have not focused on this potential role of gesture in this article, this view is not incompatible with the embodied framework we embrace here. Gestures that externalize information may serve to ground speakers' utterances in the physical environment. In doing so, such gestures may also off-load aspects of cognitive processing onto the environment (Kirsh & Maglio, 1994), thereby lightening speakers' processing load.

Implications for Research in the Learning Sciences

Our perspective aligns with several themes that are central in contemporary research in the learning sciences (see Nathan & Alibali, 2010, for discussion). The first such theme is the attempt to bridge the divide between research and practice. Practical connections to educational practice can be drawn from research on embodied mathematical knowledge and how it is expressed in gesture. An embodied perspective provides a framework for interpreting students' and teachers' behaviors as they behave and communicate in classroom settings and can also yield predictions about the effects of certain instructional practices. For example, instructional experiences that involve actions on manipulatives may lead to knowledge that children readily express in gestures that simulate those actions—a prediction we are testing in ongoing research.

A second theme of contemporary research in the learning sciences is the importance of analyzing teaching and learning in authentic settings. An embodied perspective offers a unique lens for considering aspects of learners' and teachers' behavior in ecologically valid, real-world settings, such as classrooms and tutoring sessions. Many of the examples we have presented in this article were drawn from such settings. The embodied perspective acknowledges the inherent complexities of learning and teaching in real-word settings and provides a framework for analyzing the action and communication that take place in such settings in an effort to deeply understand processes of knowledge change.

A third theme of contemporary research in the learning sciences is its emphasis on the behavior of learners in interaction with the physical, social, and cultural world, including semiotic and technical resources. Research on embodied mathematical knowledge takes seriously the *situated* nature of knowledge in the context and setting where it occurs and, in particular, its grounding in the actions of a physical body that has particular sorts of perceptual systems as it engages with objects and symbols in the physical environment, which is itself culturally situated and includes certain semiotic and technical resources. Many of the examples we have presented in this article describe how teachers and learners use gestures to ground mathematical symbols (which are culturally defined semiotic resources) in the physical environment and in the actions of the body. Gestures and speech reveal both learners' and teachers' knowledge and exhibit the ways in which physical, social, cultural and semiotic resources are recruited during learning and instruction. Thus, an embodied perspective recognizes that learning and teaching are multimodal interactions that occur in rich communicative contexts and that draw on prior knowledge as well as resources present in the physical environment and the broader cultural context.

Ties to Research on Teaching Practices and Teacher Education

In this article, we have proposed a framework that addresses both gestures produced by learners and those produced by teachers. However, we believe it is also important to acknowledge the special pedagogical nature of communication by teachers. Teachers are generally charged with communicating novel concepts and procedures to learners. Despite the integral role of gesture in instructional communication, teachers' gestures have not been a central focus of education research (Roth, 2002). This situation persists despite a rich tradition of studying how other aspects of teachers' knowledge and behavior may foster student learning (e.g., Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Hill, Rowan, & Ball, 2005; Peterson, Carpenter, & Fennema, 1989; Shulman, 1986; Zeichner, 1999).

The lack of attention to gesture in teacher education methods courses is also striking. Methods courses generally prescribe approaches for classroom instruction, addressing topics such as delivering lessons, facilitating classroom discussion, introducing and conducting activities, performing assessments, and managing the classroom (e.g., Abell, Appleton, & Hanuscin, 2010; Feden & Vogel, 2003). When one delves further into the instructional strategies offered to pre-service teachers (e.g., Manning & Bucher, 2009), one finds an overwhelming emphasis on the verbal channel. In our view, there should be a place in teacher education for the consideration of how speech and body-based resources such as gesture can work in concert to implement effective and engaging instruction that promotes deep understanding of fundamental ideas in mathematics and other content areas.

Conclusion

In this work, we have identified three ways in which teachers and students use body-based resources, specifically gestures, in teaching and learning settings: (a) *Pointing* gestures reflect the grounding of cognition in the physical environment, (b) *representational* (i.e., iconic and metaphoric) gestures manifest mental simulations of action and perception, and (c) some *metaphoric* gestures reflect body-based conceptual metaphors. In carrying out this work, we seek to advance efforts at developing empirically supported methods for improving the educational experiences of students and teachers. At the same time, we seek to advance understanding of the nature of mathematical thinking, how it changes with development and learning, and how it is fostered through instruction. We believe that an embodied account of mathematical thinking, instruction, and communication holds promise for integrating understanding of cognitive processes and behavior in the real-world social and physical interactions where learning occurs.

ACKNOWLEDGMENTS

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APPENDIX

Table A-1
Transcript of Teacher–Student Interaction

Line	Speech Transcript + Gesture Number(s)	Gesture Transcript
1	T: [What's the slope of the blue line?] 　　1	1. RH point traces up and down blue line
2	S: [Fifteen.] 　　2	2. RH point to 15 in $y = 2x + 15$ equation next to line
3	T: How do you know that?	No gesture
4	S: Because [two] [times][what] [equals], 　　　　3　　4　　5　　6 [plus fifteen]. 　　7	3. RH point to 2 in $y = 2x + 15$ 4. RH point to x in $y = 2x + 15$ 5. RH point to x in $y = 2x + 15$ 6. RH point to 15 in $y = 2x + 15$ 7. RH point traces under $+ 15$ in $y = 2x + 15$
5	T: Okay.	No gesture
6	S: Because, [the equation said, you 　　　　　　8 started off at fifteen, so our slope should be fifteen.]	8. RH point traces under 15 several times

(*Continued*)

Table A-1
(*Continued*)

Line	Speech Transcript + Gesture Number(s)	Gesture Transcript

7	T: [Ah, okay.] You're keying into
	    9
	[something else] that's [close to slope]
	    10 11
	but not quite the same.
		9. RH point in neutral space, in general direction of graph, beats up and down 2 times
		10. RH point in neutral space, in general direction of graph, beats up and down
		11. RH point in neutral space, in general direction of graph, beats up and down

8	T: [Remember] [our slope] was [how 12 13 14 much] we made for every candy bar we sold?	12. LH point in neutral space, beats up and down 13. LH point in neutral space, beats up and down 14. BH C handshapes oriented toward center in neutral space
9	S: Mmhm.	No gesture
10	T: So [we made two dollars for every 15 candy bar we sold here.]	15. BH points, LH held at origin (0,0), RH point traces from origin to end of black line ($y = 2x$), back to origin and then back to end of line again
11	T: [On the red line we had four dollars for 16 every candy bar we sold.]	16. BH points, LH held at origin (0,0), RH point traces from origin to end of red line ($y = 4x$), back to origin and then back to end of line again
12	T: [How much did we make for every 17 candy bar in the blue line?	17. BH points, LH held at y-intercept (0,15), RH point traces from origin to end of blue line ($y = 2x + 15$), back to origin, back to end of line, and back to origin again, and held through student's response and reiteration of student's response
13	S: Two.	No gesture

14	T: Two dollars]. Yeah, so even though it
	    17 (held from before)
	started at [fifteen], the slope is still gonna
	    18
	be [two dollars], 'cause we made [two
	    19 20
	dollars each time].
		18. BH points to y-intercept (0, 15)
		19. RH point along line from y-intercept (0, 15) to (1, 17)
		20. BH palms toward center in neutral space, move slightly left (metaphor for amount made each time)

(*Continued*)

Table A-1
(*Continued*)

Line	Speech Transcript + Gesture Number(s)	Gesture Transcript
15	S: 'Kay.	No gesture
16	T: And [so these slopes are actually the 21 same].	21. RH point at (0, 0), LH point at (0, 15), RH traces along line $y = 2x$ and back to (0, 0), and simultaneously LH traces along on line $y = 2x + 15$ and back to (0, 15)
17	T: But [like you mentioned], there's a 22 [fifteen in there]. 23	22. RH point to student 23. LH point to 15 in $y = 2x + 15$, taps $3x$
18	T: And I like how you mentioned that 'cause the [fifteen actually is what we call 24 our y-intercept].	24. RH point to y-intercept (0, 15), holds
19	S: Mmhm.	No gesture
20	T: So ['member all along this axis like you 25 labeled], [this is called our y-axis]. 26	25. RH point to top of y-axis, held, then LH point added to same spot 26. LH held at top of y-axis, RH point traces along y-axis to origin (0, 0)
21	T: [Whenever a line crosses through] [this 27 28 y-axis], [we call it] a y-intercept. 29	27. RH point at top of line $y = 2x + 15$, traces along line past y-axis and continues 28. RH point to y-intercept (0,15), slight back-and forth motion over point 29. RH point moves to origin (0, 0), LH point at top of y-axis, traces down along y-axis to y-intercept (0, 15)
22	T: [So this blue line crosses through the 30 y-axis at fifteen.	30. RH point at top of line $y = 2x + 15$, traces along line to just past y-axis, then holds at (0, 15)
23	T: So that's where that fifteen [is coming 31 in.]	31. RH point still held (from previous gesture) at (0, 15), LH point to 15 in equation, tap
24	T: [We call it a y-intercept.]] 32	32. RH point still held (from previous gesture) at (0, 15), LH hand cupped toward up in neutral space (*conduit metaphor*)

(*Continued*)

Table A-1
(*Continued*)

Line	Speech Transcript + Gesture Number(s)	Gesture Transcript
25	T: It's [not the same thing as slope] 33 because the slope tells us [how tilted our 34 line looks].	33. LH hand cupped toward up in neutral space, beats down/up (conduit metaphor) 34. Both palms toward center (as if holding a line in between them) over graph, depicting a steeper line, then a shallower line, then a steeper line
26	T: But the [y-intercept] is gonna tell us 35 [how high it started], or [how much 36 37 money was donated] in the first place.	35. BH points on line $y = 2x + 15$, move toward y-intercept 36. BH point at y-intercept $(0, 15)$, tap 37. BH palms toward center in left neutral space, about 6" apart
27	S: Mmkay.	No gesture
28	T: So the [y-intercept on the blue line is 38 fifteen].	38. RH point to y-intercept $(0, 15)$, moves about slightly
29	T: [What is the y-intercept on this red 39 line?]	39. RH point traces over line $y = 4x$ from top of line down to origin $(0, 0)$
30	S: Four.	No gesture
31	T: Four? And how'd you get that?	No gesture
32	S: Because we [made four dollars]? 40	40. RH point (holding marker) to line $y = 4x$, trace a short portion of the middle of the line
33	T: That would [be our slope.] 41	41. RH point in neutral space, toward center, beats once
34	S (*at the same time*): [The money]. 42	42. RH point (holding marker) to line $y = 4x$, trace a short portion of the middle of the line (*repeat of previous gesture*)
35	T: How much money did we start with though? Where does [this red line cross 43 [this y-axis]]? 44	43. RH point traces line $y = 4x$ from top down to origin $(0, 0)$ and slightly beyond, then hold 44. LH point traces y-axis from top down to origin $(0, 0)$
36	S (*questioning tone*): Zero?	No gesture

(*Continued*)

Table A-1
(*Continued*)

Line	Speech Transcript + Gesture Number(s)	Gesture Transcript
37	T: Zero. So [the y-intercept is actually 45 zero on here], [and the slope is four]. 46	45. RH point to (0, 0), tap several times 46. RH point traces from origin (0, 0) to top of line $y = 4x$
38	T: [How 'bout the y-intercept on this 47 black line?]	47. RH point traces line $y = 2x$ from top down to slightly past origin (0, 0) and hold there
39	S (*more certain tone*): Zero.	No gesture
40	T: Zero. So [both of these lines start at 48 zero] because [no one donated money in 49 those two cases][to start off with]. 50	48. BH points, RH on $y = 2x$, LH on $y = 4x$, trace to top of line and then back to origin (0, 0) 49. BH palms down, move apart in neutral space 50. BH points to origin (0, 0)

Note. T = teacher, S = student, RH = right hand, LH = left hand, BH = both hands.