Embodied Learning on Any Device: Geometric Proof in a Motion-Capture Video Game

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Abstract

An emerging literature on embodied cognition shows that mathematical ideas can be learned through action-based interventions that ground understanding of abstract principles in concrete experiences. As new theoretical perspectives like embodied cognition emerge in education, so do new ways of using technology to support the difficult tasks of teaching and learning. This research offers two advancements for innovations in learning technology. First, we describe use of a prototype video game based on theories of embodied cognition to promote learning in a difficult domain – geometry theorem proving. Second, we show the feasibility of our system to track player body positions through rapid digital image processing using the camera that comes standard with laptop computers, rather than relying on specialized hardware.

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Objectives

An emerging literature on embodied cognition shows that mathematical ideas can be learned through action-based interventions that ground understanding of abstract principles in concrete experiences (Goldstone & Son, 2005). As new theoretical perspectives like embodied cognition emerge in education, so do new ways of using technology to support the difficult tasks of teaching and learning (Lee, 2015). This research offers two advancements for innovations in learning technology. First, we describe use of a prototype video game based on theories of embodied cognition to promote learning in a difficult and very conceptual domain – geometry theorem proving. Second, we show the feasibility of our system to track player body positions through rapid digital image processing using the camera that comes standard with laptop computers, rather than relying on specialized hardware.

Theoretical Framework

Theories of grounded and embodied cognition (Barsalou, 2008; Wilson, 2002) posit that core ideas behind many mathematical theorems are firmly rooted in people's bodily interactions with the physical world (Lakoff & Nunez, 2000). Recent theoretical work underscores the bidirectionality of cognition and action (Nathan, 2014). Evidence suggests that sensorimotor activity can activate neural systems, which in turn alter and induce cognitive states (Thomas, 2013). In our prior work (authors, date), undergraduates generated proofs for mathematical conjectures after being directed to perform either task-relevant actions with their bodies that physically enacted an underlying mathematical relationship, or irrelevant actions, which were kinesthetically similar but mathematically trivial body movements. Performing relevant actions alone facilitated key mathematical insights for the associated proofs, while relevant actions combined with pedagogical language (i.e., a verbal hint that explicitly linked the actions to the proof) supported improved construction of valid proofs.

A new class of technology for education and recreation processes players' body movements to create physically interactive embodied experiences that can foster learning. Fischer et al. (2015) used digital dance mats and Kinect sensors to promote physical experiences related to the number line. Lindgren (2015) used a mixed-reality laser-scanning and floor-projection simulation to cue body movements relevant to physics principles. Other interventions have explored how hand motions detected by the Kinect foster an understanding of proportions (Trninic and Abrahamson, 2012) and how touchpads allow for dynamic interaction with algebraic symbol systems (Ottmar, Landy, and Goldstone, 2012). Smith, King, and Hoyte (2014) used the Kinect to allow students to dynamically model angles with their arms, while Hall, Ma, and Nemirovsky (2015) describe students using global-positioning system devices and mapping software to draw geometric constructions. Such technology-based embodied interventions seem well-suited to the spatial qualities of geometry in particular; however these technologies are difficult to bring into K-12 classrooms at scale.

Embodied learning has not been extended to the investigation of mathematical proof, a central competency in mathematics education (Yackel & Hanna, 2003) that is challenging for students (e.g., Dreyfus, 1999; Healy & Hoyles, 2000; Martin, McCrone, Bower, & Dindyal, 2005). We follow Harel and Sowder (1998) in defining proving as "the process employed by an individual to remove or create doubts about the truth of an observation" (p. 241). Harel and Sowder (1998) define valid, *transformational proofs* as proofs that are: *general,* and show the argument is true all cases; that involve *operational thought* by progressing through goals and subgoals, and that involve *logical inference* where conclusions are drawn from valid premises.

In the present study, we explore the potential of directing body motions using cutting-edge motion capture technology to support proof of geometric conjectures. Significantly, this technology, unlike the Kinect, digital dancepads, or laser scanning, utilizes only the camera on a standard laptop, making it incredibly portable into K-12 settings. Our research questions are: How do directed motions facilitate geometric reasoning, with and without pedagogical language? How do students respond to a motion capture technology intervention for geometry learning?

Methods

Our video game, *The Mystics*, was built for Windows using the C# language in the Visual Studio 2013 IDE. We used specialized image processing software to generate a wireframe skeleton to track the coordinates of the user's joints (Figure 1). The game (Figure 2) begins with brief setup instructions, and then the storyline starts with the player encountering an imposing tribe whose culture they must accept by performing arm movements at the tribe's welcome ceremony. Movements are designed to either be task-relevant, capturing some key relation of the subsequent conjecture (Table 1) or task-irrelevant. Sequences of tribal movements must be successfully repeated five times, and the game utilizes a standard built-in camera to determine in real time whether the player successfully executes each sequence. The player is then presented with a geometric conjecture as a challenge from the tribe and instructed to speak-aloud with a proof as to why the conjecture is true or false. Afterwards, players choose among 3-4 different proof statements commonly given for the conjecture, where only one is a valid transformational proof. Conjectures are presented in random order, and the participant receives relevant motions for some conjectures and irrelevant motions for others. There were 8 conjectures in the game; however 2 conjectures are omitted because of technical and methodological issues with the motions. Prior to conducting our research study, three rounds of user testing were conducted with three successive prototypes using high school students.

Data Sources

Eighteen middle and high school students (grades 6-11; 16 male and 2 female) attending a video game design summer camp on a university campus individually tested the game; 3 had previously taken a geometry course. Students were given a pre-assessment measuring their interest in geometry and their knowledge of geometric properties. Students then played the game. Upon finishing, the interviewer chose 2-4 conjectures that the student had received relevant motions for, and revealed that the motions had been relevant by displaying an image that showed the motions and the conjecture. Participants were given an additional opportunity to provide a justification using this interviewer hint (i.e., "pedagogical language"). Participants were then given a post-survey that assessed their interest in geometry (same items) and their triggered situational interest in the game itself. All interest items (Table 2) were drawn from Linnenbrink-Garcia et al. (2010). Finally, participants were asked what they liked about the game and what they would change.

Students' responses to each of the 6 conjectures were scored 0/1 along 3 dimensions: (1) whether they were correct in their judgment of whether the conjecture was true or false, (2) whether they formulated a valid transformational proof, and (3) whether they chose the correct multiple choice option. *T*-tests were examined to look for significant differences in performance and interest. Responses to open-ended questions were coded using emergent themes (Glaeser & Strauss, 1967). Selected transcripts from the hint sequences were analyzed using multi-modal analysis (McNeil, 1992) of gestures to reveal how explicit reflection upon directed movements impacts proof practices.

Results and Discussion

The prototype performed well for all 18 participants. It reliably calibrated each player, despite a range of ages and sizes, and adequately processed players' body positions and in-game menu selections. Students generally said they liked the motion detection and found it interesting (8 participants): "There's a lot of educational games out there, but I think motions are really interesting" and "You just used a laptop, which is really cool that you found the technology to do that." Students also liked that the motions were actually relevant to the geometry tasks (7 participants): "My favorite part was how you showed me the relationship between the movements and the actual problems… that was really a cool way that you found to interpret movements into a game." Students also thought the problems were interesting, challenging, and/or helped them learn (5 participants).

In terms of improvements, students thought the game was difficult and wanted more scaffolding to be provided and/or for their score to be shown (12 participants). Some students felt the movements were repetitive or that the motion capture needed some adjustment (7 participants): "Sometimes it would randomly calibrate in the middle of it and that kind of bugged me." Finally, some students felt the relationship between their motions and the problem should have been made explicit (4 participants).

The difficulty of the game and the lack of in-game support seemed to be a major issue. The low performance rates in Table 3 suggest that the game may be most appropriate for students who have taken at least part of a geometry course, which was not characteristic of our sample. Accordingly, on the pre-/post-intervention questionnaire regarding interest in geometry, students started out near the middle of the scale (pre-average = 3.28), and did not change as a result of playing the game (post-average = 3.36). On the survey assessing interest in the game, students rated items as slightly above "Somewhat True" (average 3.20 on a 1-5 scale). One item, "I found the math in the geometry game interesting" was rated significantly above the 3.0 mid-benchmark (average = 3.56, one sample *t*-test yielded $p = 0.028$). The difficulty of the game seemed to be an important element in driving whether interest was triggered; the correlation between average interest rating in the game and pre-test score of knowledge of geometric properties was moderate at $r = 0.433$, $t(16)=1.92$, $p=0.07$. In video games, difficulty should be only slightly above the learners' skill level to promote engagement while maintaining effort (Gee, 2003). Educational games with some game-based features (a storyline) but not others (adaptivity to knowledge level and just-in-time support) may not have the positive outcomes that coherent, fully-developed games do (Jackson & McNamara, 2013).

Since the study was underpowered for inferential statistics, owing to the small sample size, and participants were not all developmentally matched to the intended audience of the game (geometry students), we provide some preliminary descriptive findings on the effects the game had on math learning in Table 3. At debrief, all participants except one stated that they were unaware of the connection between the motions and the conjectures while playing the game. The likelihood of participants determining a correct true/false judgment was at chance performance. There was a small advantage of relevant actions for formulating a valid transformational proof (30% vs 23.9%), but not for selecting the correct multiple choice option.

Lindgren (2015) recommends that students have the opportunity to reflect upon cued movements so they can see how their actions enact the concepts they are grappling with. When participants were given the hint by the interviewer that certain motions they had performed were relevant to particular conjectures ($N = 43$ instances), participants' likelihood of formulating a valid proof increased significantly, from 23.2% to 34.9% correct; a paired *t* test yielded p = 0.024. Notably, all those who benefitted from hints had completed Algebra I.

Figure 3 shows a multimodal transcript of a student formulating a new proof for the conjecture that only one unique triangle can be formed from three angle measurements (Conjecture 1 in Table 1) after receiving a hint that his directed actions had been relevant. The student had originally incorrectly said the conjecture was true because angle measurements are unique to a triangle. Although he incorrectly maintained that the conjecture was true after the hint, his verbal proof showed a growing triangle as a means of disproving the conjecture. To make his argument, he utilized co-speech spontaneous gestures of a triangle growing outwards, indicating the directed motions left a legacy (authors, date).

Significance

The emergence of embodied theories of cognition presents new avenues for spurring learning. Typically, embodied interventions focus on early concepts in math (counting, number line) and reading (vocabulary). We demonstrate that common laptop technology can be used in an interactive video game for improving college preparatory proof practices, which supports widespread scale-up. Players responded positively to our prototype and offered constructive suggestions for improvement. We found some advantages for relevant over irrelevant actions for improving proof, and added benefits for hints that made the links from actions to concepts explicit. These findings provide partial replication of our earlier experimental (non-game) intervention. They underscore the value of coupling explicit language with actions for grounding complex ideas during learning.

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Authors (date). References of this format have been removed for blinding.

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Figures and Tables

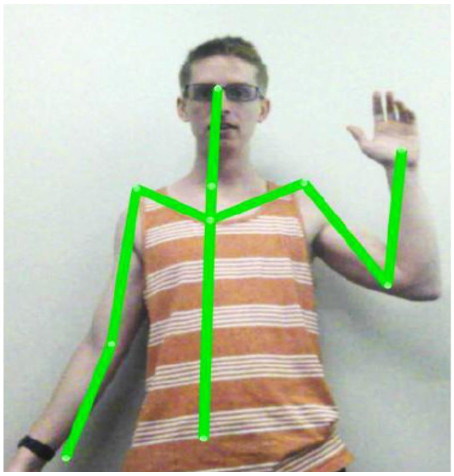


Figure 1. Wireframe skeleton that shows how the software tracks users' body movements using a laptop camera.

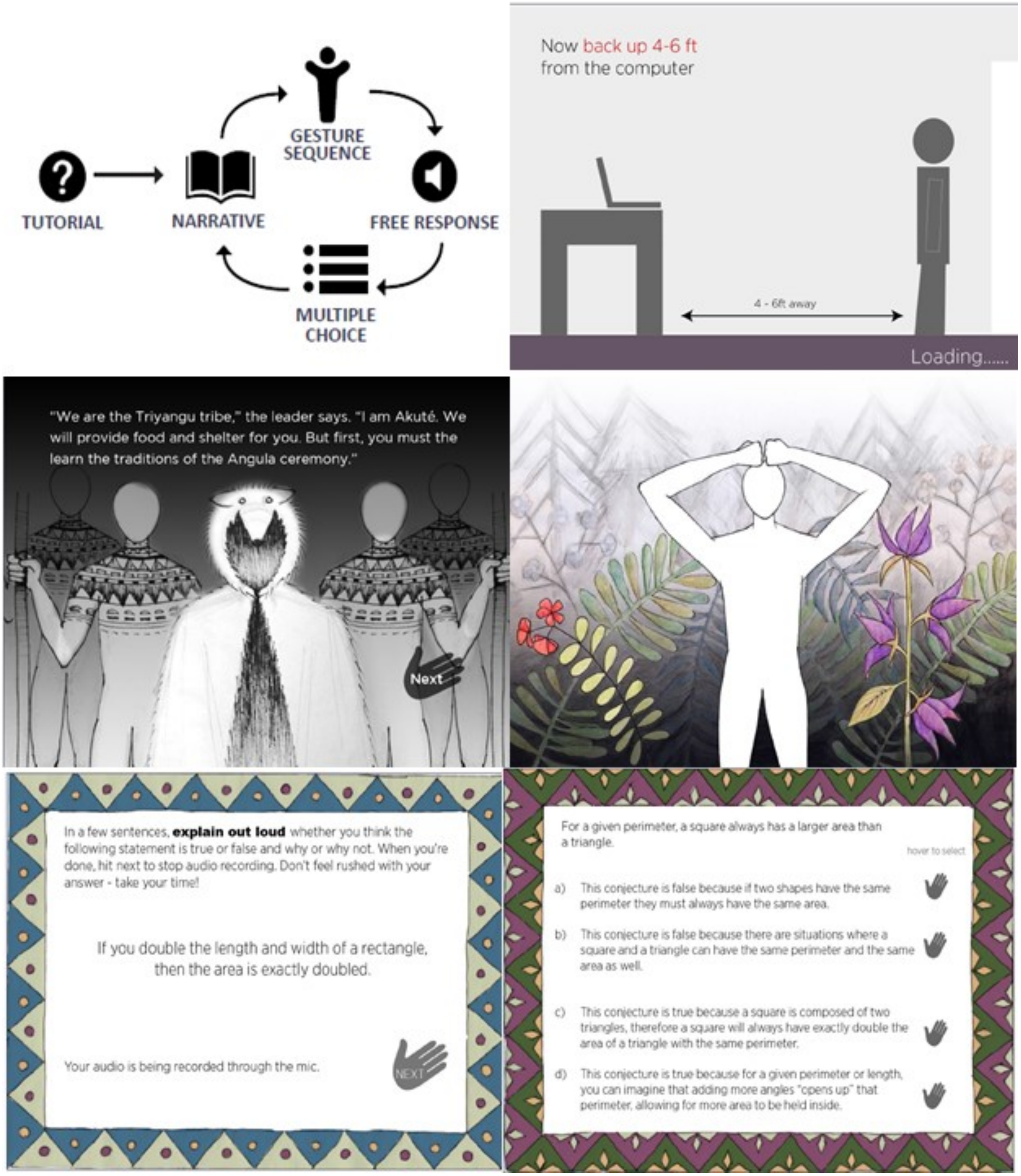


Figure 2. The top left image shows the flow of the game through 5 stages - the circular cycle is repeated for each of the conjectures. The remaining 5 images show an example screenshot from each of the 5 stages of the game - tutorial (top right), storyline (middle left), gesture sequence (middle right), free response to geometric conjecture (bottom left), multiple choice response to geometric conjecture (bottom right).

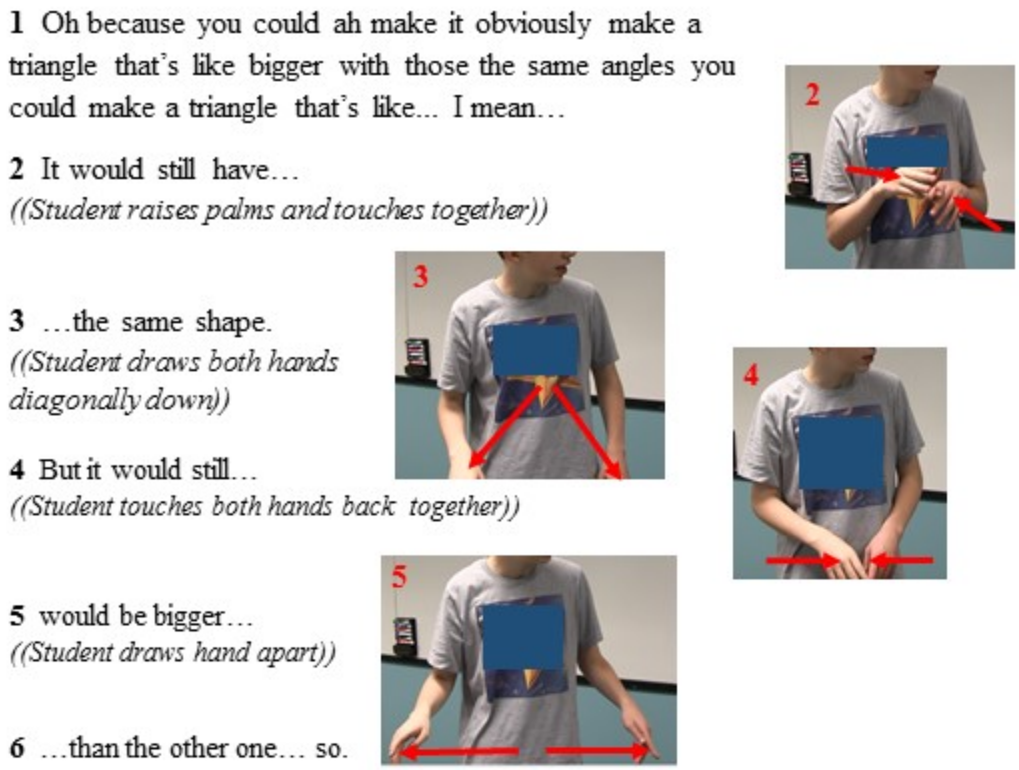


Figure 3. Transcript of student proving the conjecture "*Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements*" after receiving a hint that his directed actions had been relevant to the conjecture.

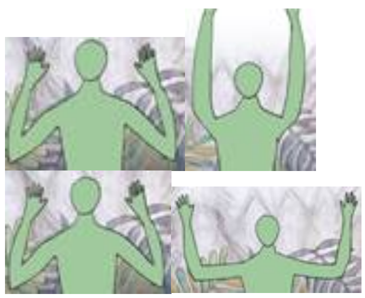
Conjecture	Relevant Motion Sequence	Intended Relevance of Directed Motions
1. Given that you know the measure of all three angles of a triangle, there is only one unique triangle that can be formed with these three angle measurements. **[FALSE]**		Triangle growing outward, forming similar triangles that had the same angles, but longer side lengths.
2. The area of a parallelogram is the same as the area of a rectangle with the same length and height. **[TRUE]**		Area of a parallelogram can be formulated from the area of a rectangle when the area is rearranged (tilted).
3. If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle. **[TRUE]**		Experience that as an angle opens up, the opposite side necessarily becomes longer.
4. If you double the length and width of a rectangle, then the area is exactly doubled. **[FALSE]**		Convey that when the width doubles, the rectangle becomes twice as wide (top) and that when the length doubles, the rectangle becomes twice as long (bottom), resulting in an area that is 4 times greater.
5. For a given perimeter, a square always has a larger area than a triangle. **[TRUE]**		Relevant motions intended to convey that a triangle (left) makes inefficient use of its perimeter, while a square (right) allows more space to fit inside.
6. The measure of a central angle of a circle is twice the measure of any inscribed angle intersecting the same two endpoints on the circumference. **[TRUE]**		Experience the 2:1 relationship between the larger central angle (top, bent arm) and the smaller inscribed angle (bottom, straight arm)

Table 1. Relevant motions students performed and associated conjectures. Irrelevant motions (not pictured) included similar arm movements chosen so that they were not directly related to the conjecture.

		Pre Average (SD)	Post Average (SD)
1)	Geometry is practical for me to know.	3.72 (0.89)	3.72 (0.89)
2)	Geometry helps me in my daily life outside of school.	2.78 (1.00)	2.78 (1.00)
3)	It is important to me to be a person who reasons mathematically.	3.94 (1.11)	3.94 (1.11)
4)	Thinking mathematically is an important part of who I am.	3.61 (1.20)	3.61 (1.20)
5)	I enjoy the subject of geometry.	3.11 (1.18)	3.11 (1.18)
6)	I like geometry.	3.33 (1.19)	3.33 (1.19)
7)	I enjoy doing geometry.	3 (1.03)	3 (1.03)
8)	Geometry is exciting to me.	2.72 (1.13)	2.72 (1.13)

1)	The geometry game was exciting.	3.72 (0.89)
2)	When I was playing, the game did things that grabbed my attention.	2.78 (1.00)
3)	The game was often entertaining.	3.94 (1.11)
4)	The game was so exciting it was easy to pay attention.	3.61 (1.20)
5)	What I was learning in the geometry game was fascinating to me	3.11 (1.18)
6)	I was excited about what I was learning in the geometry game.	3.33 (1.19)
7)	I liked what I was learning in the geometry game.	3 (1.03)
8)	I found the math I did in the geometry game interesting.	2.72 (1.13)

Table 2. The top portion of the table shows interest items given to students at the beginning and end of the session (reported Cronbach's $\alpha = 0.90$). The bottom portion of the table shows interest items given to students at the end of the session (reported Cronbach's $\alpha = 0.86$). The scale was 1-5 with benchmarks 1-Not at all True, 3-Somewhat True, 5-Very True.

Condition	**Correct True/False?**	**Correct Proof?**	**Correct Multiple Choice?**
Irrelevant Actions ($N = 46$ instances)	52.2%	23.9%	26.1%
Relevant Actions ($N = 60$ instances)	50.0%	30.0%	21.6%

Table 3. Descriptive statistics showing mean performance levels on the conjectures in the game. Because outcomes were binomial (0/1, incorrect/correct), means and sample sizes are given rather than standard deviations. Note that for true/false judgments, chance performance would be at 50% correct, and for multiple choice selections, chance performance would be 25-33% correct.