Building Cohesion Across Representations: A Mechanism for STEM Integration

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Abstract

Purpose The mechanisms of integration of science, technology, engineering, and mathematics (STEM) remain largely underspecified in the research and policy literatures, despite their purported benefits. Our novel claim is that one key mechanism of STEM integration is producing and maintaining *cohesion* of central concepts across the range of representations, objects, activities, and social structures in the engineering classroom.

Method We analyze multiviewpoint videos of multiday classroom activities from Project Lead the Way (PLTW) classes in digital electronics in two urban high schools.

Results To forge cohesion, teachers use *coordination* of representations, tools, and materials, and they use *projection* to reference places and events, past and future. Teachers also perform explicit *identification* to label central invariant relations that are the conceptual focus of their instruction. Teachers typically perform identification, coordination, and projection on the particular STEM representations used in projects in order to improve the cohesion of the conceptual content of a curriculum unit. Teachers can also represent the larger sequence of project activities within the curriculum to construct a cohesive account of how the various activities and representations relate and build upon key ideas.

Conclusions This paper found that cohesion-producing activities promote student understanding by threading conceptual relations through different mathematical representations, scientific laws, technological objects, engineering designs, learning spaces, and social structures. In these ways, cohesion can promote STEM integration in the engineering classroom.

Key words Pre–college engineering education; representational fluency; STEM integration

Introduction

A central challenge for high school engineering students is maintaining an awareness of key mathematics and science concepts as they thread through the steps of the typical project design cycle. Operating throughout the life cycle of a project, these central concepts can become unrecognizable to students, because the concepts appear in dramatically

Journal of Engineering Education http://wileyonlinelibrary.com/journal/jee
January 2013, Vol. 102, No. 1, pp. 77–116 DOI 10.1002/jee.20000

different representations and contexts, even though the concepts themselves may refer to relatively immutable relationships, such as universal physical laws and mathematical theorems. In this article we develop a framework for discussing and analyzing *cohesion* by focusing on the obstacles that inhibit students in project-based engineering classrooms from noticing these connections, and by describing specific ways teachers' pedagogical actions can help students to perceive these links. Often, the important links for students to make are those that connect the engineering activities with key concepts from science and mathematics. Therefore, cohesion-based accounts of learning and instruction can serve as an effective mechanism for fostering science, technology, engineering, and mathematics (STEM) integration in the classroom.

Representations in Engineering Education

External representations are important in engineering professional practice and educational settings because they transform concepts and processes into symbolic and visual forms that are intended to stand for ideas, objects, and relations. When we talk about representations in engineering and other STEM fields, the tendency is to think of them as narrowly encompassing formalisms (Nathan, 2012) such as equations, vector diagrams, and graphs. These standardized symbolic expressions and rules for interpreting and manipulating them are part of the professional discourse and practices in STEM, and they serve as common elements of interdisciplinary STEM collaboration (e.g., Stevens & Hall, 1998) and STEM integration (Schunn, 2009).

Given the privileged status of representations in the STEM disciplines and in society more broadly, we consider some of the traits that make conventional representational systems so important to STEM activity. One trait is their expressivity. Representations can convey complex relations in concise ways. Second is their generalizability. Representations can be used to notate a family of actual relations, thereby documenting relations that carry over different values and settings and that give mathematical models their predictive capability. A third trait is that representations promote connectivity. They can serve as shared objects across STEM disciplines (e.g., Hall, Stevens, & Torralba, 2002), and they can promote collaborative discussion and problem solving (Nathan, Eilam, & Kim, 2007; Schwartz, 1995). Schunn (2009, p. 5), for example, points to the special role of mathematics notation in STEM as "the language of physical sciences and engineering sciences" that is uniquely capable of fostering interdisciplinary collaboration.

In this article, we adopt an embodied perspective on the study of behavior that acknowledges the sociocultural and situated nature of STEM practices and classroom learning (e.g., Greeno, Collins, & Resnick, 1996; Shapiro, 2011). Central to this perspective is the view of "understanding and learning in terms of people's participation in practices of inquiry and discourse that include interactions with others and with the material, symbolic and technological resources in their environment" (Kozma, 2003, p. 206). From this standpoint, it is not adequate to regard representations solely as symbol systems that draw upon preestablished and set rules of grammar for parsing and production. An embodied perspective on STEM classrooms includes social norms of representation use, the physical and perceptual qualities of the notational systems, and the naturally perceived actions (i.e., their affordances; Gibson, 1979; Norman, 1999) these notations elicit, as well as the manners of speech and social interaction that are organized around the STEM activities (Roth, Bowen, & McGinn, 1999). An embodied perspective explicitly acknowledges ways that conventions for interpreting and applying representations vary depending on the setting, goals, and participants in the STEM activity (Greeno & Hall, 1997). For example, the social interactions that occur in the presence of a symbolic equation may be very different when that equation is presented by a teacher as part of a formal lecture, used by an individual student solving homework problems, or used by a group of students collaborating in a machine shop. In this sense, representations are not primarily specified by their denotative function and abstract qualities. An embodied perspective on the nature of STEM representations includes attending to how representations are perceived and used, and their roles within rich social and physical interactions (White & Pea, 2011).

Cohesion Across Engineering Activities and Representations

In problem-based and project-based learning as it is commonly practiced in K–12 engineering and science curricula, students must learn to perceive the continuity of central concepts in science and math as they are presented via a variety of representations, such as equations, graphs, diagrams, models, and simulations (Kozma, 2003). The range of representational forms is sufficiently vast that scholars on the sociology of science such as Latour (1990; also see Lynch, 1990) often combine any manner of externalized drawing, writing, and graphical notational system used in the service of intellectual activity under the general term *inscriptions* (e.g., Roth & McGinn, 1998).

Gainsburg (2006) notes in her ethnographic study of structural engineers that equations, diagrams, and other such formalisms make up an important though small part of the practices of engineers and engineering students. Engineering practices also include, for example, concrete depictions of important relations that significantly influence engineers' analyses and decision making. As Johri, Olds, and Roth note, "some of our most essential skills in engineering" arise out of engagements not only with formal representations, but also with tools, materials, and other people (Johri & Olds, 2011, p. 163; also see Hall & Nemirovsky, 2012). As an example, in a unit on ballistics, we can observe the teacher lecturing and writing equations that model the laws of kinematics, which are then used to derive relations between initial velocity and distance traveled (Nathan, Alibali, Wolfgram, Srisurichan, & Walkington, 2011b). These formalisms serve as representations for the prevailing models of ballistic behavior in that they stand for the behavior of projectiles. Later, we see the teacher make a hand gesture toward the classroom whiteboard while critiquing a group's design of a catapult when inquiring about the connection of the design to the kinematics laws. Through gesture, the whiteboard actually serves as a representation of the equations since, in this context, the reference to the board is intended to invoke the kinematic laws. In a later class, a similar reference to the whiteboard is used in class even though the content of the board has been erased. In this way, we can see complex representational chains where symbols and actions stand for physical phenomena, and objects stand for abstract symbol systems. Thus, the intellectual roles of referencing, analyzing, and modeling physical phenomena typically ascribed to formal representations are, more accurately, accounted for by interactions with many types of representations in a variety of settings.

Furthermore, students must come to recognize what is invariant about key math and science concepts as a concept is represented using different symbols and materials and in different social interactions (e.g., lecture or group work). For example, engineering students typically must be able to notice and maintain common conceptual threads though formal lectures on physics and math, 2-D drawings, 3-D CAD designs and simulations, fabrication of the parts of a working device in a machine shop, and on to assembling, testing, and measuring the operation of the device in a lab and in the field. Data collected from these measurements then need to be interpreted in relation to the original problem statement, analytic models, and performance goals so that changes to the construction and design can be made to improve performance, cost, or other parameters of interest. Across these many changes in representation, social organization, material presentation, and physical location, students must construct and maintain cohesion of the key mathematical concepts and physical relations that connect these phases into a cohesive whole. In our usage, *cohesion* addresses the extent to which connections are perceived between elements of the classroom environment that are relevant to students' comprehension and learning. This usage follows from research in reading comprehension and learning from text, where cohesion addresses how idea units connect to one another as one moves through a passage (Graesser, McNamara, Louwerse, & Cai, 2004).

Terminology

The central aim of this article is to explain the challenge of maintaining cohesion throughout project-based learning activities as they occur in high school engineering classrooms. In order to describe behaviors of teachers and students as they attempt to maintain cohesion across representations, events, and ideas in project-based lessons, we need to introduce some terminology. First, we need an all-inclusive term for the many depictive forms that engineering students encounter and use that encompasses the range of notational systems, objects, and tools, as well as the social configurations and spaces in which students are situated when they interact with these entities. Throughout this article we will use *representations* in this broader manner.

We also need a term to describe the process of recognizing and building connections across representations, activities, and social structures. For this we use *cohesion* (Nathan et al., 2011b; Walkington, Nathan, Wolfgram, Alibali, & Srisurichan, in press). A theoretical focus on producing cohesion in the classroom and maintaining cohesion across a host of project-based experiences means specific attention is directed at how representations in the external environment of the classroom, laboratory, or fabrication shop connect to one another. One way of producing cohesion is to provide explicit links across different representations and activities that convey their shared conceptual structure (Nathan et al., 2011b; Walkington et al., in press). For example, a teacher may point to a coefficient in an algebraic equation and then use her forearm to depict steepness of a line on a graph, linking the value of a numerical coefficient to its shape on a coordinate graph (Alibali & Nathan, 2012). Another way to promote cohesion is to link across time, by connecting representations, objects, and events that were presented earlier as they occurred in different project stages or even earlier lessons. For example, during engineering projects, past references are often made to mathematical formalisms such as Pythagoras's theorem and the quadratic equations for parabolas that model the ideal physical behavior of the system, while future references are made to alert students to pending situations, such as the conditions and expectations for testing the device.

Following prior work in this area (Nathan, Alibali, Wolfgram, Srisurichan, & Felton, 2011a; Walkington et al., in press), we recognize three primary methods teachers and students use to produce cohesion. Speakers may use *coordination* to explicitly show the association between representations, objects, and events that are simultaneously present during an activity. In the algebra example mentioned above, the teacher coordinates the numerical coefficient describing the slope of a linear function with an arm motion that also describes the slope, but in a different form. However, when the representations that we want to connect are not all present, we often need to refer to them backward or forward in time. When students or teachers use *backward projection* (Engle, 2006), they call upon their prior experiences. A teacher can subsequently reference the concept of slope or the slope parameter in an equation simply by raising her arm in an angular manner. Such backward projections can support the review of past lessons and facilitate reflection on thoughts and prior knowledge. *Forward projection* refers to instances when participants make connections between current events or representations and future activities. For example, the teacher in the catapult lesson might point out the importance of the mathematical equations to the designs students are creating by describing how the equations will be useful in future activities of analyzing and testing their devices. Forward projections are often planned connections and can be used to foreshadow the relevance of a concept to an area of application or association. Empirically, projection can facilitate learning and transfer by allowing new ideas to become grounded in prior experience and by preparing students for future learning (Walkington, Srisurichan, Nathan, Williams, & Alibali, 2012).

Finally, we need to refer to the underlying mathematical and scientific concepts as they are referenced using different names, locations, and representations. We use *invariant relations* to refer to those properties of the "deep structure" (Bransford & Johnson, 1972) that are of importance to the engineering projects and that remain consistent, even when some or all of the outward qualities have changed. For example, in an investigation of a unit on projectile motion from an engineering class in the Project Lead the Way™ course, Principles of Engineering (Nathan et al., 2011b), there is a demonstrated need to characterize the relation between the angle of ascent of a projectile and the distance traveled. This is an invariant relation, because it maintains some degree of consistency whether it is represented in a mathematical equation, graph, table of data, or in the actual performance of the physical device.

STEM Integration

With these terms in hand, we offer the following thesis: Together, projection and coordination create cohesion-producing opportunities for students that can foster STEM integration by enabling learners to thread mathematical and scientific invariant relations through disparate engineering activities, representations, and social structures. Given its importance to this study, we now review the current research on STEM integration.

Research on STEM Integration

Integration of the individual domains of science, technology, engineering, and mathematics is recognized as a central aim of engineering education reform. STEM integration breaches the historical, disciplinary "silos" of the individual STEM domains by addressing "the natural connections among the four subjects, which are reflected in the real world of research and technology development" (Katehi, Pearson, & Feder, 2009, p. 12). STEM integration is considered in policy statements to be necessary to promote advanced scientific studies, to increase and diversify the pool of capable and motivated employees in technology and science economic sectors, and to prepare technologically knowledgeable and competent citizens (Katehi et al., 2009; NRC, 2005). In developing the pioneering Integrative STEM program at Virginia Tech, Sanders and Wells also note that the ideals of STEM integration are not likely to be fulfilled by the integration of any pair of STEM fields. They specify that "Integrative STEM Education refers to technological/engineering design-based learning approaches that *intentionally* integrate content and process of science and/or mathematics education with content and process of technology and/or engineering education" (Sanders & Wells, 2006–2011, p. 1). This definition considers the pairing of technology with engineering (the design sciences) as insufficient to satisfy STEM integration, and also excludes pairing science and math (the natural sciences). Rather, it calls for STEM integration that spans the design and natural sciences.

Though definitions and descriptions of STEM integration vary, some traits have gained broad acceptance in STEM education. Foremost, curriculum content across the STEM fields is integrated rather than merely combined (cf. Dyer, Reed, & Berry, 2006; Satchwell & Loepp, 2002). To Schunn (2009), the STEM integration curriculum reveals a synergy that goes beyond the constituent parts. This content integration (Roehrig, Moore, Wang, & Park, 2012) merges fields and thereby reveals big ideas that transcend specific disciplines.

There is empirical support for positive effects on learning with curricula that provide STEM integration (Burghardt & Hacker, 2007; Fortus, Krajcik, Dershimer, Marx, & Mamlok-Naaman, 2005; Hartzler, 2000; Kolodner et al., 2003; Satchwell & Loepp, 2002; Phelps, Camburn, & Durham, 2011; Wang, Moore, Roehrig, & Park, 2011). An integrative approach to STEM education is also supported by learning sciences research on transfer of knowledge (e.g., Pellegrino, Chudowsky, & Glaser, 2001; Sheppard, Pellegrino, & Olds, 2008). Yet the effects of an integrative STEM education on student learning are uneven (Hartzler, 2000; Prevost et al., in press; Tran & Nathan, 2010a, 2010b), and high-quality implementations of STEM integration are not commonplace (Katehi et al., 2009; Nathan, Tran, Phelps, & Prevost, 2008; Prevost, Nathan, Stein, Tran, & Phelps, 2009; Prevost, Nathan, Stein, & Phelps, 2010; Welty, Katehi, Pearson, & Feder, 2008).

Regardless of the empirical results, there are powerful economic, social, and political forces driving rapid proliferation of education programs promoting STEM integration. This proliferation comes in part from federal initiatives and funding sources, such as Race to the Top (Chang, 2009), legislation such as the 2006 Reauthorization of the Perkins Career and Technical Education Act (Public Law 105-332, 1998), and national policy documents (e.g., Committee on Standards for K–12 Engineering Education, 2010; NRC, 2003). However, many of the elements that are regarded as necessary for providing and maintaining the ideals of STEM integration and achieving buy-in from stakeholders are simply not yet in place in K–12 education (Chandler, Fontenot, & Tate, 2011). These elements include teacher certification and professional development programs, successful models of cross-departmental interactions, cohesive curricula, assessments of integrative thinking, and professional teaching standards, among others.

STEM integration is central to many facets of K–12 engineering education and strongly endorsed in the research and policy literatures. Yet, our review of STEM integration research and policy reveals an important gap in our understanding; namely, that the integration process, while touted for its benefits and broad appeal, remains somewhat mysterious. Studies showing advantages of integrated curricula on student performance typically show only relative benefits over business-as-usual models rather than explaining the ways students' actions and reasoning processes have changed. How STEM integration occurs, whether integration is chiefly about instructional practices or the knowledge states of students, and what demonstrable impact integration has on performance, engagement, and learning, all remain largely underspecified. Without a clearly defined construct of STEM integration, scholars and policy makers cannot be certain that studies purported to show benefits (or those that do not) are all testing comparable interventions and outcomes. Similarly, teacher education programs lack guidance (Should everything be integrated?). The current state of knowledge also makes it difficult to specify what it is about STEM integration that actually promotes the kind of synergy called for in policy and research reports.

One possible means of achieving STEM integration in the classroom is through learning experiences that foster cohesion production. Many forms of cohesion, however, are not directed at STEM integration (such as the within-discipline example above that linked an algebra equation to an algebraic graph). However, one account of STEM integration, guided by the tenet advocated by Sanders and Wells (2006–2011) for reaching across the natural and design sciences, involves producing cohesion by linking science and mathematics with engineering ideas and representations that share the same conceptual structure. In the next section, we delve into ways cohesion is managed in pre-college engineering classrooms and report our empirical findings. Cohesion offers a viable account of how STEM integration can be brought about and shows how striving for integration influences students' and teachers' actions. A focus on cohesion production can also reveal ways that STEM integration supports meaning making within engineering practices. A cohesion-based account of instruction and learning yields key insights into some of the challenges of promoting STEM integration in the engineering classroom.

Challenges to Building Cohesion

Curricula designed around a broad set of hands-on, collaborative activities are generally assumed to be beneficial to learning, because they provide students with a varied set of experiences with ideas, representations, tools, and skills that foster rich interactions around disciplinary and interdisciplinary ideas and practices (Bransford, Brown, & Cocking, 2000; Johri & Olds, 2011). Yet, there are also ways in which these complex contexts and activities place heavy demands on students. For example, the invariant relation in the ballistics lesson describing the relationship between angle of ascent of a projectile and the distance it will travel may be instantiated in many ways: a teacher's raised arm hovering over the base of a hand-drawn triangle, a Greek symbol *theta*, a numeric measure, a tangent line meeting a plane, and the relation between the trajectory of a moving object and the Earth. In these cases *theta* is realized, respectively, by the description of the flight of a ball during a lecture, an algebraic equation, a sextant, an idealized diagram in analytic geometry, and the operation of a catapult.

Although engineering experts, curriculum developers, and teachers may perceive and understand these invariant relations across the range of representations and activities, students often struggle to make the connections (Kozma, 2003). Empirical observations of classroom learning and instruction have exposed two trouble sources. In one, students exhibit a narrow sense of the math or science concepts, which limits their perceptions, actions, talk, and ways of representing and reasoning about the concepts. In this way, their epistemological orientations – their very ideas about what constitutes knowing a conceptual relation – are tied to the specific surface structure of that concept (Kozma & Russell, 1997; Nathan et al., 2011a; Walkington et al., in press). Rather than acquiring the abstract relations reflected by the deep structure, students are fixated on the modal-specific qualities of how the representations and objects appear. These modal-specific orientations are evident when students orient their talk and actions toward the observable properties of the current activity or representation and do not consider any broad connections with other project activities. For example, during an engineering unit on bridge building, high school engineering students tended to focus on the concrete and immediate forms of their balsa wood bridges, without acknowledging how the designs and performances of their bridges were linked to previously covered concepts from physics such as tension and compression. In contrast, when cohesion is highlighted during the lessons, the objects and representations can be recognized as instantiating a previously encountered invariant relation. For example, one can interact with drawings in a multitude of ways. In the projectile motion unit, students can build cohesion of the invariant relation between angle of ascent and the trajectory of a projectile by linking equations for the angle *theta* to the design drawing. Achieving this coordination between the equation and the drawing alters the ways students view elements of the drawing, so that students talk and gesture about it in ways that start to conform to the laws of kinematics.

A second trouble source was observed when students encountered ecological shifts in the classrooms. These are broad shifts in class organization, activity structure, or venue. For example, in the bridge building lesson, students encountered an ecological shift when they transitioned from listening to a teacher lecture on statics to working in small groups to build their bridges. Such breaks of continuity can impede cohesion production. On many occasions students simply failed to see the relevance of prior phases of a project when they experienced ecological shifts (Nathan et al., 2011a).

Producing and Maintaining Cohesion in the Classroom

In their analysis of classroom interactions in pre-college engineering, Nathan and colleagues showed the important role of cohesion in mechanical and electrical engineering (Nathan et al., 2011b), bridge design and testing (Walkington et al., in press), and geometric proof (Nathan et al., 2011a). Coordination and projection were often observed working in tandem. It is likely that coordination and projection contribute to cohesion in somewhat complementary ways that foster content integration (Roehrig et al., 2012). Coordination provides explicit links that allow students to perceive the deep structure of mathematical representations and connect these ideas to concrete referents. Forward projections to future events can *orient* the learner to upcoming activities, representations, and ecological shifts, *prime* relevant prior knowledge, *facilitate* planning, and *highlight* the importance of current concepts that can promote preparation for future learning. Backward projections to past events and representations emphasize review, reflection, and seeing the big picture.

Research Question

The production and maintenance of cohesion during project-based engineering classes offers a glimpse into the ways that key STEM concepts can transcend their particular modal-specific surface structures and their discipline-specific silos in order to connect reasoning and goal-directed action across the range of classroom interactions and representations. Building these connections across representations supports integration across STEM fields. Yet analyzing classroom learning with cohesion in mind also reveals some of the inherent challenges that engineering students and teachers face in actively constructing these connections. We present analyses of videos from two urban high schools to better understand the challenges that engineering students and teachers face, and to document how these connections are made and supported *in situ* through instruction and peer interactions. This leads us to ask, How does a focus on cohesion production help us to identify successful and missed opportunities for student understanding and integration of STEM concepts in project-based engineering classrooms? By attending to cohesion, we seek to generate insights into the process by which students come to assign meaning to representations and activities in the context of collaborative, project-based learning experiences. We also seek to develop a new perspective to advance our understanding of the challenges teachers face in their efforts to promote STEM integration, and to suggest ways instruction can become more effective. We address these aims using data on the rich interactions that unfold in the complex environments of high school classrooms.

Method

Participants and Settings

This study draws from observations of the Project Lead the Way (PLTW) Digital Electronics classes at two urban high schools in the midwestern United States. Elm High School (EHS) is a large, four-year high school in a city of under half a million people, with moderate levels of cultural and socioeconomic diversity (for student demographics, see Table 1). Students in this elective class are in grades 10 to 12. The EHS classroom teacher attended several colleges and acquired a variety of post-secondary degrees, including: an associate's degree in mechanical design, bachelor's degrees in both building construction and industrial arts (with minors in math, science, and graphic design), and a master's degree in computers and technology. He also had a range of experience in business and various technical professions before he went into teaching. The EHS teacher currently has a teaching certification in vocational education. At the time of data collection, the EHS teacher had taught career and technical education for nine years, and had been teaching in the PLTW program for four years, including the courses Principles of Engineering and Digital Electronics. The EHS class was comprised of 18 registered high school students from a range of grade levels. The Digital Electronics class at EHS typically involved teacher lectures followed by students working individually on various tasks.

Table 1 Demographics of the School Sites Elm (EHS) and Redwood (RHS) High School

	RHS (%)	EHS (%)
White (Caucasian)	4.2	48.5
African-American	66.7	28.0
Hispanic	16.0	12.5
Asian	9.0	10.5
Native American	3.1	0.5
Other/No Response	1.0	0
Free/Reduced Price Lunch	72	24

Students would complete worksheets covering principles from digital electronics and would design circuits to solve real-world problem scenarios using simulation software, diagrams, and breadboards. The teacher's instructional style was didactic, meaning that when assisting students one-on-one, he would provide heavy scaffolding and often gave direct answers to student questions.

Redwood High School (RHS) is located in a large midwestern city, which serves as the hub of a metropolitan area of about 2 million people. The school and school district is more ethnically diverse than EHS and has a greater proportion of families who qualify for the federal free/reduced price lunch program (Table 1). Students in this class are in grades 10 through 12. The instructor for the Digital Electronics course has a bachelor's degree in electrical engineering and a master's in mathematics education (secondary level). He is a member of the high school mathematics department. At the time of the study, the RHS teacher had five years of experience teaching courses in the PLTW program. He is a native Spanish speaker and is bilingual, as are many of the RHS students (though Hmong is also spoken regularly in the classroom during small group work). Consequently, portions of the class discourse were in Spanish. Nineteen students registered for the RHS class we observed. The digital electronics class at RHS, although following the same PLTW curriculum, was a very different environment than EHS. The students worked in groups on project-based activities, and the teacher explicitly focused on building and using effective collaboration skills. The teacher's style of instruction was to direct questions to students in order to push them to explain their actions and their reasoning. The teacher usually avoided giving students direct answers when they ran into issues, encouraging them to reflect on their work and take initiative in figuring out solutions. This class was somewhat less orderly in RHS than in EHS, and some students struggled with the autonomy given them.

Methodological Perspective

This research is conducted from a learning sciences perspective (Nathan & Alibali, 2010; Sawyer, 2006). We assume that knowledge and action are socially constituted and are situated in the embodied, material, social, and cultural ecologies under observation (Jordan & Henderson, 1995; Kozma, 2003). We further posit that culturally constituted meanings of representations and practices stem from the scientific disciplines and school settings.

Data collection and classroom activities The data for the current investigation were obtained from video records of the classroom from two camera angles, both mobile, along with field notes, analysis of curricular artifacts (e.g., worksheets, simulation software, and electronics kits), and interviews with the classroom teachers after the videotaped lessons.

We observed EHS over four contiguous days while students participated in a unit on a voting booth security system. The problem statement framed the activity, "For privacy reasons, a voting booth can only be used if the booth on either side is unoccupied." An effective monitoring circuit is indicated by two outputs: a green light-emitting diode (LED) that is activated whenever a particular voting booth is available for use, and a red LED that lights up whenever privacy is at risk and entry is denied. The circuit design involved implementing the basic set of logical constraints and conditions into a working electronic circuit that outputs a green light when all of the conditions are met, or a red light (alarm) when any condition is violated. The process unfolded sequentially:

1. Introducing the problem in words, along with a block diagram representing the function of the monitoring system, and an equipment list;

2. discussing a completed truth table with entries composed of 1s and 0s accounting for all of the possible states of the circuit (voting booth occupancy and LED output) and a related, spatial Karnaugh map (K-map);
3. generating and manipulating a set of Boolean algebraic expressions consistent with the K-map;
4. drawing an automated optical inspection (AOI) circuit;
5. modeling the circuit in the Multisim software (developed by National Instruments, Corp.) to create computer generated circuit diagrams that can simulate the behavior of a working electronic circuit; and
6. building and debugging a working electronic circuit made of a breadboard, integrated circuits, resistors and capacitors, wires, a power source, and LEDs.

We observed RHS over three consecutive days while students participated in a unit on designing and building a digital circuit that could address the majority vote problem. For this task, students must build a circuit that correctly displays the voting outcomes of a four-member board of directors (president, vice president, secretary, and treasurer; represented, respectively, by P, V, S, and T), where ties are resolved in favor of the president. The problem statement reads, "To avoid a tie in voting, the president is given two votes, and all other members must vote. For a motion to carry, three "yes" votes were required ('yes'=logic 1, 'no'=logic 0). Otherwise, the motion fails to carry." Students were then directed to develop a truth table, determine the logic equation using Boolean algebra, simplify the Boolean expression using K-maps to provide a simplified circuit design, draw the circuit, build and test a simulation of the circuit, and wire and test the electronic circuit using breadboarding.

Procedures and units of analysis For each day's lesson, two camera views were synchronized for viewing. One camera followed the teacher, capturing his interactions with students as he assisted them on their project work. The other camera provided a close-up view of the materials or representations the students were currently working with – their Boolean equations, circuit design drawings, computer screen, or breadboard. A single verbal transcript was generated from the audio channels, and gesture descriptions and screen shots were added to key parts of the transcript, to allow for gesture analysis (cf. Nathan et al., 2007). The transcript was time synched with the synchronized, dual video stream using the Transana software system. This arrangement allowed for yoked navigation through the data so that any position in either video channel would immediately call up the appropriate place in the transcript; reciprocally, selecting any location in the transcript would call up the appropriate video frames from both cameras (see Figure 1 for an example).

Each video transcript was then segmented at two levels through multiple passes. At the macroscopic level, ecological shifts were identified as those points where a noticeable break in continuity of the interactions occurred. Examples of ecological shifts (see Table 2) include changes in venue, topic, or classroom participation structure. For example, when a class transitioned from a lecture on Boolean algebra to students working in groups at computers with simulation software, this event was coded as an ecological shift. For each ecological context, analysis was also conducted at the microscopic level (Table 2). Here, sections of the video were organized into video clips, such that each clip represented a single instructional activity – often the teacher's actions with one student or group of students

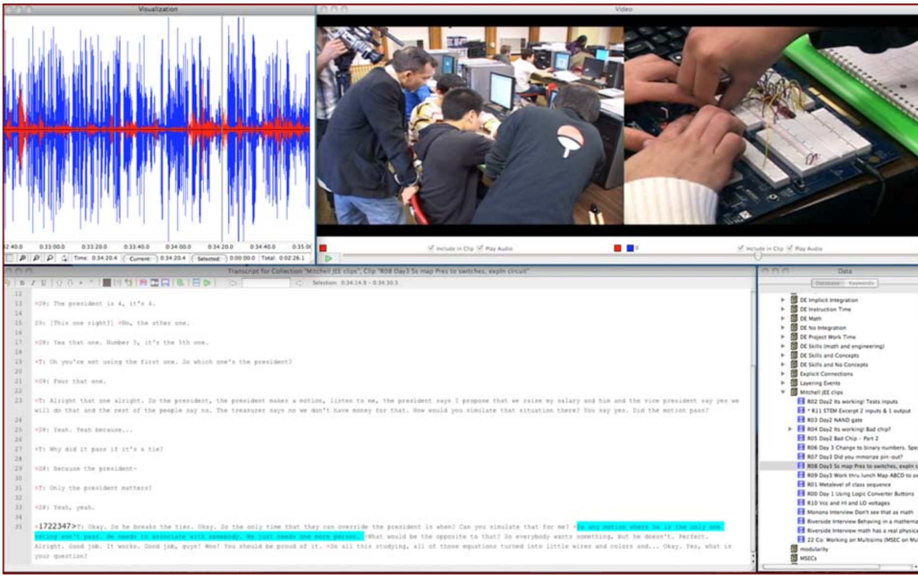


Figure 1 Example Transana screen showing two camera views and two audio signals (red and blue). The current speech is highlighted in the transcript window, bottom left. [Color figure can be viewed in the online issue.]

around a consistent set of representations. For example, a clip might include the teacher assisting two students in mapping from a circuit diagram to the breadboard.

The resulting segmented video transcripts for each day of the two high school classes then served as the focus of a series of collaborative viewing sessions. Viewing sessions involved a multidisciplinary team of research analysts from mathematics, mathematics education, electrical and computer engineering, engineering education, developmental psychology, educational psychology, and anthropology. The review team included members of the data collection teams for continuity. The review team met weekly to watch the transcribed videos. During these meetings, hypotheses were generated and documented, and repeated viewings and discussions were used to substantiate or refute claims about hypothesized data interpretations. Members would review and notate events in the data stream for subsequent collaborative meetings.

Coding system Codes for the data stream had two general sources. Codes developed from prior analyses of comparable classroom research (Nathan et al., 2011a; Walkington et al., in press) and from the research literature on learning, teaching, and practice in complex settings (Engle, 2006; Hutchins, 1995; Stevens & Hall, 1998) were proposed by members of the review team as useful top-down descriptors. Codes were also generated from assertions grounded in the data in a bottom-up fashion during review sessions or during viewing between review sessions. Special attention was paid to instances in the video where student understanding seemed to break down, where the teacher seemed to provide critical support for STEM integration or mapping between representations, and where there was evidence of student understanding of central concepts across different disciplines or representational forms.

Table 2 Coding System

Level	Codes	Coding Criteria	Source
Macroscopic: Ecological Context	Ecological Shift	Evidence of a major reorientation of classroom activity to involve different settings, activities, or participation structures	Top-down
Microscopic: Content	Identification	Identification of an invariant relation or central disciplinary concept regardless of its physical or semiotic manifestation	Bottom-up
	Projection Forward Projection Backward	Evidence that participants refer to an absent (past or future) representation or activity for content	Top-down
	Coordination	Evidence that participants link two or more co-present material or representational forms	Top-down
	Learning	Evidence of learning	Bottom-up
Meta-Level: Curriculum	Identification	Identification of an invariant relation or central concept of curriculum and instruction regardless of its physical or semiotic manifestation	Bottom-up
	Projection Forward Projection Backward	Evidence that participants refer to an absent (past or future) representation or activity for curriculum or instruction	Bottom-up
	Coordination	Evidence that participants link two or more co-present material or representational forms	Bottom-up

The codes from both the top-down and bottom-up sources were then taken under consideration during subsequent collaborative data review sessions. Criteria for accepting, rejecting, and modifying proposed codes focused on consistency with the theoretical assumptions articulated above and parsimony for a streamlined coding system. In addition, the analysis team strove for consistency across the corpus of data. This consistency was achieved by using Boolean search, which allowed the research team to find all instances in the corpus (video transcripts) of a given code, and to consider potential phenomena that might have met the coding criteria but that were omitted from a given search. This method ensured that the codes that were generated for describing cohesion production and STEM integration were not isolated incidents – they were representative of how students and teachers overcame trouble spots and supported conceptual understanding throughout the data set. Convergence on a stable coding system was reached over multiple viewing sessions and multiple passes through the data.

Codes were applied at both the micro- and macroscopic levels on the basis of the coding criteria listed in Table 2. Codes that reflected top-down considerations were used to identify ecological shifts at the macroscopic level, and coordination, backward projection, and forward projection at the microscopic level of analysis. Two additional codes emerged from bottom-up considerations from the data, both of which were at the microscopic level: identification of invariant relations, and evidence of learning. Generally, all these codes could be identified as focusing on the STEM content in the curriculum. As the viewing sessions progressed, it also became clear that there were specific activities occurring at a meta-level, which were used to represent the curriculum activities themselves, including past and future classroom units. The bottom-up process yielded a new set of meta-level codes that described cohesion production at the curriculum level.

Results

Our first set of findings suggests the conditions within which student understanding can be improved during the course of project-based learning in the classroom. The classroom transcript shown in Excerpt 1 illustrates how coordination and projection can provide cohesion of an invariant relation that transcends multiple activities. In this case they help students to see how a Boolean logical expression can be physically implemented using digital electronics, thereby supporting STEM integration of mathematics with engineering and technology. The transcript in Excerpt 2 shows that coordination and projection are not sufficient for cohesion production, and that identification of invariant relations must also occur when addressing student trouble sources in the classroom. Together, Excerpts 1 and 2 offer support for the claims that explicit identification, coordination, and projection contribute meaningfully to cohesion production in project-based engineering classes in ways that can improve student understanding. Excerpt 3 focuses on how producing and maintaining cohesion further supports STEM integration by illustrating how mathematics can be directly pulled into the classroom discourse around engineering design activities to support STEM content integration (Roehrig et al., 2012). Excerpt 4 takes a different tack by showing that the curriculum activities, such as prior and future units and learning goals, can become represented objects, much like objects from the STEM content. Cohesion production at the level of the curriculum structure utilizes the same methods as cohesion production for content that were evident in Excerpts 1, 2, and 3 – identification, coordination, and projection – but here the target elements refer to the curriculum itself. Cohesion used in this manner appears to be a powerful way to convey to students the metalevel structure of the curriculum unit in which their activities are situated. Meta-level cohesion can be used to promote reflection about the course, and can orient students toward future learning opportunities. Overall, analysis of classroom cohesion offers new insights into pedagogical processes for fostering STEM integration.

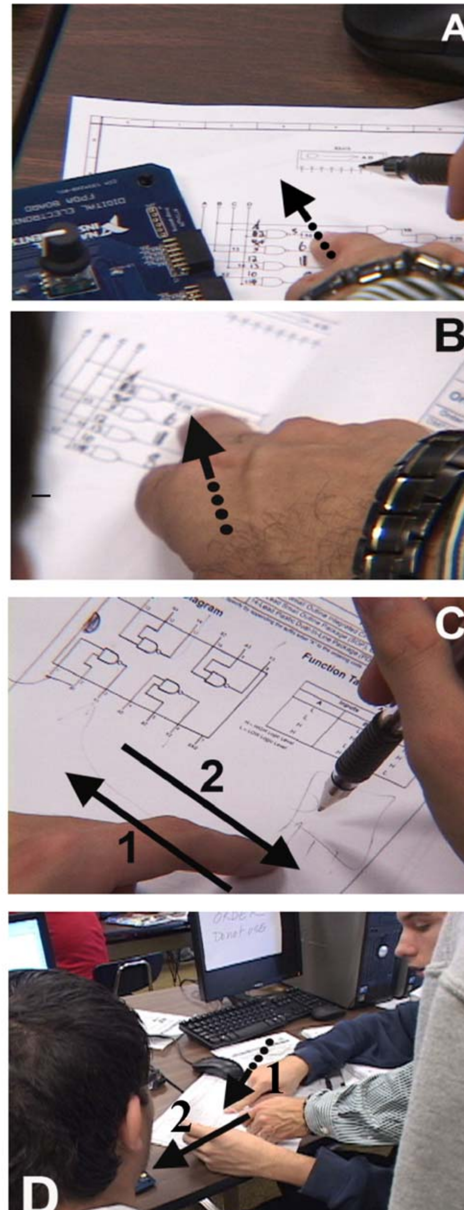
Integrating Mathematics with Engineering Design and Technology

In Excerpt 1, we track the discussion of a group of students from RHS who have confronted a trouble source (Line 2) while doing the majority vote activity. Much of their discussion focuses on using a network of logic devices called NAND gates (the symbol for a single NAND gate is shown in Figure 2a). The NAND gate enacts a mathematical function that takes two inputs and produces one output equivalent to the logical inverse of the AND operation of the inputs. The NAND function is an

1 T: Now who asks – who needs, I saw some hands up. Nobody?
2 S: Yeah, over here.
3 T: Over here? Alright, any suggestions from the other people in – did you do that in the last project at all?
4 S: No.
5 T: What does this (**A**) **indicate? So I have** – where is your – I have this gate, right?
((**A**: *Teacher points to sim diagram while student hovers pen over it. Teacher then flips to a page that has a printed chip layout diagram, and the students' hand drawn diagram of the chip*))
6 S: Yeah.
7 T: It has two – **input and one output. But it says here it needs to be connected like** that. What is that indicating?
((**B**: *Teacher points to gates on sim diagram*))
8 S: So we have to bring 3 all the way over to both of them?
9 T: Uh-huh, correct.
10 S: So do we (**C**) **short circuit it, and then bring it to 3?** Okay.
((**C**: *Student sweeps finger from hand-drawn chip diagram to printed chip layout diagram, and back*))

(* Teacher breaks from this group to help another group and then returns *)

11 T: Perfect. You answered your own question.
12 S: (**D1**) **How do we get this one?**
((**D1**: *Student indicates place on paper*))



Excerpt 1 Wiring problem at RHS. Speech transcripts are accompanied by concurrent video frames (photos) of the classroom scenes. In speech, descriptions of gestures and other actions appear in double parentheses. Speech text in bold corresponds to when an action (a gesture) in a photo begins and ends. In photos, dotted arrows show the location of pointing actions, while solid arrows show action/movement. Multiple arrows are numbered chronologically by their order of occurrence. [Color excerpt can be viewed in the online issue.]

13 T: Which one?

14 S: Uh, 17.

15 T: So this will be your next NAND, right? Do you have any available? (**D2**) **Yeah, you've got some opens there.**
((**D2**: *Teacher points to sim diagram, then sweeps hand over to circuit board*))
Okay, so same question that I asked you before. (**E**) **Here's a NAND, two input, one output.**
((**E**: *Teacher circles NAND on sim diagram, indicates two inputs on left and one output on right, using screw driver*))
(**F**) **Here you have, here's one, two input, one output. Here's another one, another one.**
((**F**: *Teacher indicates chip layout diagram*))
I don't know which one is already free here, which one is busy, which one is open. So how would you do it?

16 S: We'd have to bring the 3 (**G**) **from this one next to the 4 and the 5?**
((**G**: *Student points to integrated circuit on circuit board*))

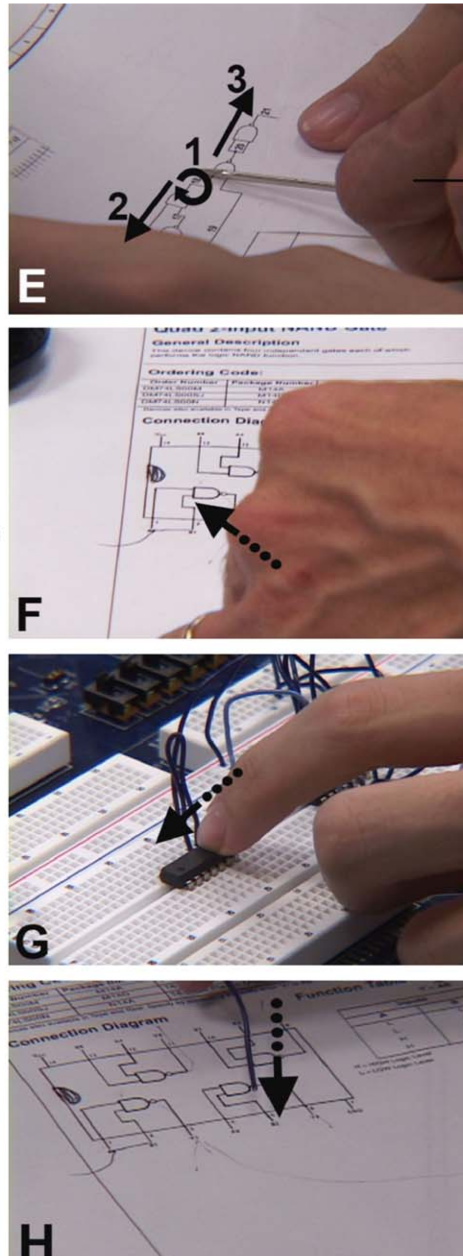
17 T: So output – right. Bring number 3, bring a wire and put it where?

18 S: (**H**) **4, 5 on this one?**
((**H**: *Student points to chip layout diagram with wire*))

19 T: Yep, you *could* do *that*.

20 S: Okay.

21 T: Mm-hmm. Perfect.



Excerpt 1 (continued)

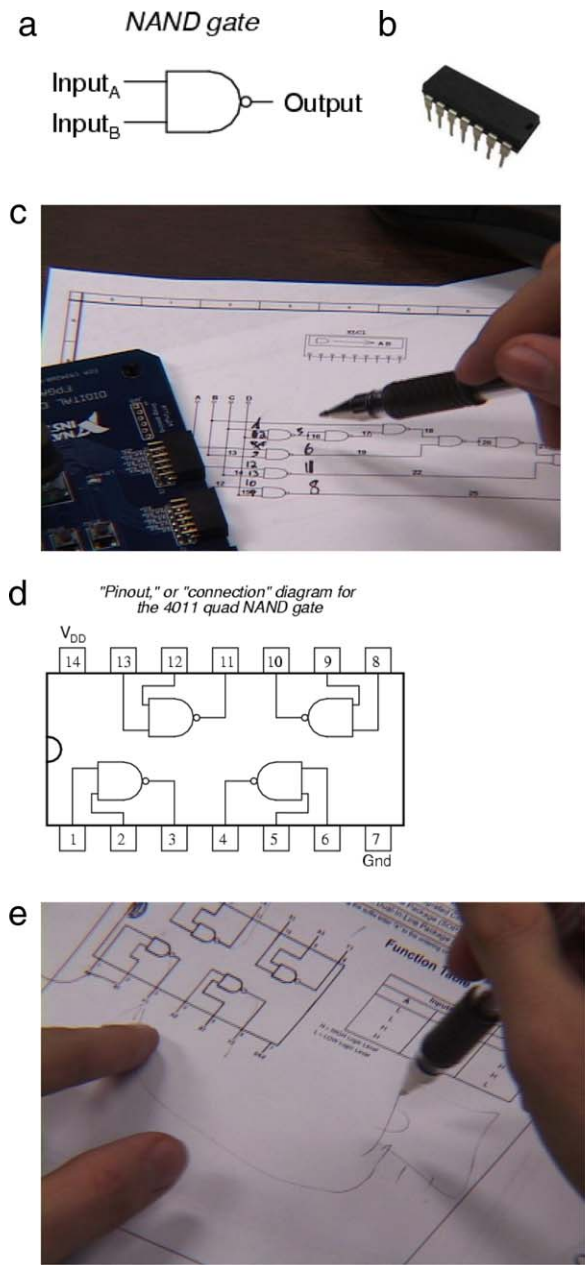


Figure 2 (a) Iconic representation of a single NAND logic gate. (b) The silicon "chip." (c) The computer generated sim diagram of the complete circuit made up of multiple logic gates. (d) The layout (or "pinout") of NAND gates for the CD4011 Quad integrated circuit (IC). (e) Student hand drawing of the chip pinout. [Color excerpt can be viewed in the online issue.]

invariant mathematical relation that must be carried across each of the different activities and representations used in the project. To enact the solution of the majority vote activity, students must design and then build a complete electronic circuit that includes NAND gates and other logic gates, as well as a power source, switches (ON for a yes vote, OFF for a no vote), and LEDs to correctly report the outcome of every possible voting combination (red if it fails, green if it passes). Prior to the majority vote activity, students derived and simplified Boolean algebraic expressions that modeled the logic of the majority vote problem, and showed its validity using truth tables. They then fed their Boolean expressions into a computer program called a logic converter, part of a larger suite of the Multisim simulation software package, which produced a computer generated sim diagram of the complete electronic circuit.

In Excerpt 1, the students do not understand how to map the full circuit from the sim diagram (Figure 2c) to the layout (the pinout) of the manufactured integrated circuit (IC; Figure 2d), a hand-drawn version of the pinout (Figure 2e), and the wiring of the silicon chip (Figure 2b) while the chip is in a breadboard. In order to meet the project goals, the students must coordinate these different-appearing representations of the same logical relations.

The problem in Excerpt 1 arises when a student who is attempting to physically wire the chips to match the schematic from the sim diagram realizes neither he nor his partners understand how the NAND gate operates. We observe how the teacher uses identification, coordination, and projection to help the students develop a more complete understanding of how the NAND function operates in each of the different representations. Following this interaction, there is evidence that one of the students improves his understanding when he subsequently demonstrates autonomy in solving the problem.

At the beginning of the exchange in Excerpt 1, the teacher asks the students to apply their prior experiences from an earlier wiring activity they did in class (Line 3). However, this attempt at backward projection fails because this student did not participate in the earlier activity (Line 4). The teacher then explicitly identifies the representation of the circuit in the sim diagram that shows the logic gates and their interconnections (Line 5). His question "What does this indicate?" is a direct query to the students to show the cross-representation mapping from elements in the sim diagram (Figure 2c) to the chip pinout (Figure 2d).

In Line 7 the teacher uses identification to specifically point out a central invariant property of the mathematical function instantiated by the NAND gate – that it takes two inputs and produces one output – and he repeats this point for emphasis. Through a series of coordinations (Line 7), the teacher establishes for the students the mapping between the sim diagram (Figure 2c) and the chip pinout (Figure 2d). He then applies that mapping to the silicon chip configuration ("it says here it needs to be connected like *that*"). He then asks students in the team to explain. Under careful direction the student responds correctly regarding how the wires will be placed to match the circuit design (Lines 8–11). First, in Line 8 a student identifies the location of one of the outputs on one of the NAND gates (the bottom left gate shown in Figure 2e, with inputs labeled *1* and *2*, and output labeled *3*) using the numerical index *3* shown on the pinout ("So we have to bring 3 all the way over…"). The student notes that this output needs to serve as the input to another gate. But it must serve as both inputs (Line 8, "…bring *3* all the way over to both of them?"), since, as the teacher noted in Line 7, the NAND gate requires two inputs. To accomplish this, the student realizes he has to short circuit, or directly connect, the two inputs of the new gate and connect that to *3* (Line 10). In doing so, the student chooses to use his own, hand-drawn pinout diagram (Figure 2e), rather than the computer generated pinout that is also immediately present on the same page (see the gesture for Line 10). This is done, perhaps, because the pinout representation generated by the student, though equivalent to the computer generated one, has direct meaning for the student. The first part of the excerpt reaches closure as the teacher validates the student's actions and explanation (Line 11).

However, the student's understanding is not robust enough to autonomously carry out the mapping later, when another NAND gate must be added to the circuit (Pin location *17*; Lines 12–14). In response to the student's request, the teacher again performs the coordination that connects the representation of inputs and outputs shown in the sim diagram to the actual chip sitting in a breadboard (Line 15). Since the pinouts are identical for each NAND gate on the chip (but with different numbered locations), the teacher provides a backward projection to their prior conversation ("same question that I asked you before") in order to make it clear to the student that the ideas and procedures they are confronting now are connected to those they addressed earlier. In Line 15 he coaches the student to do the threading through by having the student identify which openings of the pinout (and, therefore, by reference, which locations in the actual chip) are still available for wired connections ("I don't know which one is already free here, which one is busy, which one is open"). In Line 16, the student demonstrates his new understanding that the output of one NAND gate can go to the (short circuited) input of a subsequent one ("We'd have to bring the *3* from this one next to the *4* and *5*?"), forming a chain that will physically instantiate the Boolean logic. At this point, the student is focused on the chip, as indicated by his gesture to the IC in Line 16, but he is using the terminology from the pinout diagram of the index locations. In this way, the student is demonstrating he has achieved an understanding of how the representations connect to one another.

There are, in fact, multiple ways the student could act, so his correct response in Line 18 is not trivial, and it indicates that he is now autonomously directing the construction of the circuit in a way that is consistent with the sim diagram. The teacher acknowledges this point ("Yep, you *could* do *that*.") in Line 19. The teacher validates the student's reasoning and actions in Line 21 and provides some closure to the scaffolding session.

In Excerpt 1, the social interactions indicate the kind of socially mediated learning that Vygotsky (1975) identified in his construct of the zone of proximal development. Early on, the students needed tremendous support to carry out correct actions, and the student central to this interaction seemed adrift – even when he was performing nearly the same task only minutes later – when that support was removed. Through explicit coordination, projection, and clear identification of the structure of the NAND function, the teacher helped the student to build the connections that enabled him to reason in an integrative manner, where he could interpret the diagrams as carrying out the mathematical operation of NAND, and then see the pinout locations as though they were actually on the chip. Consequently, the student gained new understand through these connections so that he eventually performed the wiring autonomously. The zone of proximal development perspective highlights this subtle but important shift. The student's actions seem nearly the same in the two instances; but in the second, the student appears to have internalized the connections between the various representations of the NAND operator and the conventions for representing the organization of the chip, which made it possible for him to carry out those same actions on his own. Thus, projection, coordination, and identification contributed to the production and maintenance of cohesion across the array of representations and facilitated the integration of the mathematical relation with the technology of the circuit and the engineering design process.

Challenges of Operating with a Single Representation

Excerpt 2 illustrates one of the downsides of becoming fixated on a single STEM representation when confronting a trouble spot. Working exclusively with a single STEM representation makes it more difficult to subsequently identify the underlying invariant relation when it appears in other contexts and other symbolic or material forms of representation (Kozma, 2003).

In this episode from EHS, the teachers and students are enacting coordination between a paper schematic diagram of the circuit for the voting booth security system and a dynamic representation of the circuit in the Multisim circuit simulation environment. Prior to the portion of the excerpt shown, the student requested help to connect the switch (labeled Switch C) for a simulation of the circuit located at the top of the sim diagram to an input for a gate located at the bottom of the diagram. The teacher spends about one minute editing the Multisim diagram for the student. After the teacher finishes, the teacher notices that one gate has no input. After deleting the extra gate and reconnecting those gates that are downstream, the student adds a connection from one of the lower rows of gates up to the top of the diagram, causing the Multisim program to route the new connection around the perimeter of the circuit, which makes it hard to read and work with. The teacher then focuses on the spatial layout of the circuit.

The portion of the interaction shown in Excerpt 2 (Lines 22–30) highlights how the lack of cohesion between the key representations leaves the students with few inroads for making meaning of their actions or the actions that are performed by the simulation software. In reality, there is need to form cohesion across the original Boolean algebra expression, the functionality of the individual logic gates, the simulation of the complete circuit, and the actual wiring of the circuit board. The teacher appears to be overly directive, and fixes the problem for the students rather than facilitating the construction of connections across the representations. Although there is a lot of indexing of the parts of the sim diagram, little reference is ever made to the algebra that preceded it or the circuit board that will follow from it. The students are polite but find the process tedious and seem to give up on fixing their original schematic. At the conclusion of the episode (Lines 29–30), the students come away somewhat discouraged. When one comments "I say we should start over again," the other agrees.

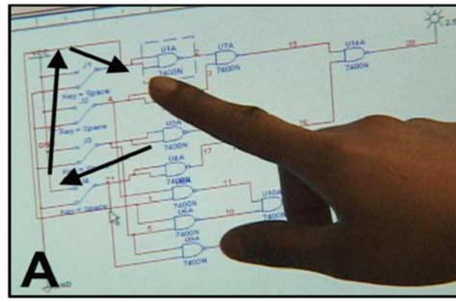
Excerpts 1 and 2 address similar activities in digital circuit design, yet document very different outcomes. In Excerpt 1 the RHS teacher explicitly identifies a central concept regarding the input-output properties of the NAND function. He invokes a rich web of relationships, using coordination, and makes backward projections with other representations and objects with which the students are familiar. The RHS teacher also uses forward projection to orient students to future activities. Though the student's need for scaffolding in Excerpt 1 is extended and recurrent, the exchange appears to eventually empower the student. He demonstrates autonomy, solving the problem at hand in a way that was not an exact imitation of the earlier directions by the teacher, which earns him the teacher's praise.

In Excerpt 2, in addition to helping the students, the EHS teacher's emphasis seems to be to produce an orderly simulation diagram with proper formatting that will allow it

22 T: How's it goin'?

23 S: Yeah (A) **it's right there**.

((**A:** *Student sweeps finger from the line which teacher pointed to, to the upper line on Multisim diagram*))

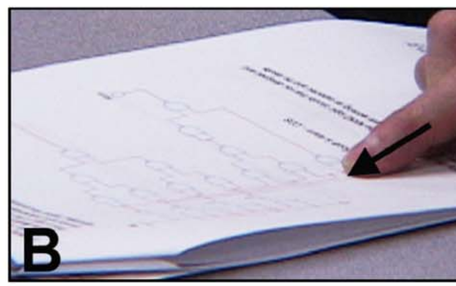


24 T: It's goin' around. That goes to C.

((Teacher deletes the line which student indicated, arranges gate locations, and adds a new connection from the identified gate to Switch C, the third switch from the top, on Multisim diagram))

Don't do that. Okay. (**B**) **From this one this one say goes to C.**

((**B:** *Teacher sweeps finger to the identified NAND gate on schematic diagram*))

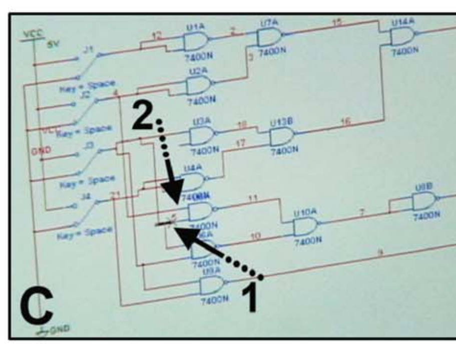


Alright.

25 S: Mmhm.

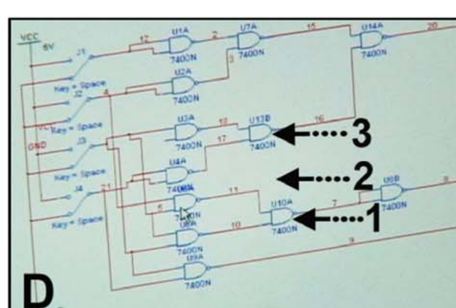
26 T: (**C**) **That's C. What happened here?**

((**C:** *Teacher points to the identified gate and the next one above it on Multisim diagram using mouse*))



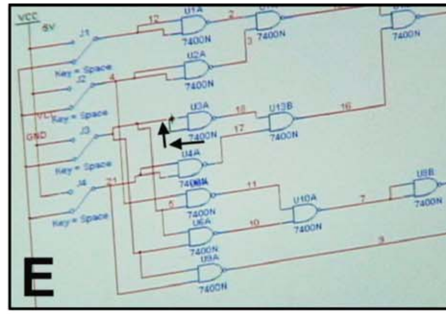
You got for some reason (**D**) **one two three**.

((**D:** *Teacher points at three different NANDs on Multisim diagram using mouse*))

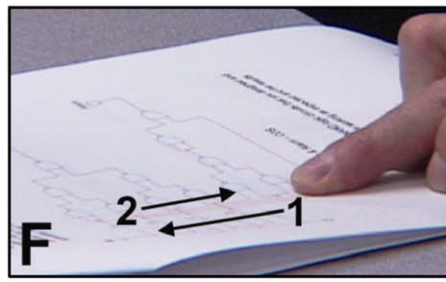


Excerpt 2 Representations at EHS (see Excerpt 1 for transcription conventions). [Color excerpt can be viewed in the online issue.]

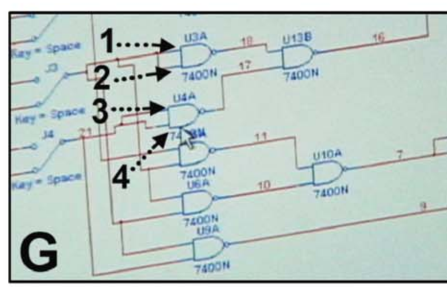
(E) This one goes back to here.
((**E:** *Teacher adds a connection to a NAND on Multisim diagram*))



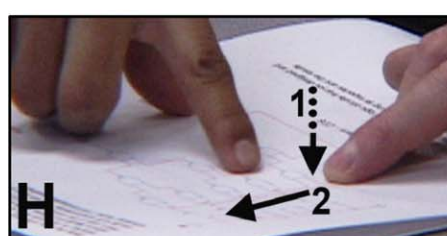
(F) After you connect the connector
((**F:** *Teacher sweeps finger from the bottom of schematic diagram to two upper gates*))



(G) one two three four. Kay.
((**G:** *Teacher points to four different locations on Multisim diagram*))



27 S: We're missing one.
(H) We're missing this one. This one that goes all the way up.
((**H:** *Student points to a gate on schematic diagram and sweeps finger from the gate to the top of diagram*))



28 T: Right. You got some lines that really need to be cleaned up but...
((Student marks the gate with finger on *schematic* diagram while editing Multisim))
29 S: I say we should start over again.
30 S: That's what I was thinking.

Excerpt 2 (continued)

to be easily read and printed all on one page. The aim is admirable, since disorganization and lack of clarity often impede students' efforts to understand their own work and engage in efficient debugging. However, the focus on one particular representation, the sim diagram, to the exclusion of all other relevant ones, establishes a modal-specific orientation that isolates that activity from the conceptual and historical chain. It reinforces attention to surface appearances without connecting to the conceptual role the diagram serves in the design and construction process. With no apparent connection to the context of the voting booth problem, Boolean relations, functionality of the individual gates, or actual wiring of the circuit, students have nothing more to connect their understanding to, and indicate little advancement in their understanding.

Fostering Integrative Thinking

Digital electronics presents a special set of challenges for STEM integration. The integrated circuits, or ICs, truly are black boxes, since they have no visible or moving parts (except, perhaps, at the level of electrons!) that reveal their function, yet they are talked about in almost exclusively functional terms. In this way, integrated circuits are very much like symbols, in the sense used by Peirce (1909) in his theory of semiotics, because they take on meaning through arbitrary but consistently applied rules. To Peirce, a symbol serves its representational role "independently alike of any resemblance or any real connection" (1909, p. 461). In digital electronics, the student must map between the symbol (in this case, the chip itself, or its name) and its meaning (the input-output relations that perform the particular logical function).

Digital circuits naturally cross multiple STEM perspectives. From a physics perspective, they adhere to both digital and analog scientific principles. The digital system is configured to operate on discrete behavioral states of the semiconductor material that makes up the chip. The analog signals include the continuous voltage and current from the power outlets that are involved in powering and grounding the circuit. From an engineering perspective, Boolean logic is used to design, simulate, model, and control behavior in the world. There are also technology perspectives, such as specific technical knowledge of chip pinouts (as we saw in Excerpt 1) and the circuit simulation software (as we saw in Excerpt 2). Students need to learn to work the technology in order to make it perform as they need. From a mathematical perspective, the devices are modeled by a truth table that specifies the output behavior for every possible combination of inputs, which are restricted to two states: ON (also represented by one or logical True) and OFF (also represented by zero or logical False). In addition, a Boolean algebra system (unfamiliar to many high school students) is used to symbolically inscribe the logical functions that produce the intended input-output relations of the circuit as a whole. In Boolean algebra, symbols for familiar arithmetic operators such as addition and multiplication are used to represent unfamiliar operations – the OR and AND functions, respectively.

Given the wide range of interdisciplinary knowledge involved, the training and orientations of the engineering teachers matter a great deal (Wang et al., 2011). At a deep structure, the mathematics of the majority vote circuit design is clear. Yet not all teachers recognize the mathematics in the task. In one of the teacher interviews at a school that is not part of the data set presented here, we asked a high school digital electronics teacher to provide his "view of how mathematics is integrated into the [digital electronics] lessons." Although he initially mentions the role of Boolean logic, his response resolves to an important distinction: "But I don't really see it. I know K- [Karnaugh] mapping is math and that whole process, but I guess I don't really see that as math" (Interview of December 12, 2010). As he elaborates, the teacher places our question in a much larger context: "I don't necessarily think that doing a bunch of that stuff is going to help [the students] out upstairs in math."

Herein lies one of the central tensions of K–12 engineering courses – and of career and technical education more broadly. Despite acceptance of the general rhetoric of policies that promote broad STEM education, the academic standards for secondary education tend to focus on college readiness (i.e., the math "upstairs") almost to the exclusion of career preparation (Rose, 2004; Symonds, Schwartz, & Ferguson, 2011; for a more in-depth treatment see Custer & Daugherty, 2009; Lewis, 2007; Nathan, Tran, Atwood, Prevost, & Phelps, 2010).

When asked about the connection to math, the RHS teacher voices a quite different view: "Each gate has a truth table and it's behaving in a certain mathematical way" (RHS interview of January 13, 2011). In his elaboration, the teacher offers a perspective on the algebraic symbols used in digital circuit design that may enhance students' overall appreciation of mathematics:

> Well for number one, the kids understand that not all of those equations are so abstract, that they have a meaning in the real physical world; that's my, my main take in that. So they say, "I know you took algebra and it was $X+2=5$, and that meant nothing. But now you see, you see what it says, what does it say $A+\text{NOT } A=1$," and it has a physical, real value. Either it's electrical voltage or maybe it's something else for another career. But it has a meaning and you need the equations and you need to make sense of them and learn how to manipulate them.

This connected view of the mathematics to the electronic devices emphasizes students' meaning making by threading a common idea across a range of representations, settings ("for another career"), and activities.

As background to the transcripts in Excerpt 3 (showing here only lines 17–48), the interaction begins with the teacher asking a student to explain how this circuit design for the majority vote activity came to be this way, hinting briefly at the role of the (Boolean) equations. This is a pertinent question. Quite often, the focus is on whether a circuit or physical device works. Occasionally, teachers and students ask how it works or why it fails. By framing the question in terms of the historical progression of the circuit design, the teacher is attending to cohesion by prompting the student to (re)produce the complex "sequence of mediators" (Latour, 1990, p. 79) that link ancient and modern symbol systems and principles of physics and Boolean algebra to the students' circuit design.

In responding, the student takes the teacher systematically though the rationale for his hand-drawn schematic, portions of which are linked through coordination to specific entries (rows) in the truth table. In the student's approach, the structure of the circuit design corresponds to the exact entries from the large (2^4) truth table. In this approach, when truth table entries for, say, the secretary (designated as *S* here) are zero, then the input *S* to the circuit simulation must be inverted to give a low signal to the logic gate when the *S* switch is ON.

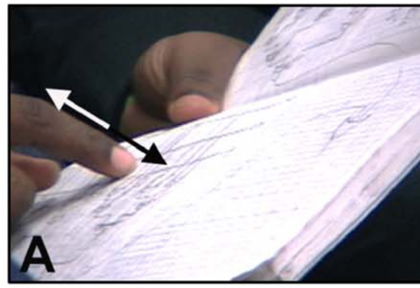
At this point, the identification process is taking place, directed by the student on this occasion, as he draws on the invariant relation of the inverter function (logical NOT) as it appears in the truth table, the sim diagram, and, ultimately, the wired electronic circuit. By employing this principle throughout, the student can map the design directly to the

17 S: Oh! See tha, I think that's the problem.

18 T: I'm not saying you cannot do it, (Student speech overlaps) but you have to really…

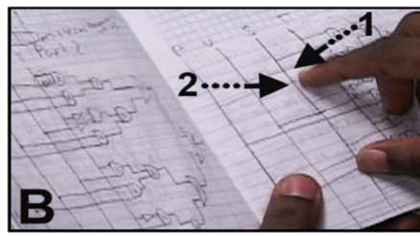
19 S: (At the same time) I think I started, no, I think, **(A) I think I started from the wrong side**.

((**A:** *Student continuously pokes at hand-drawn schematic diagram on another (right) page of his notebook*))



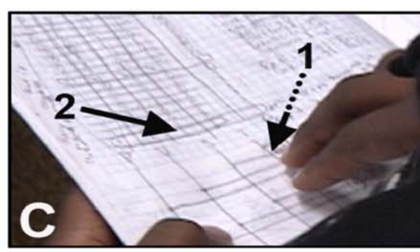
Because remember I wrote (**B**) **this backwards?**

((**B:** *Student points to some parts of the right hand-drawn schematic diagram*))



But, yeah, so, just, anyways, (**C**) **this one is the one right here**.

((**C:** *Student points with index and middle fingers to the portion of hand-drawn schematic diagram and to the row of truth table with the 1101 entry*)) one, one, zero, one. But that's what, that's all I did, cause I, like I have, I don't recall having a Boolean.



20 T: Did you simulate this? And it worked?

21 S: I simulated it and it worked. Uh, my vote, my vote is right here.

((Student then moves to the Multisim on the computer))

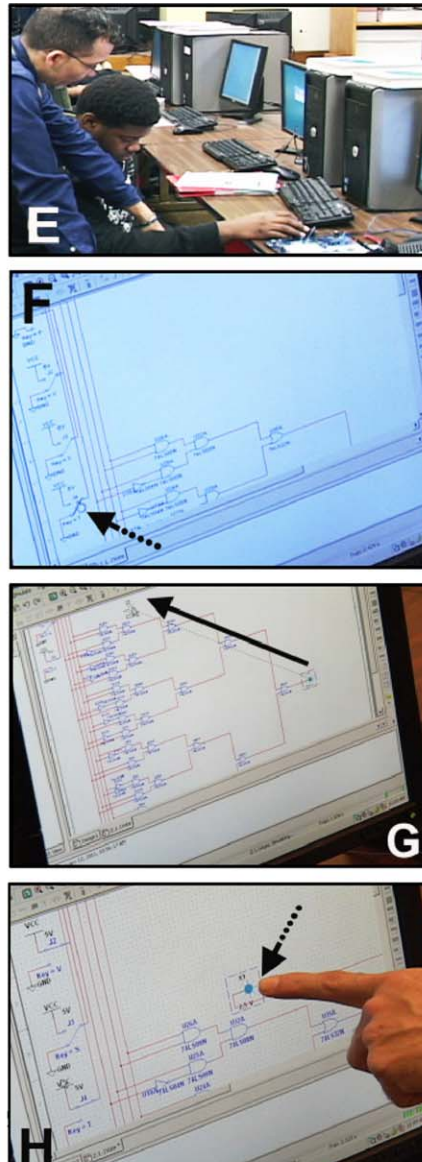
22 T: Oh! Where did you find all of those? I thought they were all-.

23 S: My vote's right here.

24 Okay, so give me tha, give me that one, one, zero, one. Just to see if it's working. (**D**) **Turn it on**.

((**D:** *Teacher points to the computer screen))*

Excerpt 3 STEM integration of math and circuit design at RHS (see Excerpt 1 for transcription conventions). [Color excerpt can be viewed in the online issue.]



25 S: Alright, I want to make sure. The president

26 T: So, that's, everybody (**E**) **voting one, except the** secretary.

((**E:** *Teacher points to student's notebook*))

27 S: Okay. (**F**) **That's connected.**

((**F:** *Student points at a switch on Multisim diagram using pointer*))

28 T: That's a zero, so that's a no. Uh huh

29 S: H'okay. You said the secretary (Teacher speech overlaps) voted.

30 T: (At the same time) The secretary voted no, uh huh, and everybody else should be voting yes.

((Student sets the Multisim switches to ON ON OFF ON to simulate the input values 1101, for votes of President, Vice President, Secretary, and Treasurer, respectively.))

31 S: Alright.

32 T: So the motion should pass.

33 S: Alright it passed.

34 T: Okay

35 S: It's very far away so you can't see it.

((Student scrolls up and down to look at the whole Multisim diagram because the diagram is off the right edge of computer screen))

36 T: Right. One thing you can do is this. If you want to see the result

37 S: Uh-huh

38 T: You can just (**G**) **pull this guy near your switches.**

((**G:** *Teacher drags the final output LED closer to switches on Multisim diagram*))

39 S: Okay

40 T: So you don't need to see the rest of your cırcuit. ((Teacher selects and drags the final output LED to the left portion of the Multisim circuit, so it is visible without having to scroll the screen horizontally))

41 S: Okay. Uh-huh.

42 T: So now (pause), (Student overlaps speech) you can do your

43 S: (At the same time) Because I was trying to look to the (inaudible)

44 T: switches, just hit the keys and **(H) see the results here**.

((**H:** *Teacher points to the final output LED on Multisim diagram*))

Excerpt 3 (continued)

45 S: Okay, (overlaps with teacher speech) just (inaudible), so what'd you want now?

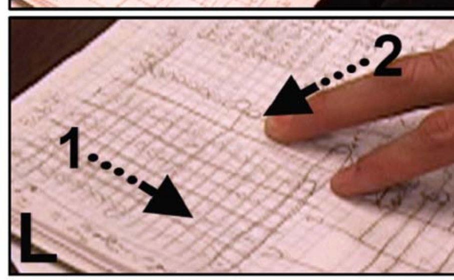
46 T: Well, I want you, I want you to explain how you came up **(J)** **with the circuit**

((**J:** *Teacher points with hand to the Multisim diagram on computer screen*)) without your **(K)** **Boolean expression.**

((**K:** *Teacher points with hand to student's notebook*))

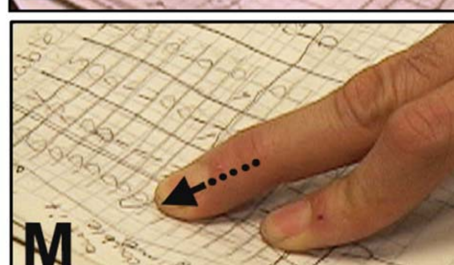
And you're explaining that you just **(L)** **went and got your first one**

((**L:** *Teacher attempts to point to Entry 1101 in the truth table and its resulting value*))



(M) and then you connected this, put an inverter here.

((**M:** *Teacher points to an input value in the truth table*))



47 S: Uh huh. Cause uh, cause if you have an inverter, it ha, it's automatically a ze– , an, an, it's NOT.

48 T: It's doable, yeah, that's, that's an even better understanding.

Excerpt 3 (continued)

combinatorial structure of the truth table, where every input takes on every possible combination of Yes and No votes (when switches are ON and OFF) from the four fictitious executive board members. Grasping this, the teacher notes, "So you went straight." Though the student continues, the teacher wants to check the circuit design against the truth table by matching the simulated behavior for the specific case of the secretary voting No to the expected output, which evidently works correctly (Lines 20–34).

Excerpt 3 starts (Line 17) with a statement from a student in the group reporting a problem with the design and offering a hypothesis about what went wrong (Line 19). Although the circuit design that is hand drawn is flawed (photo for Line 19), the simulation the group constructed appears to work for every voting combination (Lines 21–34). During this process, there is coordination between the truth table entries, voting context, and specific components or subgraphs of the schematic diagram performed by the teacher and by the student (Lines 19, 21, 25, and 33).

Eventually, the teacher reiterates his request for the rationale of the circuit (Line 46), making a coordination in the process between a zero entry in the truth table and the location of that input in the schematic where the input value needs to be flipped from Yes to No ("put an inverter here"). The teacher also uses projection to repeat the student's rationale (Line 46). The teacher also acknowledges (Line 48) that the student's reasoning reflects a rather sophisticated understanding of the interrelations of the task ("that's an even better understanding").

Orchestrating STEM integration is a high priority in contemporary discussions of career and technical education and engineering education. In Excerpt 3, a student displays an integrated understanding of the project from a mathematical as well as an engineering perspective. Mathematically, the student attends to the combinatorial structure

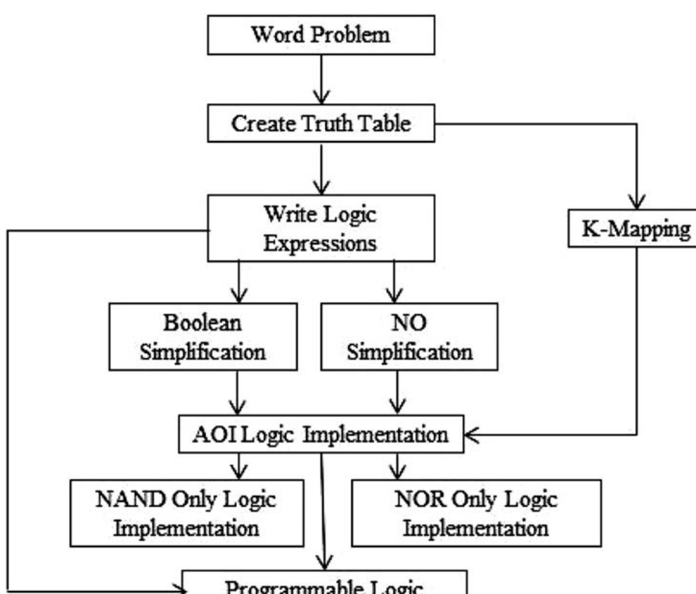
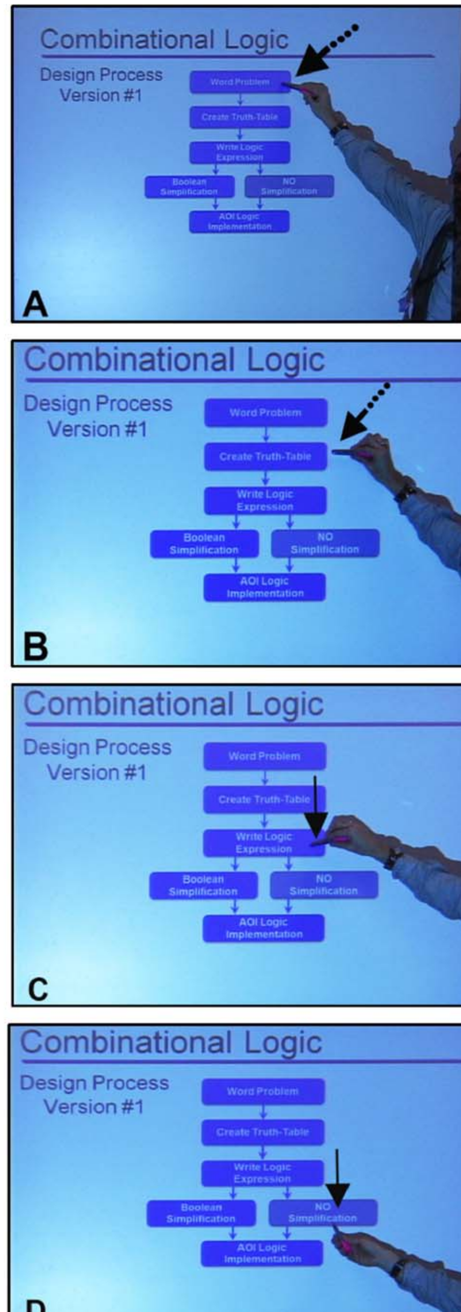


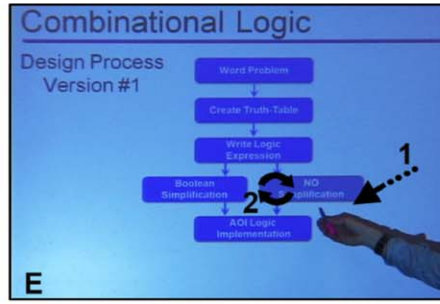
Figure 3 Project flow chart that teacher displays on overhead (Excerpt 4), giving the sequence of project activities.

1 T: This is where we are in the process.
2 T: (A) **All eyes up here please. We started with a word problem**.
((**A:** *Teacher points with pen to the top box on slide labeled "Word Problem"*))
There was no variables, there was no 0's, there was no 1's there, right?
3 T: We got (**A**) **through the process and we say,**
((**A**: *Teacher again points with pen to the Word Problem box on slide*)) okay, we're going to define variables. President, Vice President, all of those are variables. That totals the (**B**) **number of combinations, and**
((**B**: *Teacher points with pen to the next box on displayed slide labeled "Create Truth Table"*))
the requirement of the problem told us where do we want an output to be a 1 and when to be a 0.
4 T: (**C**)**That gave us what?**
((**C**: *Teacher sweeps pen from the "Create Truth Table" box on slide to the box labeled "Write Logical Expression"*))
(D) The logic expression.
((**D**: *Teacher sweeps pen from the "Write Logical Expression" box on displayed slide to the box labeled "NO Simplification"*))

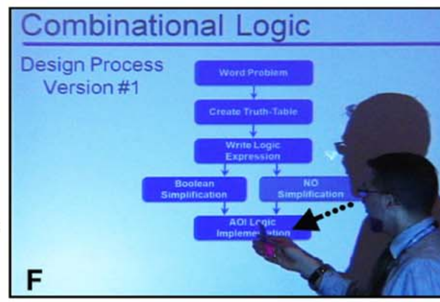


Excerpt 4 Metalevel lecture at RHS (see Excerpt 1 for transcription conventions). [Color excerpt can be viewed in the online issue.]

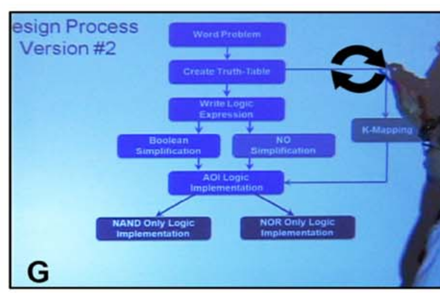
5 T: **(E1) We did a non-simplified version of this,**
((**E1:** *Teacher points with pen to the "NO Simplification" box on slide*
(E2) we swat – we sweat through that.
((**E2:** *Teacher circles an area on the "NO Simplification" box with pen*))
And **(F) we did the** AOI logic implementation.



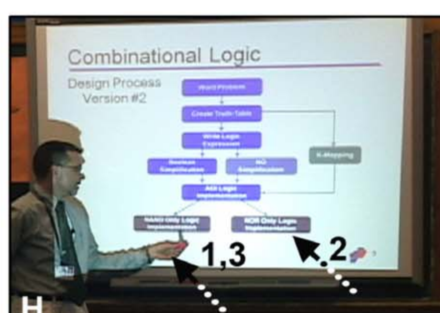
((**F**: *Teacher points to the box labeled "AOI Logic Implementation"*)).
((Teacher touches a computer key to show an alternative route in the flow chart, where an arrow goes directly from the "Create Truth Table" box to the "AOI Logic Implementation" box))
6 T: Then we learned Boolean simplification and we learned K-mapping, **(G) and we repeat the process**.



((**G**: *Teacher makes a cyclical gesture on the arrow between the "Create Truth Table" box and the box labeled "K-Mapping."*
(F) And what happened to our circuits? What happened to our expression? It got smaller, right?
((**F**: *Teacher points at the "AOI Logic Implementation" box*))



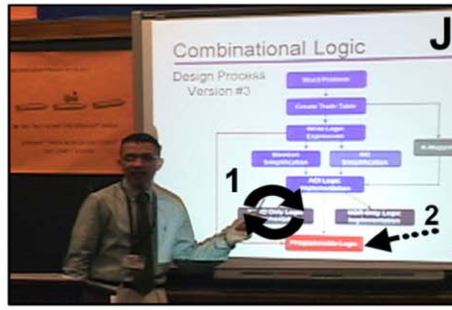
7 T: So this is where we are right now. **(H) We're going to try to do an implementation** using only NAND gate.
((**H**: *Teacher points from the box labeled "NAND Only Logic Implementation" to the box labeled "NOR Only Logic Implementation," and back*))
And that's going to be the end of this project for this semester.
8 T: But next semester, we're going to learn another step that's going to make it much easier.



Excerpt 4 (continued)

((Teacher touches a computer key to show an alternative route in the flow chart, where an arrow goes directly from the AOI Logic Implementation box to the box labeled "Programmable Logic"))
Even for today's project, **(J1) implementing it like this,**
((**J1**: *Teacher circles the "NAND Only Logic Implementation" box with pen*))
there is going to be a whole bunch of wires, okay? But once we get to this step, **(J2) doing a programmable device,**
((**J2**: *Teacher points to the "Programmable Logic" box*)) a logic device, then we're done with most of the wiring.

9 T: We'll probably just need one wire…
((Not Shown: *Teacher's left hand forms an ASL letter G shape))*
…**(K) to the LED**
((**K**: *Teacher's left hand arcs up and then points down*))
and we're done with it. So that will be very sweet, you're going to write basically a program **(L) that is going to do in the memory – we're going put that into the memory**
((**L**: *Teacher repeatedly pushes out both hands from chest*)) that of the programmable device,
(M) and that will do all what the gates are doing right now.
((**M**: *Teacher waves both hands as though erasing something*))
But we haven't learned that yet.



Excerpt 4 (continued)

of the truth table and uses it to guide the design of his digital circuit. In his explanation of the design, the student tightly connects the mathematical ideas and formalism to the digital circuitry. Both the student and teacher contribute to the maintenance of cohesion in this example, employing identification, coordination, and backward and forward projection. This example illustrates how content integration, interpreted through the lens of cohesion production, can occur. The example also shows how context integration can be maintained, because elements of the representations are often referred to as entities from the original problem (e.g., votes from specific officers).

Cohesion Production at the Curriculum Level

One of the novel contributions of this research is the finding that the curriculum itself can be represented – for example, with graphical elements in a flow chart that denotes specific portions of the class curriculum as it is occurring. In Excerpt 4, the RHS teacher uses the flow chart (Figure 3) to give an overall picture of the elements and sequence of the course curriculum to help students realize what they had already done, what they were working with, and what is to come next. We see that cohesion production can operate at the curriculum level in much the same way that it operates at the content level in Excerpts 1, 2, and 3. We illustrate this briefly to show how treating the curriculum as an object in this way can effectively provide a metalevel discussion of the class that can foster STEM integration.

In Excerpt 4, the teacher from RHS starts off giving a monologue with accompanying slides showing the flow chart of the sequence of curriculum activities that represent past, present, and future classroom activities (Figure 3). At this level, the invariant relation is the curriculum objective of moving from a verbal problem statement of a problem to a functioning digital circuit. He frames his lecture using meta-level terms ("This is where we are in the process"; Line 1) and shows how this invariant relation is manifest in several different ways over time.

In Line 2, the teacher makes an overt backward projection to the word problem, which sets the context of this and other circuit design tasks. He makes a pointing gesture to the top box in the flow chart to refer to a generic word problem and then uses that shared referent for coordination with other representations (variables, 0s, and 1s) in other steps in the curriculum sequence. He then relates the combinatorial process for generating all of the conditions of a problem context to the structure of the truth table (Line 3), and uses backward projection to link the curricular goal of writing logical expressions to past events that have occurred in the course that involved writing those Boolean expressions with and without simplification (Line 4).

In Line 5, the teacher reminds students of the great effort ("we sweat through that") involved in generating and wiring a nonsimplified version of a logical relation using AOI (an abbreviation of the AND-OR-inverter operations that are the building blocks of Boolean algebra). He uses his pointer (his hand holding a pen) to make a circle inside the box to underscore his point. The cyclical move is repeated when the new K-mapping pathway is introduced (Line 6) to show the relationship between the two methods for simplifying the circuit design, K-mapping and Boolean simplification. They can be used to arrive at the same goal, but by very different means (one spatial and the other symbolic). He describes the simplified expressions as smaller by using both speech and gestures that can convey something collapsing.

Line 7 includes a coordination act that places the class into the temporal sequence of the flow chart ("This is where we are right now") when the teacher points to the box labeled "NAND Only Logic Implementation." He also uses forward projection here and in Line 8 to make the students aware of where the class is going next. The projection also includes reference to a nonpresent, but shared experience of working with wires to highlight the complexities of hardware-based solutions more generally (Line 8). In the near future, the wiring process, he predicts, will be even simpler ("just need one wire"; Line 9), with the logical functions carried by programming rather than arrangements of logic gates. This other method of digital computing is represented as an alternate pathway that takes the curriculum sequence to a new place (and a new box, which now appears in the chart that is displayed), namely, programmable logic. He concludes the account of where they have been, where they are, and where they are going, by relating students' experiential state to their knowledge state ("But we haven't learned that yet").

Excerpt 4 illustrates how teachers can produce cohesion at the meta-level. Phases of learning, alternative pathways, the expected knowledge state of the students – even the classroom itself – are placed in temporal and spatial relation to the overall curricular sequence. This meta-level coordination and projection of elements of the curriculum contrasts with the content-level coordination and projection that we observed in Excerpts 1, 2, and 3, where the focus was on threading central STEM concepts throughout the various representations and activities. The lecture in Excerpt 4 uses cohesion of an invariant relation at the curriculum level (i.e., moving from a generic verbal problem statement to a functioning digital circuit along any of a number of pathways). This brief monologue (running just over one and a half minutes) presents to students a big picture view of how mathematics, technology, and engineering design can be integrated over a range of design projects, including some that the students have not yet experienced.

Summary of Results

Cohesion production is a versatile and powerful process that can promote STEM integration at content and curriculum levels. Our analyses suggest that cohesion production can do more for learners than just connect ideas to representations; it can also change how students perceive these objects (e.g., when students start to see the pinouts of the chips they are wiring), and what meaning they hold for students. In Gibson's (1979) ecological psychology, the term *affordance* is introduced to convey the array of potential actions that arise as one perceives and responds to objects in the immediate environment. (Think of how one's hands naturally align as they approach a door that has either a door knob or push plate.) Cohesion production can literally change the affordances of the objects and representations with which students interact. Consider the physical affordances of a chip positioned on a breadboard: it can be wired in myriad ways that ignore the problem context; but it can also serve as a toothpick holder, high-tech drink coaster, projectile, and so on. Yet not all such possibilities fit a given context. Rather, people tend to respond to a far more restricted set of perceived affordances and cultural constraints (Norman, 1999); that is, those actions and conventions that arise on the basis of what is perceived, is expected from past experience, and matches an agent's current goals.

Our ongoing investigation suggests that the perceived affordances can themselves be altered through cohesion production. A final example from our data set illustrates this phenomenon. At EHS, some of the teams selected their wires to color code different aspects of the circuit, thereby distinguishing wires that relayed analog considerations of power and ground and digital considerations of functional inputs and outputs. The technique seems almost trite because it has no impact on the functioning of the circuit. Yet in terms of STEM integration, it is quite profound. From a perceptual standpoint, chips have an extremely simple design (Figure 2b). They are rectangular, they are uniformly black except for a regular pattern of silver pins along two sides and a notch along one edge, they have some markings that may be quite cryptic, and they produce no discernable movement to reveal when they are operational, at rest, or malfunctioning. They show tremendous symmetry and might easily be handled as such. But they are not actually symmetric, and attending to the particular arrangement of logic gates (and there are many different manufactured arrangements) is critical for successfully building the circuit. Color-coded wiring is an effective way to alter the perceived symmetry of the chip, and it imposes a durable form of coordination by mapping the structure of the chip to the specific analog and digital relations. In this way, the color-coded circuit is explicitly integrated with what Latour (1999) would call the sequence of STEM mediators. Though the functioning of the circuit is unchanged, establishing and maintaining cohesion through a color-coding system can alter its perceived affordances, changing the ways that students see and interact with it and changing what it stands for among agents within the current context. In this way, cohesion – and the processes that enact it in the educational environment – serves as a potential mechanism for effective STEM integration.

Discussion and Conclusions

Our main contribution is to explicate how STEM integration in the project-based engineering classroom can be viewed as the production and maintenance of cohesion of invariant relations across the broad range of representations that exist in the engineering classroom. Cohesion is carried out through four observable mechanisms: (1) Identification directs students' attention to the deep conceptual structure of representations so they notice the central invariant relations that are threaded through the various representations, objects, settings, and social configurations of the project-based classroom; (2) coordination supports students' reasoning and meaning making by constructing clear links across representations and activities; (3) forward projection facilitates planning, highlights pending importance, and prepares students for future learning opportunities; and (4) backward projection prompts students to engage in reflection and emphasizes making connections between new and prior knowledge. Our analyses of multi-viewpoint videos of multiday classroom interactions show how cohesion serves an integrative role that can repair trouble spots and foster greater understanding and learning. Our findings also show how impediments to producing cohesion can hinder the integration process.

Limitations

The sample used in this investigation was small, and therefore general claims must be treated with caution. Our focus was to develop a theoretical perspective, along with the analytic methods to develop illustrative accounts of cohesion production that reveal new insights about the challenges and opportunities for learning in the pre-college engineering classroom. While some of these findings have been replicated with different content and school sites, a great deal of empirical work remains to be done to warrant the generalizability of these findings. Their value for generating hypotheses is most readily apparent at this stage of the research.

Cohesion

Earlier work (Nathan et al., 2011b; Walkington et al., in press) established the importance of coordination and projection for producing and maintaining cohesion in the project-based

STEM classroom. The findings reported here extend that work in several ways. First, this analysis alerted us to the importance of explicit identification of the invariant relations. In Excerpt 1, learning was in evidence with identification, coordination, and projection in place. In contrast, learning appeared to be absent in Excerpt 2, which contained many instances of coordination and projection, but lacked a clear identification of the invariant relation. Although we cannot draw conclusions as to its causal role, the contrast points out the value of identification for cohesion production because it explicitly identifies the (often) underlying invariant relations being threaded through the various representations and activities. Second, these data illustrate how cohesion production can be a mechanism for enacting STEM integration in the classroom by making cross-representation and cross-disciplinary connections between the natural sciences and mathematics and the design sciences of engineering and technology. Third, in addition to the accounts of content-level cohesion production, we observed how cohesion also occurs at the level of the curriculum itself through identification, projection, and coordination of reified elements of the curriculum sequence. This curriculum-level use of cohesion production presented students with the big picture of their learning sequence in a manner that is not readily apparent when focusing on project-level particulars. Finally, we proposed that cohesion production is an effective method for promoting STEM integration because building the cross-representation connections changes the perceived affordances of the objects and representations in the STEM environment.

Challenges of Teaching for STEM Integration

STEM integration is an oft-cited objective of pre-college engineering education, which leverages the close relation of professional practice to make firm connections across the traditional silos of the individual STEM subject areas. As noted in a recent report of the National Academies, "in the real world, engineering is not performed in isolation – it inevitably involves science, technology, and mathematics. The question is why these subjects should be isolated in schools" (Katehi et al., 2009, pp. 164–165).

Teachers face numerous challenges in promoting STEM integration in the classroom. Certainly the current accountability climate provides few rewards for efforts directed to this curriculum objective. Teachers must possess the multidisciplinary content knowledge to recognize the many potential points of integration. Teachers also need the pedagogical content knowledge (Shulman, 1986) to know how to strategically select among points of integration and address them in productive ways. The study shines a light on the inherent complexities of project-based engineering education settings. Contemporary educational reform promotes the use of rich and authentic contexts and practices (e.g., Bransford et al., 2000; NRC, 2009). Yet learners must also navigate through the many ecological shifts and seemingly disparate representations and activities that inhabit such educational settings. STEM integration poses new demands on both the student and teacher that must be explicitly addressed through curriculum design and classroom instruction.

Acknowledgements

We gratefully acknowledge the technical assistance of Dan Klopp in conducting this research and preparing this article. We are indebted to the teachers and students at our research sites. We benefited greatly from our discussions with members of the Tangibility for the Teaching, Learning, and Communicating of Mathematics project at San Diego State University, University of Wisconsin–Madison, and Vanderbilt University's Peabody

College. This work was made possible through the financial support of the National Science Foundation REESE Program (DRL-0816406) and the wonderful resources and staff of the Wisconsin Center for Education Research.

References

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the Learning Sciences. (Special issue on embodiment in mathematics)*, *21*(2), 247–286.

Bransford, J., Brown, A., & Cocking, R. (2000). *How people learn: Brain, mind, experience and school*. Washington, DC: National Academy Press.

Bransford, J. D., & Johnson, M. K. (1972). Contextual prerequisites for understanding: Some investigations of comprehension and recall. *Journal of Verbal Learning and Verbal Behavior*, *11*, 717–726.

Burghardt, D., & Hacker, M. (2007, February). *Engineering professional development*. Paper presented at the National Symposium to Explore Effective Practices for Professional Development of K–12 Engineering and Technology Teachers, Dallas, TX.

Chandler, J., Fontenot, A. D., & Tate, D. (2011). Problems associated with a lack of cohesive policy in K–12 Pre-college engineering. *Journal of Pre-College Engineering Education Research*, *1*(1), 40–48.

Chang, K. (2009, November 23). *White House begins campaign to promote science and math education*. The New York Times. Retrieved from http://www.nytimes.com/2009/11/24/science/24educ.html

Committee on Standards for K-12 Engineering Education. (2010). *Standards for K-12 engineering education?* Washington, DC: National Academies Press.

Custer, R. L., & Daugherty, J. L. (2009). *Professional development for teachers of engineering: Research and related activities*. Washington, DC: National Academic of Engineering. Retrieved from http://www.nae.edu/Publications/TheBridge/16145/16204.aspx.

Dyer, R. R., Reed, P. A., & Berry, R. Q. (2006). Investigating the relationship between high school technology education and test scores for algebra I and geometry. *Journal of Technology Education*, *17*(2), 7–17.

Engle, R. A. (2006). Framing interactions to foster generative learning. *Journal of the Learning Sciences*, *15*(4), 451–498.

Fortus, D., Krajcik, J., Dershimer, R. C., Marx, R. W., & Mamlok-Naaman, R. (2005). Design-based science and real-world problem-solving. *International Journal of Science Education*, *27*(7), 855–879.

Gainsburg, J. (2006). The mathematical modeling of structural engineers. *Mathematical Thinking and Learning*, *8*(1), 3–36.

Gibson, J. J. (1979). *The ecological approach to visual perception*. Boston: Houghton Mifflin.

Graesser, A., McNamara, D. S., Louwerse, M., & Cai, Z. (2004). Coh-Metrix: Analysis of text on cohesion and language. *Behavioral Research Methods, Instruments*, *and Computers, 36*, 193–202.

Greeno, J. G., & Hall, R. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan*, *78*(5), 361–7.

Greeno, J., Collins, A. M., & Resnick, L. (1996). *Cognition and learning*. In D. Berliner & R. Calfee (Eds.), *Handbook of educational psychology* (pp. 15–46). New York: Macmillan.

Hall, R., & Nemirovsky, R. (2012). Introduction to modalities of body engagement in mathematical activity and learning [Special issue]. *Journal of the Learning Sciences*, *21*, 1–9.

Hall, R., Stevens, R., & Torralba, A. (2002). Disrupting representational infrastructure in conversations across disciplines. *Mind, Culture, and Activity*, *9*, 179–210.

Hartzler, D. (2000). *A meta-analysis of studies conducted on integrated curriculum programs and their effects on student achievement* (Unpublished doctoral dissertation). Indiana University, Bloomington, IN.

Hutchins, E. (1995). *Cognition in the wild*. Cambridge, MA: MIT Press.

Johri, A., & Olds, B. (2011). Situated engineering learning: Bridging engineering education research and the learning sciences. *Journal of Engineering Education*, *100*(1), 151–185.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *Journal of the Learning Sciences*, *4*(1), 39–103.

Katehi, L., Pearson, G., & Feder, M. (Eds.) (2009). *Engineering in K-12 education: Understanding the status and improving the prospects*. Washington, DC: National Academies Press.

Kolodner, J. L., Camp, P. J., Crismond, D., Fasse, B., Gray, J., Holbrook, J., Puntambekar, S., & Ryan, N. (2003). Problem-based learning meets case-based reasoning in the middle-school science classroom: Putting Learning by Design™ into practice. *Journal of the Learning Sciences*, *12*(4), 495–547.

Kozma, R. (2003). The material features of multiple representations and their cognitive and social affordances for science understanding. *Learning and Instruction 13*, 205–226.

Kozma, R., & Russell, J. (1997). Multimedia and understanding: expert and novice responses to different representations of chemical phenomena. *Journal of Research in Science Teaching*, *43*(9), 949–968.

Latour, B. (1990). Drawing things together. In M. Lynch & S. Woolgar (Eds.), *Representation in scientific practice* (pp. 123–152). Cambride, MA: MIT Press.

Latour, B. (1999). *Pandora's hope. Essays on the reality of science studies*. Cambridge, MA: Harvard University Press.

Lewis, T. (2007). Engineering education in the schools. *International Journal of Engineering Education*, *23*(5), 843–852.

Lynch, M. (1990). The externalized retina: Selection and mathematization in the visual documentation of objects in the life sciences. In M. Lynch & S. Woolgar (Eds.), *Representation in scientific practice* (pp. 153–186). Cambride, MA: MIT Press.

Nathan, M. J. (2012). Rethinking formalisms in formal education. *Educational Psychologist*, *47*, 125–148.

Nathan, M. J., & Alibali, M. W. (2010). Learning sciences. *Wiley Interdisciplinary Reviews – Cognitive Science*, *1* (May/June), 329–345.

Nathan, M. J., Eilam, B., & Kim, S. (2007). To disagree, we must also agree: How intersubjectivity structures and perpetuates discourse in a mathematics classroom. *Journal of the Learning Sciences*, *16*(4), 525–565.

Nathan, M. J., Tran, N., Phelps, L. A., & Prevost, A. (2008). The structure of high school academic and pre-engineering curricula: Mathematics. *Proceedings of the American Society of Engineering Education (ASEE) 2008* (Paper no. AC 2008–2566, pp. 1–19). Pittsburgh, PA: ASEE Publications.

Nathan, M. J., Alibali, M. W., Wolfgram, M., Srisurichan, R., & Felton, M. (2011a, April). *Modal engagements in high school STEM classrooms: Tracking the 'where' and 'what' of mathematics across ecological contexts*. Paper presented at the American Educational Research Association Annual Meeting, New Orleans.

Nathan, M. J., Alibali, M. W., Wolfgram, M., Srisurichan, R., & Walkington, C. (2011b, May). *Threading mathematics through symbols, pixels, sketches and wood: Tailoring high school STEM instruction to build cohesion across modal engagements*. Paper presented at the Annual Meeting of the Jean Piaget Society, Berkeley, CA.

Nathan, M. J., Tran, N. A., Atwood, A. K., Prevost, A., & Phelps, L. A. (2010). Beliefs and expectations about engineering preparation exhibited by high school STEM teachers. *Journal of Engineering Education*, *99*(4), 409–426.

Norman, D. A. (1999). Affordances, conventions and design. *Interactions*, *6*(3), 38–43.

National Research Council [NRC]. (2003). *Improving undergraduate instruction in science technology, engineering, and mathematics*. Washington, DC: National Academy Press.

National Research Council [NRC]. (2005). *Rising above the gathering storm: Energizing and employing America for a brighter economic future*. Washington, DC: National Academies Press.

National Research Council [NRC]. (2009). *Learning science in informal environments: People, places, and pursuits*. Washington, DC: National Academies Press.

Peirce, C. S. (1909). A sketch of logical critics. In Peirce Edition Project (Ed.), *The essential Peirce. Selected philosophical writings* (vol. 2, pp. 460–461). Bloomington and Indianapolis: Indiana University Press. Retrieved from http://www.helsinki.fi/science/commens/terms/symbol.html

Pellegrino, J. W., Chudowsky, N., & Glaser, R. (2001). *Knowing what students know: The science and design of educational assessment*. Washington, DC: National Academy Press.

Phelps, L. A., Camburn, E., & Durham, J. (2009, June). *Engineering the math performance gap*. Research Brief. Madison, WI: Center on Education and Work, University of Wisconsin–Madison.

Prevost, A., Nathan, M. J., Stein, B., & Phelps, L. A. (2010). The enacted curriculum: A video based analysis of instruction and learning in high school engineering classrooms. *Proceedings of the American Society of Engineering Education (ASEE) 2010* (Paper no. AC 2010-1121). Louisville, KY: ASEE Publications.

Prevost, A., Nathan, M. J., Stein, B., Tran, N., & Phelps, L. A. (2009). Integration of mathematics in pre-college engineering: The search for explicit connections. *Proceedings of the American Society of Engineering Education (ASEE) 2009* (Paper no. AC 2009-1790, pp. 1–27). Austin, TX: ASEE Publications.

Prevost, A. C., Nathan, M. J., Phelps, L. A., Atwood, A. K., Tran, N. A., Oliver, K., & Stein, B. (in press). Academic connections in precollege engineering contexts: The intended and enacted curricula of Project Lead the Way™ and beyond. In J. Strobel, S. Purzer, & M. Cardella (Eds.), *Engineering in pre-college settings: Research into practice*. Rotterdam, The Netherlands: Sense Publishers.

Public Law 105-332. (1998). Retrieved from http://www.gpo.gov/fdsys/pkg/PLAW-105publ332/pdf/PLAW-105publ332.pdf.

Roehrig, G. H., Moore, T. J., Wang, H.-H., & Park, M. S. (2012). Is adding the E enough?: Investigating the impact of K–12 engineering standards on the implementation of STEM integration. *School Science and Mathematics*, *112*(1), 31–44.

Rose, M. (2004). *The mind at work: Valuing the intelligence of the American worker*. New York: Penguin.

Roth, W. M., & McGinn, M. K. (1998). Inscriptions: Toward a theory of representing as social practice. *Review of Educational Research*, *68*(1), 35–59.

Roth, W.-M., Bowen, G. M., & McGinn, M. K. (1999). Differences in graph-related practices between high school biology textbooks and scientific ecology journals. *Journal of Research in Science Teaching*, *36*(9), 977–1019.

Sanders, M. E., & Wells, J. (2006–2011). Integrative STEM Education Course Syllabi & Instructional Materials: STEM Education Foundations; STEM Education Trends & Issues, STEM Education Seminar. Retrieved from http://www.soe.vt.edu/istemed/

Satchwell, R. E., & Loepp, F. L. (2002). Designing and implementing an integrated mathematics, science, and technology curriculum for the middle school. *Journal of Industrial Teacher Education*, *39*(3), 41–66.

Sawyer, R. K. (2006). *Cambridge handbook of the learning sciences*. New York: Cambridge University Press.

Schunn, C. (2009, March). *What needs to be put back into S, T, and E? Well chosen M*. Presentation to the NSF/Hofstra CTL Middle School Math Infusion STEM Symposium, Singer Island, FL.

Schwartz, D. L. (1995). The emergence of abstract representations in dyad problem solving. *Journal of the Learning Sciences*, *4*(3), 321–354.

Shapiro, L. (2011). *Embodied cognition*. New York: Routledge.

Sheppard, S. D., Pellegrino, J. W., & Olds, B. M. (2008). On becoming a 21st century engineer. *Journal of Engineering Education*, *97*(3), 231–234.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*, 4–14.

Stevens, R., & Hall, R. (1998). Disciplined perception: Learning to see in technoscience. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 107–149). Cambridge, U.K.: Cambridge University Press.

Symonds, W., Schwartz, R., & Ferguson, R. (February, 2011). Pathways to prosperity: Meeting the challenge of preparing young Americans for the 21st Century. Report issued by the Pathways to Prosperity Project, Harvard School of Education. Available at http://www.gse.harvard.edu/news_events/features/2011/Pathways_to_Prosperity_Feb2011.pdf

Tran, N. A., & Nathan, M. J. (2010a). An investigation of the relationship between pre-college engineering studies and student achievement in science and mathematics. *Journal of Engineering Education*, *99*(2), 143–157.

Tran, N. A., & Nathan, M. J. (2010b). Effects of pre-college engineering studies on mathematics and science achievements for high school students. *International Journal of Engineering Education*, *26*(5), 1049–1060.

Vygotsky, L. S. (1975). *Thought and language*. Cambridge, MA: MIT Press.

Walkington, C., Srisurchan, R., Nathan, M. J., Williams, C. & Alibali, M. W. (2012). *Grounding geometry justifications in concrete embodied experience: The link between action and cognition*. Paper presented to the American Educational Research Association Annual Meeting Vancouver, BC.

Walkington, C. A., Nathan, M. J., Wolfgram, M., Alibali, M. W., & Srisurichan, R. (in press). Bridges and barriers to constructing conceptual cohesion across modalities and temporalities: Challenges of STEM integration in the precollege engineering classroom. In J. Strobel, S. Purzer, & M. Cardella (Eds.), *Engineering in pre-college settings: Research into Practice*. Rotterdam, The Netherlands: Sense Publishers.

Wang, H.-H., Moore, T. J., Roehrig, G. H., & Park, M. S. (2011). STEM integration: The impact of professional development on teacher perception and practice. *Journal of Pre-College Engineering Education Research*, *1*(2), 1–13.

Welty, K., Katehi, L., & Pearson, G. (2008). Analysis of K–12 engineering education curricula in the United States: A preliminary report. *Proceedings of the American Society for Engineering Education Annual Conference and Exposition. Pittsburgh, PA.*

White, T., & Pea, R. (2011). Distributed by design: On the promises and pitfalls of collaborative learning with multiple representations. *Journal of the Learning Sciences*, *20*(3), 489–547.

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