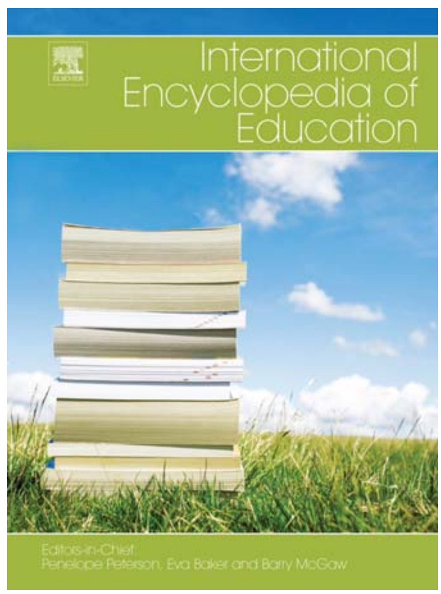

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Technology Supports for Acquiring Mathematics

M J Nathan, University of Wisconsin-Madison, Madison, WI, USA

Introduction

Technology has become essential to the practice of mathematics. Technology allows some areas of mathematics to flourish, such as the close relationships between fractal geometry with computer graphics, and statistics with computer programs (Ben-Zvi and Garfield, 2004; Frame and Mandelbrot, 2002). Technology also plays a special role in the learning of mathematics. Technology supports for mathematics education largely reflect the form and shifts of underlying theories of learning and intellectual behavior. Information processing (IP) theory and theories of cognitive skill acquisition and concept learning have been dominant influences in mathematics education for the last half century, and these are presented in the first section titled 'Skills and concept learning.' While these traditional cognitive perspectives on knowledge and learning continue to play a significant role in governing the types of technological resources for mathematics education (as well as shaping curriculum, instruction, and assessment practices more broadly), increasingly, designs of educational technology are being influenced by emerging theories of learning and practice. The second section titled 'Mathematical discovery' focuses on technologies that support learning by discovery. In the third section titled 'Collaborative problem solving,' collaborative learning technologies are presented that often draw on situated, social, and cultural perspectives of learning and constructivist views of knowledge. The fourth section titled 'Embodied cognition' reviews some of the newest technologies in craft and fabrication, inspired by emerging views of embodied cognition. Some systems appear in multiple places within this simple taxonomy. In the final section titled 'Challenges facing technology supports for acquiring mathematics,' some of the persistent challenges facing technologies for mathematical learning are discussed.

Skills and Concept Learning

There is a rich history of tools specifically designed for mathematics instruction, including formal systems of manipulatives such as Dienes' blocks, pattern boards, and Cuisenaire rods. Manipulatives are concrete objects that are designed to help students learn math concepts by representing the quantities, operations, and relations between quantities without requiring that the learner use or comprehend the written (i.e., formal) representations of the same concepts (Uttal, 2003).

Generally, the use of math manipulatives as educational aids is predicated on a view of learning as the acquisition of skills and abstractions (Chao *et al.*, 2000). The mental tool view frames learning as the acquisition of skills and symbolic structures that parallel the physical states and actions of the objects. Learning from this perspective typically favors highly structured and consistent practice to enable the acquisition and speedup of procedures, which may eventually become automated. In the abstraction view, math learning is facilitated through generalization across a range of varied experiences that all model a common concept. This form of learned behavior tends to exhibit slower and more deliberate responses that are highly accurate, along with more frequent use of rudimentary strategies. Even with these trade-offs in mind, the research literature does not consistently show an advantage for manipulatives for math learning among primary grade students (e.g., Hiebert, 1989; Sowell, 1989; Uttal *et al.*, 1997).

Computer-based manipulatives and other instructional tools have emerged to capture the concrete qualities of materials along with the added control and flexibility of digital media. One of the earliest and most influential was Logo, which can be regarded as the forbearer of computers as microworlds, and objects-to-think-with. In a somewhat parallel fashion, graphing calculators emerged as hand-held tools for math activity and math instruction. These early forms also spawned computer algebra systems and more free-form modeling tools, such as Geometric Supposer and Geometer's Sketchpad.

Hand-Held Graphing Calculators

The first graphing calculator was introduced by Casio in 1985. However, graphing calculators exhibited a much greater influence on classroom learning in the early 1990s, with contributions from Hewlett-Packard and Texas Instruments (TI). The TI-82 (released 1993) and the ubiquitous TI-83 (1996; see **Figure 1**) transformed secondary and tertiary math education by putting into students' hands an affordable, portable, and accessible device that allowed them to analyze, program, and visualize mathematical procedures and structures. By the year 2000, over 80% of high-school mathematics teachers in the US who were surveyed, reported using hand-held graphing calculators in their classrooms (Hudson *et al.*, 2002).

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Figure 1 Classroom teacher and students using the TI-83-Plus hand-held graphing calculator by Texas Instruments. Photo courtesy of Texas Instruments Education Technology.

Hand-held graphing technology has the potential to change the nature of mathematics instruction and learning, as well as alter the very content of the mathematics that gets taught in schools (Waits and Demana, 1994, 2000). However, reviews of US textbooks exhibit a fairly simplistic use of the technology (Burrill, 2004; Senk *et al.*, unpublished).

Internationally (in studies of Great Britain, Sweden, France, Australia, New Zealand, South Africa, Israel, Netherlands, and the United States), hand-held graphing-technology use generally facilitates learners' concept development and its use is predictive of higher performance gains and measures of problem-solving skills. Those who use graphing calculators show a better understanding of functions, applied problem solving in algebra, and interpreting graphs (Burrill *et al.*, 2002).

Studies suggest that the increased conceptual learning that accompanies use of graphing calculators does not come at the expense of building facility in procedural skills (Burrill, 2004). Frequent use of graphing calculators tends to accompany more graph use and greater flexibility with representations, solution strategies, and reasoning with real data. More broadly, classrooms with graphing calculator use tend to foster a more constructivist climate, with more conjecturing, more frequent use of multiple solutions, and higher levels of discourse than those classrooms with infrequent calculator use. Using graphing technology for nonroutine activities, such as mathematical discovery and complex problem solving, tends to support increased conceptual understanding and higher achievement, while use of technology for routine calculations does not (Dugdale *et al.*, 2004).

Computer Algebra Systems

The next major breakthrough was the incorporation of computer algebra systems (CASs) into calculators. The CASs allow users to perform mathematical operations on symbolic expressions in support of problem solving, generalization, and reasoning about functions. The CAS made its way into hand-held calculators in the late 1980s with the HP-28, but it was when both Casio (the FX2.0, released in 1996) and TI released new models (the TI-92, released in 1995, and the TI-89 in 1998) that the technology became more affordable and more prevalent among high-school and then middle-school classes.

The promise is that CAS allows learners and teachers to focus on conceptual aspects of expressions and functions (Heid, 1988; Pierce and Stacey, 2007) rather than getting caught up with the mechanics of symbol manipulation, a benefit that may help low-performing students, in particular (Kuzler, 2000). The research does support this view, generally, though, as with other forms of calculators and technology, more generally, these influences are mediated by the types of lessons and instructional approaches. Scholars have shown that calculus students within conceptually oriented classrooms who used CAS demonstrated more conceptual knowledge than those who had skills-oriented lessons (Heid, 1988; Palmiter, 1991). CAS implemented in conjunction with other standards-based practices such as sense making and group discussions also supports college students' understanding and reasoning with symbolic expressions (Keller and Russell, 1997).

Geometric Supposer

In addition to support for skills in computation and symbol manipulation, there are powerful tools such as Geometric Supposer for supporting skills and concept development in geometric reasoning. Geometric Supposer is designed to support exploration and discovery of properties of Euclidean (plane) geometry by providing primitive operations for drawing, analyzing, measuring, and manipulating diagrams. Generalization across cases (induction) is supported by repeating operations on arbitrary exemplars, an experience which may assist students in formulating deductive proof. Geometric Supposer is one of the clearest examples of a system that instantiates many of the ideals of the constructivist philosophy of mathematics education because it allows direct access and construction to otherwise abstract objects, procedures, and concepts.

Although students typically struggle with using geometric diagrams, year-long use of Geometric Supposer has resulted in improvements with diagrams and students' understanding of the objects to which they refer, as well as the variety of ways that diagrams can be viewed and described (Yerushalmy and Chazan, 1990).

Cognitively-Based Tutoring

Some of the most exciting and well-researched technology-based systems come from the class of intelligent tutoring systems (ITSs). Drawing from cognitive science, artificial intelligence (AI), and expert systems, ITSs typically institute an expert module that contains, usually in rule-based form, knowledge of the skills involved in successful problem-solving behavior, and previously identified bugs, or common errors, in the rules of learners. In the ITS instructional model, the learner engages in problem solving while the expert module tracks learner behavior. Erroneous steps, recognized through pattern matching by the rule-based expert module, are usually met with immediate feedback, in keeping with the theory of the acquisition of cognitive skill. The expert module can also provide context-specific hints, or help, and offer an expert-level solution to the problem (Anderson, 1988).

While there are many ITSs, those derived from Anderson's (1996) ACT theory – the Cognitive Tutors – are of particular interest because of their strong theoretical and empirical support, and because the essential architecture has proven so resilient over time and across a range of mathematical domains and age groups. Cognitive Tutors (**Figure 2**) provide differentiated instruction in pre-algebra, algebra I and II, geometry, and integrated mathematics to 500 000 students in around 2600 US middle schools and high schools. Current systems incorporate classroom curricula (often 3 days per week), technology-based instruction and practice (2 days per week), and teacher professional development. Students engage in explanation-based reasoning, as well as goal-directed problem solving.

A number of empirical studies have established the effectiveness of this approach (see Ritter *et al.*, 2007, for a review). For example, Algebra I students in Pittsburgh, PA and Milwaukee, WI, who used the tutor showed superior gains from their peers overall, particularly on performance-based tests of problem solving and uses of multiple representations (Koedinger *et al.*, 1997).

Mathematical Discovery

As educational researchers more fully embraced expanded and alternative views of knowledge and learning such as constructivism (e.g., Cobb *et al.*, 1992) and situated cognition (e.g., Brown *et al.*, 1989), these have been reflected in the designs of educational technologies. New technologies were designed to be student centered, to draw directly from students' own knowledge of mathematical and physical

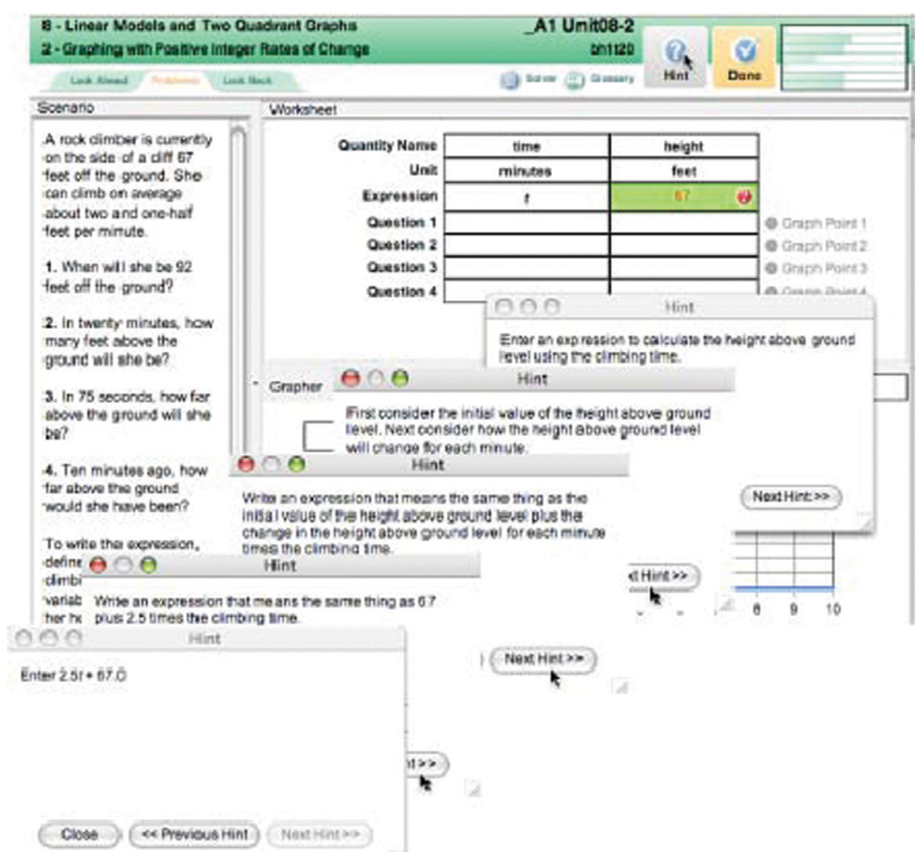


Figure 2 A series of hints, followed by direct instruction provided to a student using the Algebra Cognitive Tutor developed by Carnegie Learning, Inc. Adapted from Koedinger, K. R. and Aleven, V. (in press). Exploring the assistance dilemma in experiments with Cognitive Tutors. *Educational Psychology Review*.

phenomena, and to put more responsibility of discovering and articulating connections between representations and across representation, math concepts, and procedures. Technologies that emphasize the interconnections among representations and the distributed nature of knowledge as it exists between the student and the technology also lead to different types of activities and they reconceptualize the aims of math education as one of learner-centered making meaning.

Logo

Logo (Papert, 1980) is both a computer programming language and a microworld, a designed learning environment to promote mathematical reasoning and problem-solving skills through an innovative process of directing the actions of a mathematical creature called the Logo turtle. The turtle can move forward or backward, stop, and rotate to the left or right, and raise and lower its pen in response to programmed commands. Although the original turtle was a physical robot that ran along the floor or paper, in later versions it was replaced by a graphical turtle on a computer screen.

The Logo environment offers a way for the child to externalize mathematical ideas and procedures and project them onto the actions and properties of the turtle and the Logo programming language (Eisenberg, 2003). Yet, it also becomes an object-to-think-with (Resnick *et al.*, 1996) and has been used to conceptualize many areas of mathematics, including modern algebra and group theory, computer science, cybernetics, as well as Euclidean and non-Euclidean geometry (Abelson and diSessa, 1981).

Logo has long been a tool for doing mathematics and mathematics instruction, and there is a large collection of empirical studies investigating its impact on mathematics learning, teaching, and discovery. For example, fourth graders familiar with Logo programming were better able to apply what they learned and elaborate on their procedural interpretations of geometry concepts than those taught from an inquiry-based approach (Lehrer *et al.*, 1989). In other studies, Logo improved students' use of geometric models in other areas of mathematics, generalization and abstraction of geometric operations, and improved complex reasoning along with more general cognitive skills (Battista and Clements, 1991; Clements and Battista, 1991, 1992; Lehrer and Littlefield, 1993).

As is the case with educational technology, more generally, the effects of Logo have as much to do with the teaching and the engagement of the students, as the technology itself (Kozma, 1991, 1994; though also see Clark, 1983, 1994).

The essential ideas conveyed in Papert's (1980) original work, *Mindstorms*, inspired a broad range of technological designs for learning and instruction, including: StarLogo, which uses concepts of parallel computation to introduce participants to the computational and cognitive aspects of modeling complex, dynamic systems (e.g., Colella *et al.*, 1999); and the NetLogo Project (reviewed below) at Northwestern University and The University of Texas at Austin (Wilensky, 1999; Wilensky and Stroup, 1999), which supports distributed computing.

Function Probe

Function Probe (Confrey and Mahoney, 1991, 1996) was an early and highly influential technology design for math education to come under the constructivist paradigm. Function Probe integrated a user-friendly calculator with tabular, graphical, and symbolic representations of phenomena to promote the active construction of mathematical understanding for high-school algebra, trigonometry and functions, and high school and college pre-calculus, as well as integrated math and science instruction. Data could be entered in by students or imported using external sensors. Operations on one representation were reflected as changes in other, linked representations, thus promoting representational fluency (Nathan *et al.*, 2002). The system follows one of Kaput's (1989) observations, that mathematical meaning making is actually built upon the ability to translate within and among various representations, and that fundamentally, meaning is based on a "relational semantics" between "linking representations" including internal mental representations and physical systems as well as tables, symbols, and graphs (p. 168). Function Probe also provided a modeling environment that allowed students to articulate and explore their own conceptions of mathematical and physical events as a means toward advanced understanding (Confrey and Doerr, 1994).

ANIMATE

The ANIMATE system presented another alternative to cognitive skill acquisition and the intelligent tutoring system paradigm. It was presented as an unintelligent tutoring system to emphasize that the program contained no exert module and made no attempts at modeling or tracing the knowledge states of the student (Nathan, 1990). Instead, ANIMATE assumed knowledge was distributed among the interactions between the student and the system.

The focus of ANIMATE was on student discovery of quantitative representations for modeling and solving algebra story problems involving systems of equations, including those for distance-rate time (e.g., collision and overtake) problems, combined work, and compound interest. In ANIMATE, a student constructed the algebraic equations that drove an animation of the referent story problem situation (e.g., planes flying at different rates and leaving at different times). Because of the direct causal link between the formal expressions of the algebraic solution (the mathematical symbols and structures to be learned) and the animation (the situation-based meaning of the mathematical expressions), animated actions that were inconsistent with the student's mental model of the story situation suggested errors in the proposed solution representations, the nature of which were highly constrained by the type of misbehavior. This interlinking of students' thinking and the control of the system is illustrated conceptually in **Figure 3**. Students first proposed and then iteratively debugged their algebraic representations and tested them until an acceptable situation was depicted in the animation. Neither the student nor the system could solve the problems and make meaningful connections on their own. In this way, the intelligence was not localized solely in the computer program or in the learner, but rather from the intelligent interactions needed to bring the mathematics and animation in line with the student's mental model of the story problem.

Students who used ANIMATE learned to reason explicitly about the situations described in typical word problems, and performed better on paper-and-pencil transfer tasks. The ANIMATE users also tended to spontaneously correct their own algebraic errors during problem solving in far greater frequency than control subjects (Nathan *et al.*, 1992; Nathan, 1998).

Dynamic Geometry Systems

Dynamic Geometry computer software such as Geometer's Sketchpad (Finzer and Jackiw, 1998) and Cabri Géomètre (Laborde, 2000), along with Geometric Supposer mentioned above, offer alternatives to conventional proof-based explorations of geometry by supporting direct manipulation, tracing, and visual forms of thinking without the prior stage of re-representing the intended actions into natural or formal languages.

Classroom observations of Cabri use show that junior high school students in Japan using Cabri are more inclined to explore the propositions and theorems directly through construction and manipulation than under the traditional curriculum, they better visualize the geometric claims, and develop a better sense of what is to be proved (Namura, 1999).

While experimental results are scarce, studies do show that systems such as Cabri and Sketchpad help to promote the proper classroom environment, activities, and forms of interactions that foster deductive reasoning among students (Jones, 2000).

Dynamic Statistics Packages

Several recent dynamic systems allow students to delve into data analysis activities and statistical forms of reasoning without formal knowledge of probability and statistics or the conventions of Cartesian graphs. Data-oriented statistics instruction has moved into mainstream education where it is recognized as a critical methodological tool for reasoning about data with variability, engaging in scientific reasoning, making judgments and evaluating claims, and being an informed citizen. This

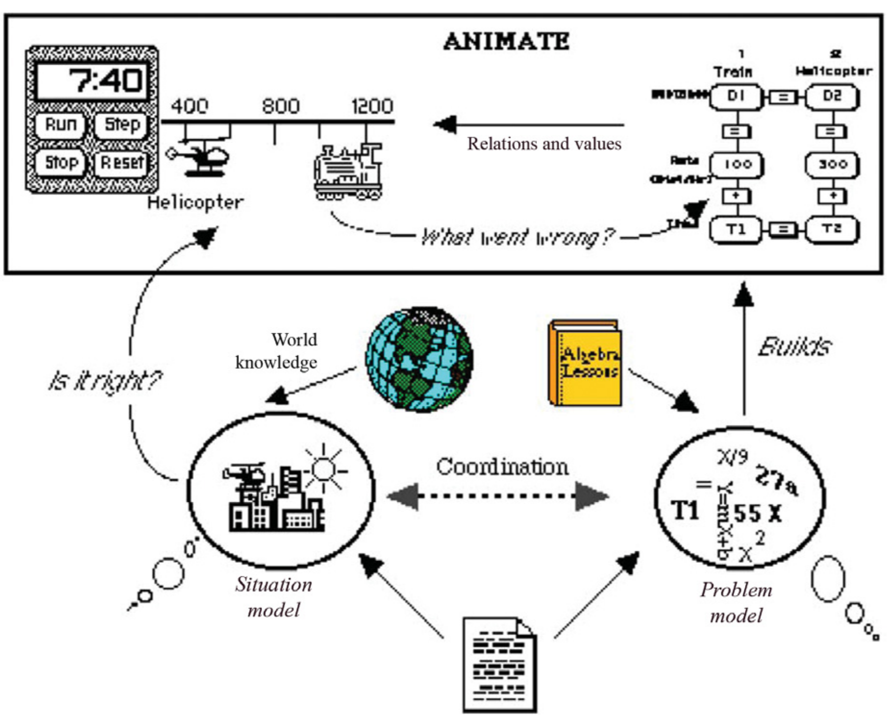


Figure 3 The conceptual underpinnings of learning to model story problems with the ANIMATE system.

reflects the growing appreciation of the need to understand and promote statistical literacy and reasoning more broadly (Ben-Zvi and Garfield, 2004; Cobb, 1993; Rumsey, 2002).

TinkerPlots (Konold, 2002) lets students approach data analysis questions conceptually and in a constructivist manner by providing functionality for organizing and representing data graphically and for progressively constructing and deconstructing graphs in order to develop a more grounded interpretation of the meanings intended by the representations. It also provides several visual methods for displaying variability and co-variation between variables, important ideas for analysis and modeling that are often difficult for students to grasp (Konold and Pollatsek, 2002). In a similar way, Fathom (Erickson, 2000; Finzer and Erickson, 1998) supports discovery of patterns through visually rich exploratory data analysis.

While no formal tests of the effectiveness of these systems compared to other methods have been reported, the approaches they offer are consistent with many of the recent prescriptions for mathematics education offered within current learning theory and standards-based educational reform.

Collaborative Problem Solving

Anchored Instruction uses extended collaborative groups and digital technology to present complex, real-life scenarios – often presented in extended, interactive videos – to situate problems and their solutions in meaningful contexts. The rich and engaging narratives serve as the anchors and give the ensuing mathematical problem-solving activities a meaningful connection to the world. Anchored instruction has been used for a variety of age groups, ability levels, and cultures.

The Adventures of Jasper Woodbury

In math education, the pioneering work on anchored instruction was implemented in *The Adventures of Jasper Woodbury* (CTGV, 1992, 1997). Jasper was developed by the Cognition and Technology Group at Vanderbilt University as a series of 12 open-ended, videodisk-based problem-solving activities that often took groups of elementary-, middle- and high school students several days or more to formulate extended solutions to the series' challenges. Generally, solutions required groups of students to make multiple passes through the information, a variety of plans and computational procedures, and clarifying assumptions. As one might expect, the rich contexts, problem-solving sessions, and clarifying assumptions resulted in complex solutions, no two of which were identical.

Experimental evaluations showed that students using the Jasper program exhibited comparable performance levels on basic mathematical concepts as matched controls, but superior performance on more complex single- and multistep word problems and multistep planning tasks (CTGV, 1992).

Teaching Enhanced Anchored Mathematics

Enhanced Anchored Mathematics is a form of anchored instruction that situates problems in authentic and meaningful contexts specifically to advance the problem-solving skills of low-achieving and special-education students, adolescents with emotional disabilities, and even re-incarcerated adults (Bottge *et al.*, 2003, 2007; Bottge and Watson, 2002). The anchored activities tie in many of the aspects of traditional industrial arts.

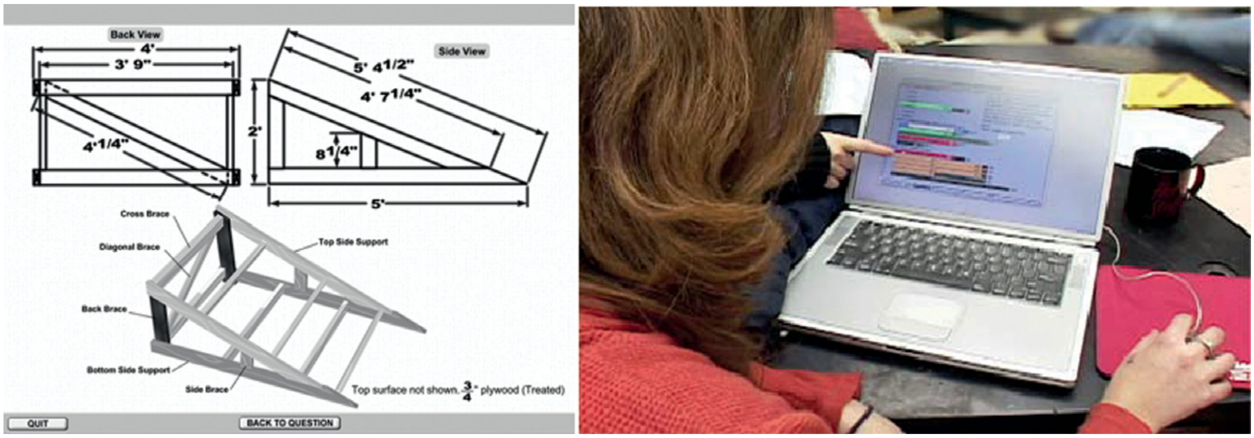
In Fraction of the Cost (Bottge *et al.*, 2002; see **Figure 4**), for example, students collaborate to build a life-size skateboard ramp with wood. Problems and information needed to solve the problems, along with irrelevant information, are naturally embedded in the narrative context. Students must use a variety of planning, budgeting, construction, and computational skills. Later, students face transfer problems such as building a working hovercraft.

In empirical research, classroom observers witnessed high and sustained levels of engagement among students of all math abilities, including those with a history of frequent off-task behaviors as they participated in multi-step mathematical reasoning, planning, and problem solving (Bottge *et al.*, 2002, 2003, 2007).

Networked Devices and Participatory Simulations

As technology has progressed to enable networked and distributed interactions, designers have used technology to facilitate students' roles in socially active, life-sized, computational simulations of dynamic systems. Many applications built in earlier decades have recently been reconfigured for networked, interactive uses. Hand-held graphing calculators, discussed earlier in the context of concept learning and skill acquisition, now also serve as one of the central means by which students access distributed networks. Technicians at TI, principally the Classnet Team, are largely responsible for one of the most widely used networking systems, the HubNet hardware, which serves as the central computer architecture through which distributed information is processed and aggregated. Networked systems used by the NetLogo Project at Northwestern University, The University of Texas at Austin, and the SimCalc Project at SRI and the University of Massachusetts at Dartmouth all draw on the TI graphing calculator platform.

Participatory simulations are among the most innovative of these new systems. They provide an individual, first-person perspective from inside the system itself, and



(a) (b)

Figure 4 (a) A schematic diagram showing the dimensions needed to build a skateboard ramp for the Fraction of the Cost activity from Teaching Enhanced Anchored Mathematics (TEAM). Adapted from Stephens, A. C., Bottge, B. A., and Rueda, E. (2009). Ramping up on fractions. *Mathematics Teaching in the Middle School*, **14**(6), 520–526. (b) A student using the TEAM system. Courtesy of Bottge.

of its emergent behaviors (Colella *et al.*, 1998; Resnick and Wilensky, 1998). The emergent behavior of the system at a macroscopic level and its relation to individual participant's microscopic actions can then become the object of collective discussion and analysis. For example, NetLogo is a programmable modeling environment for simulating complex systems dynamics that occur naturally in biological and social phenomena, such as traffic gridlock and flocking behavior. HubNet allows NetLogo to run participatory simulations in the classroom, where a whole class can enact the behavior of a system even while each student controls only a small part of the system from a networked computer or TI graphing calculator. Thus, participatory simulation activities support new forms of classroom interactions that explore and model complex mathematics typical of biological and social phenomena.

SimCalc

The SimCalc Project (Roschelle and Kaput, 1996; Hegedus, 2005) uses hand-held computer technology to democratize access to the mathematics of change and variation, including ideas underlying calculus (Kaput, 1994, 1997). The MathWorlds software, along with the hand-held technology supports computation, but also can represent mathematical ideas in ways that are important for conceptual understanding.

As with earlier tools, such as Function Probe and ANIMATE, the original versions of MathWorlds allows students to primarily control the motions of animated characters by constructing and modifying mathematical functions represented in graphical, tabular, or algebraic forms. Students can build up their understanding of the mathematics by seeing how changes in formalisms lead to changes in the corresponding animation. As an important extension from earlier dynamic systems that support proportional and linear functions, students are also asked to model stories that correspond to piecewise linear functions of familiar situations that represent different phases of action (such as resting).

As SimCalc has matured, its capabilities to support classroom connectivity have become more central (Kaput and Hegedus, 2002). SimCalc leverages the revolutionary aspects of wireless networked technology in several ways. It allows teachers or students to collect and display student responses, and thereby support large-scale forms of classroom interaction. As a mobile form of technology, SimCalc flexibly supports new kinds of social and participatory structures than were previously possible with tethered systems and those that require students to gather around a single monitor and keyboard (**Figure 5**). At the core of this connectivity is the use of hub technology that rapidly and wirelessly communicates to the teacher's computer.

SimCalc has been shown to be effective in randomized control-group studies at the middle-school and high-school levels. In a multiyear study of middle-school classrooms in Texas, SimCalc use led to statistically significant gains each year (Roschelle *et al.*, 2008). Detailed analyses showed that SimCalc students exhibited their gains on the most advanced math concepts, while showing no concomitant loss on basic material.

Serious Games

Games have been a long-running source of inspiration for mathematics activities and instruction, and gaming technology stepped easily into this practice as personal computers came on the scene and advances were made in interactivity (e.g., *How the West was Won*; Burton and

Figure 5 A teacher oversees students working with Simcalc.

Brown, 1979) and computer graphics (*Green Globs*; Dugdale, 1982). With the advent and popularity of video games, we have seen emergence of a variety of studies showing learning gains in mathematical reasoning from video game playing (e.g., Okagaki and Frensch, 1994; Subrahmanyam and Greenfield, 1994; Wenglinsky, 1998). While important, these are often unconscious and indirect effects of learning rather than the intended purpose behind the game design.

Some educational scholars have argued that education has much to learn from video game design (Gee, 2003/2007; Shaffer, 2007; Squire, 2006). Serious games exploit many of the compelling attributes of video games that have stoked their popularity. These are advanced computer-based environments with sophisticated graphics and sound that engage the learner/player with a compelling narrative about the video environment and support self-motivated progress, but are specifically designed to support learning rather than entertainment.

While a few, early serious game systems exist (e.g., *Quest Atlantis*, *Civilization III*, *The Triple A Game Show*, *Revolution*, and *Mad City Mystery*) it is still early to point to a body of research literature showing consistent learning gains in mathematics from this approach. Still, for many reasons, both theoretical and pedagogical, this is a promising and rapidly evolving area of study (Barab *et al.*, 2005; Squire and Jenkins, 2003).

Embodied Cognition

One of the traditional criticisms of educational computing is that it distances students from physical, hands-on activities and experiences, and perpetuates a view of the math learner as a disembodied information processor. Within the embodied cognition view (e.g., Barsalou, 2008; Glenberg, 1997; Lakoff and Nuñez, 2001), however, technology should allow people's thoughts and actions to mediate the relationship between real-world phenomena and formal representations.

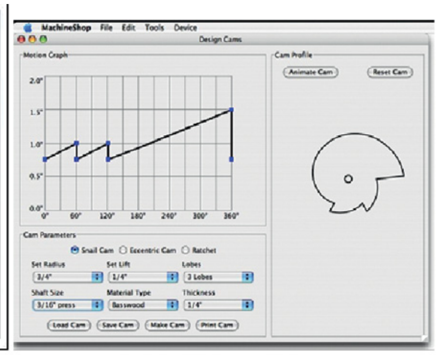
CamMotion (Boyd and Rubin, 1996) fosters this relationship by providing users ways to extract and analyze data directly from digitized video of objects and events. HyperGami (Eisenberg and Eisenberg, 1998, 1999; Eisenberg and Nishioka 1997) lets students create customized 3-D polyhedral forms on the computer screen that are printed as flat, colorful patterns, but that fold to became tangible models, such as penguins.

Recent advances in new and powerful output devices permit students to design (on the computer) and then print objects in sturdy materials such as wood, acrylic, foam core, wax, and plaster. Advances in materials science and developments in plastics, liquid crystals, and optical fibers also invite new ways of using the hands and body to engage in mathematics, and in so doing, recast the very notion of educational technology (Eisenberg *et al.*, 2005).

This greatly expands the range of mathematical objects and techniques available to students. For example, laser printing can be used to make sliceforms that can be slotted together to form mathematical objects in 3-D that are both educational and esthetic (**Figure 6**(**a**)). The approach can be used to support proof by construction. It can also be used on fabric, where mathematical patterns can inspire fashion. Laser printing is also instrumental to the MachineShop program (Blauvelt and Eisenberg, 2001, 2006), which directs a laser-cutting device to make toothed gears, cams, and levers from wood based on custom-designed mathematical functions that can then be assembled to make devices, including toys and models of dynamic systems (see **Figure 6**(**b**) and **6**(**c**)).

These approaches to tangible mathematics – and there are many more to be reviewed – recall a time when mathematics and art were closer than they are today for many students. Mathematically inspired crafts also encourage a culture of display (Eisenberg *et al.*, 2005). Thus, mathematical craftwork takes the idea of grounding the meaning of mathematics one large step forward, by inviting students to personalize mathematics and enrich our immediate surroundings with beautiful and interesting mathematical entities that we design and construct. Combined with the aims that focus on developing students' spatial reasoning and conceptual and procedural advancement, this is a valuable reminder of what math education can become.

(a)



(b)

(c)

Figure 6 (a) A sliceform ellipsoid constructed from a set of slotted wooden pieces. (b) A custom-designed cam from MachineShop. (c) Some mechanical automata designed and built by students. Adapted from Figure 3 of Eisenberg, M., Eisenberg, A., Blauvelt, G., Hendrix, S., Buechley, L., and Elumeze, N. (2005). Mathematical crafts for children: Beyond scissors and glue. *Proceedings of Art+Math=X Conference*, pp. 61–65, CO: Boulder.

Challenges Facing Technology Supports for Acquiring Mathematics

New views of learning and behavior are contributing to new forms of technology as the interactions between technological tools and tool users are reconceptualized. While *Jasper Woodbury* series, Cognitive Tutors, and SimCalc are notable exceptions, the empirical research base for many of the technological innovations reviewed is still thin. Most evaluations could benefit from qualitative investigations that document implementation fidelity and the learning process alongside more conventional quantitative studies of assessment performance.

Technology enacts a ratchet effect (Tomasello, 1999) on mathematics education, with intellectual advancements supporting the democratization of mathematics for all learners (Kaput, 1994). Yet new technologies introduce new costs to education. First is the cost of the technologies themselves, as well as adequate technical support. Second, new technology calls for new forms of teacher support that must be ongoing and systemic to the educational institutions and it must provide direct connections between the technology and the mathematical content that it is designed to support. To adequately meet the potentials these new opportunities afford, teachers need additional training. Third, the new technologies present entirely new areas of mathematics (e.g., dynamic systems and inferential statistics) and are shifting prior topics into earlier grade levels (e.g., algebra and calculus). Finally, assessments of student learning, as well as the curriculum and professional standards will need to be revised to keep pace, or fall seriously out of date.

See also: Classroom uses of Technology to Manage Instruction.

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